Statistical Test Based on Finding the Optimum Lag in Cyclic Autocorrelation for Detecting Free Bands in Cognitive Radios

Hao Feng, Yanbo Wang, Shiju Li
Institute of Information and Communication Engineering
Zhejiang Univ., Hang Zhou, 310027, China
E-mail:fenghao666@hotmail.com, happyper@hotmail.com, Leesj@cise.zju.edu.cn

***Abstract*—Cognitive radios sense the radio spectrum in order to find free bands for transmission. In this paper, a novel method is proposed for detecting free bands by exploiting a statistical test based on finding the optimum lag in cyclic autocorrelation of primary user signals within the framework of cyclostationarity theory. This approach has a satisfying detecting performance under low SNR conditions and is computational efficient. Meanwhile, due to signals' cyclostationarity, the method also has a good anti-interference property which can be utilized to detect the primary user signals in the presence of interference. Finally, simulations are performed to verify those properties of the proposed method.**

Keywords-cognitive radio; cyclic autocorrelation; optimum lag; statistical test

I. INTRODUCTION

Spectrum, as a precious resource, has currently been managed by the government in the manner that one band is generally allocated to a single user. However, the usage of most of the assigned bands vibrates sporadically between 15% and 85% in geography with a high variance in time [1]. Facing this situation, FCC has focused its attention on cognitive radio, which was defined by Mitola [2]. Cognitive radio system senses the radio spectrum in order to detect the frequency bands that are not occupied by primary users (licenced users), and uses them in an agile manner. The fundamental requirement on cognitive radio is that its transmissions should not interfere with primary users' transmissions. Hence, detection of free bands must be executed reliably, even in low signal to noise ratio (SNR) communication environment.

Generally speaking, we model the free band decision as a binary hypothesis problem [1], which can be expressed as:

$$\begin{aligned} H_0: &\quad x(t) = n(t) \\ H_1: &\quad x(t) = s(t) + n(t) \end{aligned} . \tag{1}$$

We decide a frequency band to be unoccupied if there is only noise in it, as defined in H_0; on the other hand, once there exists primary user signal besides noise in a specific frequency band, as defined in H_1, we say the frequency band is occupied. According to the model shown in (1), several research directions for designing cognitive radio detectors have been proposed [1] [3], which aim at making decisions reliably and quickly. The most popular ones are: energy detector, matched filter and cyclostationary feature detector.

Although energy detector is easy to implement and needs not any prior knowledge about the signal, it is susceptible to unknown noise levels [4]. Matched filter is usually assumed as the optimal detector if knowledge of primary user signal is provided, since it can maximize the output SNR [1] [3]. Nevertheless, it has to precisely demodulate the primary user signals, which requires precise coherency with primary user signal by performing timing and carrier synchronization. Even those procedures have been perfectly done, matched filter detection is still vulnerable to interference, in addition of which the waveform of primary user signals may be distorted.

For cyclostationary feature detection, Gardner has systematically explored cyclic spectral correlation of modulated signals, and proved that the cyclostationary feature detection has a satisfying performance even in low SNR conditions [5] [6] [7]. In addition, Dandawate has developed χ^2 statistical test to detect the presence of cyclostationarity in signals [8]. However, these methods require significant amount of calculation and memory space.

In this paper, we propose a novel method for detecting free bands based on searching the optimum lag in the cyclic autocorrelation of primary user signals. This method takes into account that how to minimize the amount of computation and improve the detecting performance if priori knowledge about the primary user signals is provided. The key of this method is to set equivalence relationship between the optimum lag and the lag that maximizes the absolute value of cyclic autocorrelation of the primary user signal. Using the computed optimum lag instead of N lags for the test statistic, the detector achieves a better performance with much less computation.

This paper is organized as follows. A brief review of cyclostationarity and χ^2 statistical test are given in section Ⅱ. In section Ⅲ, we propose our method by analyzing the expression representing probability of detection to find the optimum lag for developing the test statistic. Section Ⅳ shows the simulation results, and compares the performance of the proposed detector with that of energy detector and matched filter. Finally, we give the conclusion in section Ⅴ.

