Diversity Backpressure Routing with Mutual Information Accumulation in Wireless Ad-hoc Networks

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***Abstract*—We suggest and analyze algorithms for routing in ad-hoc networks that exploit mutual-information accumulation at receiving nodes, and are capable of routing multiple data streams (commodities) when only *mean* channel state information is present, and that only locally. The algorithm is a generalization of the DIVBAR algorithm, which in turn is based on backpressure routing. Packets are transmitted by each node on the links seeing the largest "backpressure", a measure for the differential queue lengths for a specific commodity at the considered nodes times the success probability for packets on that link. In contrast to traditional DIVBAR, nodes store and exploit partially received packets, thus increasing the probability of successful reception at retransmission, where the information is stored in separate "partial queues" at each node. We present two variants of our algorithm: DIVBAR-RMIA, which clears the partial queues whenever a packet is firstly decoded by one or more receiving nodes; DIVBAR-MIA, which retains the information about a specific packet in the partial queues until the packet has reached its destination. We prove that DIVBAR-RMIA performs strictly better than conventional DIVBAR (under some mild assumptions about the channel states), and that DIBVAR-MIA performs at least as well as DIVBAR-RMIA. Simulations not only confirm these results, but also demonstrate the impact of packet entropy on the achievable throughput.**

***Index Terms*—DIVBAR Algorithm, (Renewal) Mutual Information Accumulation, Backpressure, Lyapunov Drift**

I. Introduction

Wireless multi-hop ad-hoc networks have drawn significant attention in recent years, due to their flexibility and low cost, and their resulting importance in factory automation, sensor networks, security systems, and many other applications. A fundamental problem in such networks is the routing of data packets, i.e., which nodes should transmit which packets in which sequence. Optimum routing has been well-explored for routing of a single packet (see, e.g., [1], Chapter 22 and references therein).

However, simultaneous routing of multiple packet streams intended for multiple destinations (i.e., multiple commodities) is much more difficult, as different commodities are competing for the limited resources. While traditional routing algorithms still can be used with some ad-hoc modifications, designing *optimum* routes and resource allocations becomes much more difficult. This particularly holds true when it is not desirable and/or possible to have information about the packet arrivals or complete channel state information at the central node that can then design the optimum routes.

The most promising approach to solve these problems is the class of *stochastic routing algorithms* [2]. In particular, the backpressure algorithm [3] [2] establishes a metric for each commodity on each available link that takes into account the differential queue lengths (number of packets of the particular commodity at a node) as well as the channel state of a link. The packet with the best metric will be transmitted from each node. Thus, backpressure algorithms achieve routing without ever designing an explicit route, and without requiring centralized information. Based on this principle, [4] developed the *diversity backpressure* (DIVBAR) algorithm, which was shown to be throughput-optimal in stochastic networks under certain assumptions, most notably that any packet not correctly received by a node has to be completely retransmitted.

The efficiency of retransmission can be greatly enhanced by mutual-information accumulation (MIA), where receiving nodes store partial information of packets that cannot be decoded at the previous transmission attempt. MIA using Fountain codes was suggested for ad-hoc networks in [5] and [6], and shown to reduce transmission time and energy consumption. (Deterministic) routing with MIA for single-commodity networks was analyzed in [7].

In this paper, we propose and analyze two new algorithms that combine the concepts of DIVBAR with MIA [8]. The first version, DIVBAR-RMIA ("R" stands for "renewal") clears out the partial information at all nodes every time the corresponding packet is successfully decoded by at least one receiving node in the network, not necessarily the destination. In the second version, DIVBAR-MIA, all received partial information about a packet remains stored at all the nodes in the network until that packet has reached its destination. We prove that both algorithms can achieve larger throughput limits than the regular DIVBAR algorithm. We also provide simulation results that quantify the amount of improvement, and discuss the impact of packet entropy on the achievable throughput. [9] has done some parallel work on exploring the routing in wireless networks with AWGN channels. In contrast, our work investigates the routing in wireless networks with fading channels, where only the mean channel state information is known.

The remainder of the paper is organized as follows: Section II presents the network model and describes the DIVBAR algorithm as well as our two new algorithms. Section III provides a theoretical analysis of the throughput and proof that our algorithms outperform DIVBAR. Section IV presents simulation results, and Section V concludes the paper.