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II. DTECTION OF CYCLOSTATIONARITY: A RECAP

A. Fundamentals of cyclic autocorrelation

A discrete-time zero-mean cyclostationary process $x(t)$ is characterized by the property that its autocorrelation: $R_x(t,\tau)=E\{x(t)x(t+\tau)\}$ exhibits periodicity, and accepts a Fourier series expansion as

$$R_x(t;\tau)=\sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t} \tag{2}$$

where the Fourier coefficient $R_x^{\alpha}(\tau)$ is defined as the cyclic autocorrelation, and satisfies

$$R_x^{\alpha}(\tau)=\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R_x(t;\tau)e^{-j2\pi\alpha t}. \tag{3}$$

In equation (3), the cyclic frequency α belongs to the set $A=\{\alpha : R_x^{\alpha}(\tau)\neq 0\}$, and T is the data length.

In practical implementation, given finite length of data, we actually use the consistent estimator of $R_x^{\alpha}(\tau)$, which is

$$\begin{aligned}\hat{R}_x^{\alpha}(\tau)&=\frac{1}{T}\sum_{t=0}^{T-1}x(t)x(t+\tau)e^{-j2\pi\alpha t}\\&=R_x^{\alpha}(\tau)+\varepsilon(\alpha,\tau)\end{aligned} \tag{4}$$

where $\varepsilon(\alpha,\tau)$ is the estimation error, and the error goes to zero as $T\to\infty$.

B. Time domain asymtotically otptimal χ^2 test

In the following, a brief description of the χ^2 statistical test developed by Dandawate is to be shown. This method is directed toward the situation that there is no prior knowledge available about the signal to be detected.

Firstly, the method constructs a sequence of lags and a set of cyclic frequencies of interest, which are denoted as $\{\tau_n\}_{n=1}^{N}$ and A respectively. Additionally, following equation (4), a $1\times 2N$ vector is formed:

$$\begin{aligned}\hat{\mathbf{r}}_x(\alpha)=\Big[&\mathrm{Re}\{\hat{R}_x^{\alpha}(\tau_1)\},\cdots,\mathrm{Re}\{\hat{R}_x^{\alpha}(\tau_N)\},\\&\mathrm{Im}\{\hat{R}_x^{\alpha}(\tau_1)\},\cdots,\mathrm{Im}\{\hat{R}_x^{\alpha}(\tau_N)\}\Big]\end{aligned} \tag{5}$$

which contains the real and imaginary part of the estimated cyclic autocorrelation for N lags at a specific cyclic frequency. Now the binary hypothesis test problem show in (1) can be expressed as

$$\begin{aligned}&H_0:\alpha\notin A,\forall\{\tau_n\}_{n=1}^{N}\Rightarrow\hat{\mathbf{r}}_x(\alpha)=\boldsymbol{\varepsilon}_x(\alpha)\\&H_1:\alpha\in A, \textit{for some } \{\tau_n\}_{n=1}^{N}\Rightarrow\hat{\mathbf{r}}_x(\alpha)=\mathbf{r}_x(\alpha)+\boldsymbol{\varepsilon}_x(\alpha)\end{aligned}. \tag{6}$$

Here the $\boldsymbol{\varepsilon}_x(\alpha)$ is the estimation error vector which is asymptotically normal distributed and satisfies

$$\lim_{T\to\infty}\sqrt{T}\boldsymbol{\varepsilon}_x(\alpha)\triangleq N(0,\Sigma_x(\alpha)). \tag{7}$$

The $2N\times 2N$ matrix $\Sigma_x(\alpha)$ shown in (7) is the covariance matrix of $\hat{\mathbf{r}}_x(\alpha)$, and is computed as

$$\boldsymbol{\Sigma}_x(\alpha)=\begin{bmatrix}\mathrm{Re}\left\{\dfrac{\mathbf{Q}+\mathbf{Q}^{(*)}}{2}\right\} & \mathrm{Im}\left\{\dfrac{\mathbf{Q}-\mathbf{Q}^{(*)}}{2}\right\}\\ \mathrm{Im}\left\{\dfrac{\mathbf{Q}+\mathbf{Q}^{(*)}}{2}\right\} & \mathrm{Re}\left\{\dfrac{\mathbf{Q}^{(*)}-\mathbf{Q}}{2}\right\}\end{bmatrix} \tag{8}$$

where the (m,n) th entries of the two covariance matrices $\mathbf{Q}$ and $\mathbf{Q}^{(*)}$ are defined as

$$\begin{aligned}\mathbf{Q}(m,n)&=S_{2f_{\tau_m,\tau_n}}(2\alpha,\alpha)\\\mathbf{Q}^{(*)}(m,n)&=S_{2f_{\tau_m,\tau_n}}^{(*)}(0,-\alpha)\end{aligned}. \tag{9}$$