II. Network Model And Backpressure Algorithms

A. Network Model

Consider a wireless network with N nodes, where multiple commodities $c = 1, \cdots C$ (data streams) are transmitted, possibly via multi-hop. Each link in the network is denoted by an ordered pair (n, k), for $n, k \in \{1, \cdots, N\}$, where n is the sender and k is the receiver; all packets destined for a particular node c are categorized as commodity c packets irrespective of their origin. Exogenous input data arrives randomly to the network in units of packet, each of which has a fixed amount of information (called entropy or packet length) denoted as H_0. Packets arriving at each node are stored in a queue waiting to be forwarded, except at the destination, where they leave the network immediately upon arrival. The transmission power of each node is constant.

Time is slotted and normalized into integer units $t = 0, 1, 2, 3, \cdots$. The exogenous packet arrival rate $a_n^{(c)}(t)$ is i.i.d. (independent and identically distributed) over timeslots and upper bounded by $A_{\max}$; each node is allowed to transmit at most one packet per timeslot, i.e., the transmission rate at each timeslot $b_{nk}^{(c)}(t)$ satisfies $\sum_{c=1}^{N} \sum_{k=1}^{N} b_{nk}^{(c)}(t) \leq 1$, $n, k, c \in \{1, 2, \cdots, N\}$, where n is the current node, k is the receiver, and c is the destination. The timeslot length is assumed to be equal to the coherence time of the channel, so that we can adopt the common block-fading model: within a timeslot duration, instantaneous channel gains are constant, while they are i.i.d. over timeslots, for each link. Mean channel state information (CSI) about each link, which evolves (on a timescale much slower than the coherence time of the channel) within a finite state space $\mathcal{S}$, is known locally, i.e., at the node from which the link is emanating; however, instantaneous CSI (i.e., channel gains for a specific timeslot) are never known at any transmitting nodes. Each link uses capacity-achieving codes, so that a packet is received correctly if $\log_2(1 + \gamma(t)) \geq H_0$, where $\gamma(t)$ is the SNR in timeslot t, whose distribution depends on mean channel state. When a packet is transmitted, it can possibly be received by multiple nodes simultaneously ("broadcast effect"); in this case, only one of these nodes gets the responsibility of further forwarding the packet.

Our goal is to design a routing algorithm that can support an exogenous input rate as large as possible, while subject to a possible tradeoff with delay. In a multi-hop, multi-commodity wireless network, the time average input rate can be represented by an input rate matrix $\left(\lambda_n^{(c)}\right)$, $n, c \in \{1, 2, \cdots, N\}$. We describe the set of all the supportable input rate matrices by the *network capacity region*.

B. DIVBAR Algorithm

Ref. [4] proposed the Diversity Backpressure (DIVBAR) Algorithm, a simple online algorithm, which can be shown to be optimum subject to the above constraints, where the fading channels give rise to (known) link success probabilities. In every timeslot t, each node n observes the queue backlogs in each of its potential receiver nodes $k \in \mathcal{K}_n(t)$, and observes the channel success probabilities associated with the receivers. Then DIVBAR uses the backpressure concept to route packets in the direction of maximum differential backlog. Specifically, DIVBAR chooses a commodity packet to transmit by computing certain metrics. After getting the feedback indicating the successful transmission or not from all the receivers, node n forwards the packet to the receiver corresponding to the largest differential backlog among those successful receivers.