Here, $S_{2f_{\tau_m,\tau_n}}(2\alpha,\alpha)$ and $S_{2f_{\tau_m,\tau_n}}^{(*)}(0,-\alpha)$ denote the unconjugated and conjugated cyclic spectra of $f(t,\tau)=x(t)x(t+\tau)$ respectively. Using frequency smoothed cyclic periodograms to estimate these cyclic spectra, we get

$$\begin{aligned}\hat{S}_{2f_{\tau_m,\tau_n}}(2\alpha,\alpha)=&\frac{1}{TL}\sum_{s=-(L-1)/2}^{(L-1)/2}W(s)\\&\cdot F_{\tau_n}\left(\alpha-\frac{s}{T}\right)F_{\tau_m}\left(\alpha+\frac{s}{T}\right)\end{aligned} \tag{10}$$

$$\begin{aligned}\hat{S}_{2f_{\tau_m,\tau_n}}^{(*)}(0,-\alpha)=&\frac{1}{TL}\sum_{s=-(L-1)/2}^{(L-1)/2}W(s)\\&\cdot F_{\tau_n}^{*}\left(\alpha+\frac{s}{T}\right)F_{\tau_m}\left(\alpha+\frac{s}{T}\right)\end{aligned} \tag{11}$$

where W is a normalized spectral window of odd length L and $F_{\tau}(f)=\sum_{t=0}^{T-1}x(t)x(t+\tau)e^{-j2\pi ft}$.

Define generalized likelihood function to be the test statistic which is denoted as $T_r(\alpha)$, and constructed as

$$T_r(\alpha)=T\cdot\hat{\mathbf{r}}_x(\alpha)\hat{\boldsymbol{\Sigma}}_x^{-1}(\alpha)\hat{\mathbf{r}}_x^{'}(\alpha). \tag{12}$$

If $\alpha\notin A$, which means that α is not a cyclic frequency of $x(t)$, then

$$\lim_{T\to\infty}T_r(\alpha)\triangleq\chi_{2N}^2 \tag{13}$$

and the false alarm probability $P_f=P\{T_r(\alpha)>\Gamma\mid H_0\}$; while if $\alpha\in A$, then

$$\lim_{T\to\infty}T_r(\alpha)\triangleq N\left(T\cdot\mathbf{r}_x(\alpha)\boldsymbol{\Sigma}_x^{-1}(\alpha)\mathbf{r}_x^{'}(\alpha),4T\cdot\mathbf{r}_x(\alpha)\boldsymbol{\Sigma}_x^{-1}(\alpha)\mathbf{r}_x^{'}(\alpha)\right) \tag{14}$$

and the detection probability $P_d=P\{T_r(\alpha)>\Gamma\mid H_1\}$, where Γ denotes the threshold.

III. STATISTICAL TEST BASED ON FINDING THE OPTIMUM LAG IN CYCLIC AUTOCORRELATION

In wireless communication cases, systems typically have some knowledge about certain properties of the transmitted signal, such as waveforms, modulation type, symbol or chip rate and carrier frequency, etc., which can help us to design detectors that can work in low SNR conditions. The following discussion is developed in such a premise. Hence, what the spectrum sensing should do is to decide whether the received data displays cyclostationarity at the cyclic frequencies.

In contrast with Dandawate's method which constructs N lags for computation, we aim at obtaining better performance using a single lag in this paper.