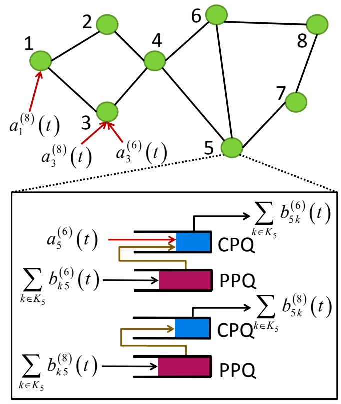

Fig. 1. Network Model: two kinds of queues are used for storing information

C. DIVBAR with Mutual Information Accumulation

DIVBAR is implemented based on a repeating transmission scheme, i.e., for each transmission, packets either are successfully received at another node, or have to be completely retransmitted in a later timeslot. In this paper we suggest to avoid the inefficiencies of complete retransmission by enabling the Mutual Information Accumulation (MIA) technique into the transmission scheme. MIA, which was first suggested for wireless ad-hoc networks by one of us [6], can be realized by rateless codes or Fountain Codes [10]. These codes encode and transmit the source information in infinitely long code streams, and the receiver can recover the original source information from the portions of the code streams received in an unordered manner, as long as the total accumulated received information is at least as large as the entropy of the source information. As is shown in Fig. 1, to implement the two algorithms, each node has set up two kinds of queues: the *compact packet queue* (CPQ) and *partial packet queue* (PPQ). CPQ stores the packets that have already been decoded being categorized by their commodities; while the partial information stored in PPQ is distinguished by the packets they belong to. As soon as the information of a specific packet accumulated in PPQ exceeds the entropy of that packet, the packet is decoded and moved out of PPQ, and then put into CPQ if the current node gets the forwarding authorization from the sender.

As we will show in Sec. III, *MIA increases the network capacity;* this is no contradiction to the throughput optimality of DIVBAR, since the allowing of MIA increases the policy space. The intuition on this throughput enhancement is: each transmission might not have to transmit a whole packet in each timeslot but take advantage of the information already accumulated at the receiver node. Only the differential amount of information is required for the decoding at the receiver. This will increase the success probability of each transmission, and therefore increases the average transmission rate.

In this work, we introduce two algorithms combining the DIVBAR algorithm and the MIA technique. DIVBAR-RMIA makes transmission decisions according to a backpressure metric and keeps transmitting the packet using MIA, but clears the accumulated information every time after any successful decoding occurs.[1] DIVBAR-MIA is designed to be related to DIVBAR-RMIA in the sense that it has the same packet transmission period as DIVBAR-RMIA, but any imperfectly decoded packet is stored at each node until that packet reaches its destination.

III. Theoretical Formulation

In this section we prove that the throughput performance achieved by DIVBAR-RMIA and DIVBAR-MIA is better than that of regular DIVBAR. The proof consists of four steps: (i) definition of the network capacity region with RMIA; (ii) the network capacity region with RMIA operation is strictly larger than the region with pure packet retransmission; (iii) DIVBAR-RMIA is throughput-optimal under the RMIA assumption, (iv) DIVBAR-MIA can support any input rate matrix that can be supported by DIVBAR-RMIA. In other words, DIVBAR-MIA is at least as good as DIVBAR-RMIA, which in turn is better than regular DIVBAR.

A. Network capacity region under RMIA

We start by re-stating the network capacity region under conventional retransmission, which was derived in [4]:

Theorem 1. *The network capacity region Λ^* consists of all rate matrices $\left(\lambda_n^{(c)}\right)$ for which there exist a stationary randomized policy that chooses probability $\omega_n^{*(c)}(s)$, $\theta_{nk}^{*(c)}(\Omega_n)$, and forms the time average transmission rate matrix $\left(b_{nk}^{*(c)}\right)$, for all n,k,c, all mean channel states $s \in \mathcal{S}$, and all subsets Ω_n of node n's neighbors $\mathcal{K}_n$, such that:*

$$b_{nk}^{*(c)} \geq 0, b_{cn}^{*(c)} = 0, b_{nn}^{*(c)} = 0, \text{ for } n \neq c, \tag{1}$$

$$\sum_k b_{kn}^{*(c)} + \lambda_n^{(c)} \leq \sum_k b_{nk}^{*(c)}, \text{ for n} \neq c, \tag{2}$$

$$b_{nk}^{*(c)} \leq \sum_{s\in\mathcal{S}} \pi_s \omega_n^{*(c)}(s) \sum_{\Omega_n \in \mathcal{K}_n} q_{n,\Omega_n}^*(s)\, \theta_{nk}^{*(c)}(\Omega_n), \tag{3}$$

where π_s is the probability of the mean channel state s; $\omega_n^{*(c)}(s)$ is the conditional probability that node n transmits a commodity c packet, given the mean channel state s; $\theta_{nk}^{*(c)}(\Omega_n)$ is the conditional probability that node n forwards the commodity c packet to node k, given that exactly the set of nodes Ω_n successfully receive the packet; $q_{n,\Omega_n}^*(s)$ is the probability that Ω_n is the set of successful receivers of a packet transmitted by node n, given the mean channel state s.

[1]Parallel to our work, [9] suggests an approach working in non-fading AWGN network under a somewhat similar assumption

The superscript $*$ in Theorem 1 indicates that they are defined for the repetition transmission scheme.

Proof: See Ref. [4]. ∎

Theorem 1 is based on the repeating transmission scheme without MIA. However, the following corollary makes an analogous statement, namely that a stationary policy achieves the capacity region also holds true for a network with RMIA transmission scheme (however, note that only the *structure* of the solution is the same, while the actual *values* of the transmission rate etc. are different; note that there is no superscript * on the variables in Corollary 1):

Corollary 1. *Under the RMIA transmission scheme, the network capacity region Λ consists of all the rate matrices $\left(\lambda_n^{(c)}\right)$ for which there exists a stationary randomized policy that chooses probability $\omega_n^{(c)}(s)$, $\theta_{nk}^{(c)}(\Omega_n)$, and forms the transmission rate matrix $\left(b_{nk}^{(c)}\right)$, for all n,k,c, all channel state $s \in \mathcal{S}$, and all subsets Ω_n of node n's neighbors $\mathcal{K}_n$, such that constraints with the same structure as (1)-(3) are satisfied:*

$$b_{nk}^{(c)} \geq 0, b_{cn}^{(c)} = 0, b_{nn}^{(c)} = 0, \text{ for } n \neq c, \tag{4}$$

$$\sum_k b_{kn}^{(c)} + \lambda_n^{(c)} \leq \sum_k b_{nk}^{(c)}, \text{ for n} \neq c, \tag{5}$$

$$b_{nk}^{(c)} \leq \sum_{s\in\mathcal{S}} \pi_s \omega_n^{(c)}(s) \sum_{\Omega_n \in \mathcal{K}_n} q_{n,\Omega_n}(s)\, \theta_{nk}^{(c)}(\Omega_n), \tag{6}$$

where the variables with no superscript $*$ are under RMIA scheme.

Proof: For the necessity part, the key point is to prove that the right hand side of constraint (6) also exists under the RMIA transmission scheme. Let $q_{n,\Omega_n}^{(c)}(s,t)$ represent the number of packets sent by node n and successfully received by set of nodes Ω_n up to time t when the mean channel state is equal to s; let $\omega_n^{(c)}(s,t)$ represent the number of timeslots that node n decides to transmit a packet from the head of its CPQ up to time t when the mean channel state is equal to s. We mainly need to demonstrate that $\lim_{t\to\infty} q_{n,\Omega_n}^{(c)}(s,t)/\omega_n^{(c)}(s,t)$ exists and is well-defined as $q_{n,\Omega_n}^{(c)}(s)$. As for other probability variables, note that $\omega_n^{(c)}(s)$ and $\theta_{nk}^{(c)}(\Omega_n)$ are determined by the policy decisions.

If we define $T_{s,\Omega_n}(j)$ as the number of timeslots used for transmitting the jth packet that is successfully received by Ω_n, it follows that

$$\sum_{j=1}^{q_{n,\Omega_n}^{(c)}(s,t)} T_{s,\Omega_n}(j) \leq \omega_n^{(c)}(s,t) < \sum_{j=1}^{q_{n,\Omega_n}^{(c)}(s,t)+1} T_{s,\Omega_n}(j). \tag{7}$$

Then we have

$$\frac{1}{\frac{1}{q_{n,\Omega_n}^{(c)}(s,t)} \sum_{j=1}^{q_{n,\Omega_n}^{(c)}(s,t)+1} T_{s,\Omega_n}(j)} < \frac{q_{n,\Omega_n}^{(c)}(s,t)}{\omega_n^{(c)}(s,t)}$$

$$\leq \frac{1}{\frac{1}{q_{n,\Omega_n}^{(c)}(s,t)} \sum_{j=1}^{q_{n,\Omega_n}^{(c)}(s,t)} T_{s,\Omega_n}(j)}. \tag{8}$$