In the following, we develop the method by finding the optimal lag τ_{opt} instead of using $\{\tau_n\}_{n=1}^{N}$ to calculate the test statistic. In the situation that the cyclic frequency α of primary user signal is known to the receiver, denote

$$\mu = T \cdot \mathbf{r}_x(\alpha)\boldsymbol{\Sigma}_x^{-1}(\alpha)\mathbf{r}_x^{'}(\alpha) \tag{15}$$

and equation (14) can be rewritten as

$$\lim_{T\to\infty} T_r(\alpha) \triangleq N(\mu, 4\mu). \tag{16}$$

The objective of finding τ_{opt} is to maximize the probability of detection P_d at a given threshold denoted as Γ. From this point of view, the maximum detection probability can be deduced as

$$\begin{aligned} P_{d\max} &= \max_{\tau}\left\{P(T_r > \Gamma \mid H_1)\right\} \\ &= \max_{\tau}\left\{\int_{\Gamma}^{\infty} \frac{1}{2\sqrt{2\pi\mu}} \exp\left(\frac{(t-\mu)^2}{8\mu}\right) dt\right\}. \\ &= \frac{1}{2}\max_{\tau}\left\{ erfc\left(\frac{\Gamma-\mu}{2\sqrt{2\mu}}\right)\right\} \end{aligned} \tag{17}$$

Here the function $erfc(x)$ denotes the complementary error function, whose property determines:

$$\begin{aligned} \max_{\tau}\left\{erfc\left(\frac{\Gamma-\mu}{2\sqrt{2\mu}}\right)\right\} &= erfc\left\{\min_{\tau}\left(\frac{\Gamma-\mu}{2\sqrt{2\mu}}\right)\right\} \\ &= erfc\left\{\frac{\Gamma - \max_{\tau}\{\mu\}}{2\sqrt{2\max_{\tau}\{\mu\}}}\right\} \end{aligned}. \tag{18}$$

From above analysis, we find that maximizing the detection probability is equivalent to maximizing μ.

In the single lag case, $\mathbf{r}_x(\alpha)$ reduces to a 1×2 vector：

$$\mathbf{r}_x(\alpha) = \left[\mathrm{Re}\left\{R_x^{\alpha}(\tau)\right\}, \mathrm{Im}\left\{R_x^{\alpha}(\tau)\right\}\right] \tag{19}$$

and $\boldsymbol{\Sigma}_x(\alpha)$ reduces to a 2×2 matrix:

$$\boldsymbol{\Sigma}_x(\alpha) = \begin{bmatrix} \mathrm{Re}\left\{\dfrac{Q+Q^{(*)}}{2}\right\} & \mathrm{Im}\left\{\dfrac{Q-Q^{(*)}}{2}\right\} \\ \mathrm{Im}\left\{\dfrac{Q+Q^{(*)}}{2}\right\} & \mathrm{Re}\left\{\dfrac{Q^{(*)}-Q}{2}\right\} \end{bmatrix} \tag{20}$$

with

$$\begin{aligned} Q &= S_{2f_\tau}(2\alpha, \alpha) \\ Q^{(*)} &= S_{2f_\tau}^{(*)}(0, -\alpha) \end{aligned}. \tag{21}$$

For real process $x(t)$, using the theory of cyclic cumulant statistics analysis [8] [9], the leading diagonal elements of $\Sigma_x(\alpha)$ has the same magnitude, whose values are denoted as K; while the minor diagonal elements are zeros.

Therefore, according to the property of diagonal matrix, we have:

$$\begin{aligned} \max_{\tau}\{\mu\} &= \max_{\tau}\left\{T \cdot \mathbf{r}_x(\alpha)\boldsymbol{\Sigma}_x^{-1}(\alpha)\mathbf{r}_x^{'}(\alpha)\right\} \\ &= \frac{T}{K} \cdot \max_{\tau}\left\{\mathbf{r}_x(\alpha)\mathbf{r}_x^{'}(\alpha)\right\} \\ &= \frac{T}{K} \cdot \max_{\tau}\left\{\left|\mathbf{r}_x(\alpha)\right|^2\right\} \\ &\Leftrightarrow \max_{\tau}\left\{\left|\mathbf{r}_x(\alpha)\right|\right\} \end{aligned}. \tag{22}$$

From (17) (18) (22) and (19), we get

$$\max_{\tau}\{P_d\} \Leftrightarrow \max_{\tau}\left\{\left|R_x^{\alpha}(\tau)\right|\right\} \tag{23}$$

which denotes that finding the optimum lag which makes the probability of detection P_d maximum is equivalent with finding the lag which makes $\left|R_x^{\alpha}(\tau)\right|$ reach its maximum value. Hence, we can obtain the optimum lag τ_{opt} by searching where $\left|R_x^{\alpha}(\tau)\right|$ forms a peak as a function of τ.