Because of the renewal operation after each successful decoding, $T_{s,\Omega_n}(j)$ are i.i.d. over different packets sent from node n to set Ω_n. Thus, the denominators of the lower and upper bounds in (8) approach $\mathbb{E}\{T_{s,\Omega_n}(j)\}$ with probability 1 as $t \to \infty$. Then it follows that

$$\lim_{t\to\infty} \frac{q_{n,\Omega_n}^{(c)}(s,t)}{\omega_n^{(c)}(s,t)} = \frac{1}{\mathbb{E}\{T_{s,\Omega_n}(j)\}} \triangleq q_{n,\Omega_n}^{(c)}(s) \text{ with prob. 1.} \tag{9}$$

Then we have

$$b_{nk}^{(c)} = \sum_{s\in\mathcal{S}} \pi_s \omega_n^{(c)}(s) \sum_{\Omega_n \in \mathcal{K}_n} q_{n,\Omega_n}(s)\, \theta_{nk}^{(c)}(\Omega_n). \tag{10}$$

Other elements of the proof, which are similar to the proof in Theorem 1, are omitted for space reasons and given in [11].

For the sufficiency part, also see [11]. We can prove that for any input rate matrix within the interior of the network capacity region, i.e., $\exists\varepsilon > 0$, and $\boldsymbol{\lambda} + \varepsilon\mathbf{1} \in \Lambda$, there exists an integer $D > 0$, and $B = N^2 D \left[1 + (A_{\max} + N)^2\right]$, such that

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} \mathbb{E}\left\{Q_n^{(c)}(\tau)\right\} \leq \frac{B}{\varepsilon}. \tag{11}$$

■

B. Network capacity region: RMIA versus repetition transmission scheme

We now show that the network capacity region with RMIA is strictly larger than that with the repetition transmission scheme, if $F_{s,R_{nk}}(x)$ - the cdf of the transmission rate over link (n,k) when mean channel state is equal to s - is continuous. Note that the latter assumption is natural for a transmission "at capacity" of a continuous distribution of the instantaneous channel state on link (n,k). Then we have the following theorem:

Theorem 2. *If $0 < F_{s,R_{nk}}(H_0) < 1$, $n,k \in \{1,2,\cdots,N\}$, $s \in \mathcal{S}$ the network capacity region with RMIA transmission scheme is strictly larger than the region with repetition transmission scheme, i.e., $\Lambda \supset \Lambda^*$.*

Proof: The proof consists of two steps: (i) show that the right hand side of (6) with RMIA is strictly larger than that of (3) with the repetition transmission scheme, which implies that the time average transmission rate can reach a strictly larger value with RMIA; (ii) show that with a strictly larger achievable transmission rate due to RMIA, the capacity region is strictly larger. In the appendix, we sketch the proof for a single link, single commodity network; for the (more complex) proof of general networks, see [11]. ■

C. Optimality of DIVBAR-RMIA

With enlarged capacity region Λ of the RMIA transmission scheme, it is possible to design an throughput optimal online routing algorithm that can support all the input rate matrices within Λ. Define an *epoch* as the period of transmitting one packet until any successful decoding occurs, or one timeslot when the the sender keeps silent. It follows that

Theorem 3. *DIVBAR-RMIA is throughput optimal under the RMIA transmission scheme assumption, i.e., DIVBAR-MIA can stably support any rate matrix within capacity region Λ.*

Proof: Based on the observation of backlog state $\mathbf{Q}(t)$ and by accumulating partial information, DIVBAR-RMIA is designed to minimize the upper bound of the multi-step Lyapunov Drift [2] over one epoch, which is smaller or equal to the bound under any stationary randomized policy that supports the input rate matrix. We can prove that for any input rate matrix $\boldsymbol{\lambda}$ satisfying $\boldsymbol{\lambda}+\varepsilon\mathbf{1} \in \Lambda$, where $\varepsilon > 0$, there exists an integer $D > 0$, and $B = N^2 D\,[1 + (N + A_{\max})]$ such that

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} \mathbb{E}\left\{Q_n^{(c)}(\tau)\right\} \leq \frac{B + 2N^2 D\,(D+1)\,(N + A_{\max})}{\varepsilon}. \tag{12}$$

A detailed proof is given in [11]. ■

D. DIVBAR-MIA versus DIVBAR-RMIA

DIVBAR-RMIA clears the partial packet information on all nodes as soon as the corresponding packet is firstly decoded by at least one receiver. However, the eliminated partial packet information could be useful for the future decoding, and therefore, clearing it might result in lowering the transmission success probability.

DIVBAR-MIA retains the partial information until the packet reaches its final destination. Moreover, the epoch of DIVBAR-MIA is set the same as that of DIVBAR-RMIA. From the discussion above, it is intuitively clear that the following theorem holds:

Theorem 4. *DIVBAR-MIA algorithm can at least support any input rate that can be supported by DIVBAR-RMIA algorithm.*

Proof: Without the renewal operation, the total amount of partial packet information available to use under MIA is no less than the partial packet information that can be used under RMIA. Since the epoch of DIVBAR-MIA is the same as that of DIVBAR-RMIA, the successful receivers under DIVBAR-MIA at least include the successful receivers under DIVBAR-RMIA by the end of each epoch, which means DIVBAR-RMIA can result in more forwarding choices. In [11], the upper bound of the Lyapunov drift under DIVBAR-MIA is shown to be less than or equal to that under DIVBAR-RMIA due to the above fact. Then we can show that DIVBAR-MIA can support any input rate that can be supported by DIVBAR-RMIA. ■

IV. SIMULATIONS RESULTS

Example Matlab simulations are carried out in an ad-hoc wireless network shown in Fig. 2. All the links in the network are independent non-interfering links, each of which is subject to Rayleigh fading (independent between links and timeslots), while the mean channel states are constant. The numbers on each link represents the mean SNR over that link.

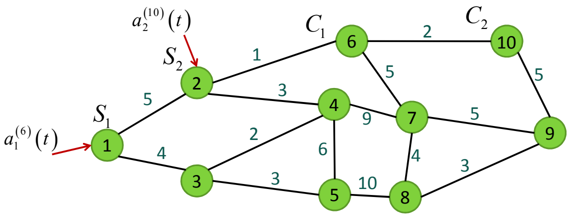

Fig. 2. Ad hoc network being simulated

Simulations are conducted comparing throughput performance of the three algorithms: DIVBAR-MIA, DIVBAR-RMIA, and regular DIVBAR, as described in Sec. II. Fig. 3 shows the time average occupancy (total time average backlog in the network) vs. input rate in units of kbit. The maximum supportable throughput corresponds to the input rate at which the occupancy goes towards very large values (due to a finite number of simulated packets, it does not approach infinity in our simulations). As is shown in the figure, the throughput under the DIVBAR-MIA algorithm is generally the largest among the three algorithms; the throughput under DIVBAR-RMIA algorithm is equal or slightly smaller than that of DIVBAR-MIA; DIVBAR-MIA and DIVBAR-RMIA algorithms are larger than that of the regular DIVBAR algorithm. These observations are in line with the derivations of Sec. III.

Simulation of the throughput comparison is carried out under different packet entropy conditions. The entropy contained in each packet is denoted by H_0 as is shown in the figure in units of kbit. When $H_0 = 1$ kbit, Fig. 3 shows that the throughput under the three algorithms are nearly identical. This phenomenon is caused by the fact that the packet length is generally small compared to the transmission ability of the links in the network. Therefore nodes in the network can usually achieve a successful transmission over a one hop link at the first attempt, which results in that (R)MIA has little benefit. However, as H_0 increases to 2 kbit and 4 kbit, the success probability in a single attempt decreases. Nodes under regular DIVBAR increases the chance of successful transmission just through trying more times, while DIVBAR-MIA and DIVBAR-RMIA accumulate information in each attempt, which will facilitate future transmission. Thus the throughput difference between DIVBAR and DIVBAR-(R)MIA becomes increasingly obvious.

Another interesting aspect shown in Fig. 3 is that all algorithms exhibit an optimum packet entropy. For DIVBAR, the throughput first increases as H_0 increases from 1 to 2 kbit, but decreases as H_0 increases from 2 to 4 kbit. The phenomenon is caused by the tradeoff between increased amount of information delivered through one successful transmission of a packet and the lowered success probability of each transmission. For very large H_0 value, the throughput tends to 0.

A similar phenomenon occurs in DIVBAR-MIA, though for a more complicated reason related to multi-user diversity. Consider a two hop network with 4 parallel relays. All the links for the first hop have mean SNR equal to 7, while the mean SNR for the second hop links is 5000. In this scenario

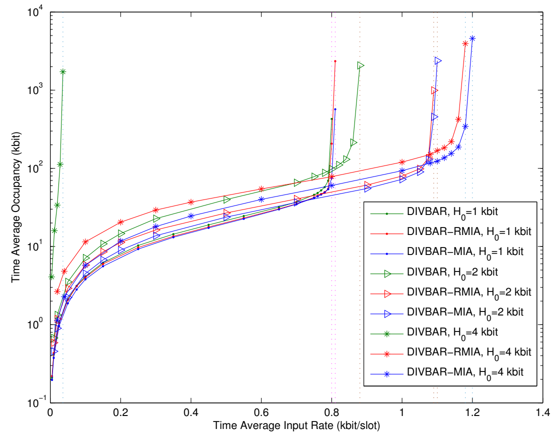

Fig. 3. Throughput performance comparison among DIVBAR-MIA, DIVBAR-RMIA and DIVBAR algorithms with different packet lengths

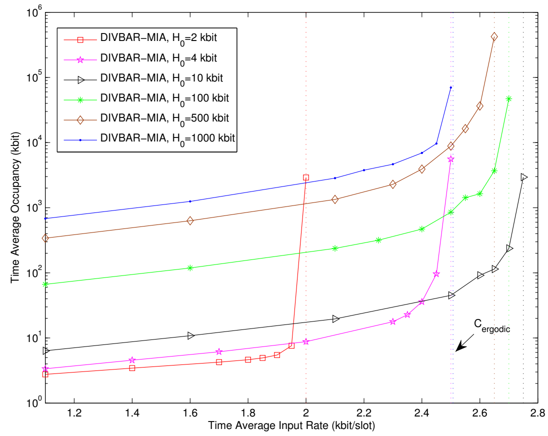

Fig. 4. Throughput evolvement under DIVBAR-MIA algorithm due to the change of multi-user diversity effect

the network throughput is dominated by the first-hop links, which facilitates analysis of the multi-user diversity effect. As is shown in Fig. 4, the throughput first increases as packet entropy increases from 2 to 10 kbit, then decreases as the entropy increases from 10 to 1000 kbit. When packet entropy is small, the packet always flows along the best one of the 4 random links in the first phase, which indicate a multi-user diversity gain. However, as the packet entropy increases to large values, each packet "sees", on each link, many channel realizations during its transmission. Due to law of large number, the transmission performance of each link gradually converges to its mean performance, and therefore, the multi-user diversity gain is gradually lost; the throughput converges to the ergodic capacity, as is shown in the $H_0 = 1000$ kbit case. Thus, there is a tradeoff between the higher effectiveness of MIA and the loss of multi-user diversity when increasing packet size. Thus, properly adjusting the packet size is essential to maximizing the network capacity region.

V. Conclusions

In this paper, we proposed two routing and scheduling algorithms: DIVBAR-RMIA and DIVBAR-MIA, which exploit mutual information accumulation technique as the physical layer transmission scheme. After setting up proper network model, including designing the queue structure of each network node to implement RMIA or MIA, we analyzed the network's throughput property under the mutual information accumulation assumption and proved that they have superior throughput performance compared to conventional DIVBAR. Simulation results confirmed this. Both traditional DIVBAR and our new algorithms were shown to have an "optimal" packet entropy that maximizes throughput, though for different reasons.

Appendix

Proof of Theorem 2 for the Single-link Case

For the single link, single commodity case, the capacity region reduces to a one dimensional set, and Ω_n reduces to a the destination node c. Therefore, we can simplify notation: $\omega_n^{(c)}(s)$ by $\omega(s)$, $q_{n,\Omega_n}^{(c)}(s)$ by $q(s)$, $\theta_{nk}^{(c)}(\Omega_n)$ by θ, $T_{s,\Omega_n}(\tau)$ by $T_s(\tau)$, and $F_{s,R_{nk}}(x)$ by $F_{s,R}(x)$.

With repetition transmission, $q^*(s)$ is just the transmission success probability over the single link:

$$q^*(s) = \Pr\{R(s,t) \geq H_0\} = 1 - F_{s,R}(H_0), \tag{13}$$

where $R(s,t)$ is the amount of information transmitted in timeslot t when mean channel state is equal to s. With RMIA, according to (9), we have

$$\begin{aligned} q(s) &= \frac{1}{\mathbb{E}\{T_s(j)\}} \\ &= \frac{1}{\sum\limits_{m=1}^{\infty} m \Pr\left\{\sum\limits_{\tau=1}^{m-1} R(s,\tau) < H_0 \leq \sum\limits_{\tau=1}^{m} R(s,\tau)\right\}} \\ &= \frac{1}{\sum\limits_{m=1}^{\infty} m \left[\Pr\left\{\sum\limits_{\tau=1}^{m-1} R(s,\tau) < H_0\right\} - \Pr\left\{\sum\limits_{\tau=1}^{m} R(s,\tau) < H_0\right\}\right]} \\ &\triangleq \frac{1}{\sum\limits_{m=1}^{\infty} m \left[F_{s,R}^{(m-1)}(H_0) - F_{s,R}^{(m)}(H_0)\right]} \\ &= \frac{1}{\sum\limits_{m=0}^{\infty} F_{s,R}^{(m)}(H_0)}, \end{aligned} \tag{14}$$

where $F_{s,R}^{(m)}(x)$ is the cdf of $\sum\limits_{\tau=1}^{m} R(s,\tau)$; $F_{s,R}^{(0)}(H_0)$ is 1.

Consider the following inequality:

$$\begin{aligned} F_{s,R}^{(m)}(H_0) &= \Pr\left\{\sum_{\tau=1}^{m-1} R(s,\tau) + R(s,t) < H_0\right\} \\ &= \int_0^{H_0} F_{s,R}^{(m-1)}(H_0 - x) f_{s,R}(x)\,dx \\ &< F_{s,R}^{(m-1)}(H_0) F_{s,R}(H_0) \\ &< \cdots < (F_{s,R}(H_0))^m, \end{aligned} \tag{15}$$

where $f_{s,R}(x)$ is the pdf of $R(s,t)$.

Combining (14), (15), it follows that

$$q(s) > \frac{1}{\sum\limits_{m=0}^{\infty} (F_{s,R}(H_0))^m} = 1 - F_{s,R}(H_0) = q^*(s). \tag{16}$$

Define $\Delta_q(s) \triangleq q(s) - q^*(s)$. With repetition transmission, for a input rate λ such that $\lambda + \varepsilon \in \Lambda^*$, $\varepsilon > 0$, there exists a supporting stationary randomized policy which chooses $\omega^*(s)$, θ^*. When RMIA is used, because $q(s) > q^*(s)$, the transmission rate is strictly larger than the rate without RMIA operation, if the same stationary randomized policy is adopted. Then the transmission rate difference is $\Delta_b \triangleq \sum\limits_{s\in S} \pi_s \omega^*(s) \Delta_q(s) \theta^*$. Define $\eta_{\min} = \min\{\Delta_q(s)/q(s), s \in S\}$, then we have

$$\begin{aligned} \Delta_b &= \sum_{s\in S} \pi_s \omega^*(s) \Delta_q(s) \theta^* \\ &\geq \eta_{\min} \sum_{s\in S} \pi_s \omega^*(s) q^*(s) \theta^* = \eta_{\min}\lambda \end{aligned} \tag{17}$$

According to (17) and considering that $\eta_{\min}$ only depends on the mean channel state, Δ_b does not approach to 0 as $\varepsilon \to 0$. Thus, if we let $\varepsilon \to 0$, the input rate $\lambda + \Delta_b$, which can be supported with RMIA, falls outside of Λ^*. Thus, the capacity region with RMIA is strictly larger than that without RMIA for the single link single commodity network, i.e., $\Lambda \supset \Lambda^*$.

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