According to the optimum lag, we construct the vector

$$\hat{\mathbf{r}}_x^{\alpha}(\tau_{opt}) = \left[\mathrm{Re}\left\{\hat{R}_x^{\alpha}(\tau_{opt})\right\}, \mathrm{Im}\left\{\hat{R}_x^{\alpha}(\tau_{opt})\right\}\right] \tag{24}$$

which also satisfies the binary hypothesis test shown as (6) (7). With finite data length, the consistent covariance matrix estimator: $\hat{\Sigma}_x(\alpha)$ is computed via (10) (11) (20) (21). From generalized likelihood ratio, we construct the test statistic:

$$T_r^{\alpha}(\tau_{opt}) = T \cdot \hat{\mathbf{r}}_x^{\alpha}(\tau_{opt})\hat{\boldsymbol{\Sigma}}_x^{-1}(\alpha)\hat{\mathbf{r}}_x^{'\alpha}(\tau_{opt}). \tag{25}$$

Now the proposed method can be summarized as follows:

1) According to the prior knowledge about the modulation type of the primary user signal, read the theoretical cyclic autocorrelation function $R_x^{\alpha}(\tau)$ stored previously in the receiver's memory.

2) Substitute the parameters of the primary user signal, as part of the prior knowledge, into $R_x^\alpha(\tau)$ and search the peak of $|R_x^\alpha(\tau)|$ in order to confirm where the peak appears. Get the corresponding lag and denote it as τ_{opt}.

3) From the received data and the optimum lag obtained above, compute $\hat{R}_x^\alpha(\tau_{opt})$ as in (4) and $\hat{\mathbf{r}}_x^\alpha(\tau_{opt})$ as in (24).

4) Calculate the 2×2 covariance matrix estimator $\hat{\mathbf{\Sigma}}_x(\alpha)$ using (10) (11) (20) (21).

5) Compute the proposed test statistic $T_r^\alpha(\tau_{opt})$ according to (25).

6) For given probability of false alarm P_f, using central χ^2 table for 2 degrees of freedom, get a threshold Γ so that $P_f = P(\chi_2^2 > \Gamma)$.

7) Declare that there is a free band available for secondary user to transmit signals at a specific time, if $T_r^\alpha(\tau_{opt}) < \Gamma$; otherwise, declare that the band is occupied by the primary user at that time.

The first two steps can be implemented off-line. Thus, when the cognitive receiver begins to work, it only needs τ_{opt} supplied by the previous steps and executes from step 3).

This method inherits the merits of Dandawate's method such as: it is asymptotically optimal in the generalized likelihood sense and the variance normalization in the test statistic makes thresholding easier and standard by looking up standard central χ_2^2 table.

Additionally, using a single lag instead of N lags, the amount of computation decreases significantly. Through calculation, we find that Dandawate's method needs about $(12T\cdot L+12L+4)\cdot N^2+(3T+2)N$ times of multiplications and $(4T\cdot L-L+5)\cdot N^2+2(T-1)N$ times addition; while the on-line process of the proposed method only needs about $12T\cdot L+3T+12L$ times multiplication and $4T\cdot L+2T-L$ times addition, approximately $1/N^2$ of the former, where T, L and N represent the received data length, the smoothing window length and the lags' number respectively.

Further more, the detecting performance of the proposed method is also better than Dandawate's method, which is supported by simulation shown in section Ⅳ. In fact, due to the change of $|R_x^\alpha(\tau)|$ according to the change of τ, $|R_x^\alpha(\tau)|$'s value usually becomes zero or near to zero at many lags, even α is a cyclic frequency of the primary user signal. In this sense, when the primary user signal occupies the frequency band, some lags belonging to $\{\tau_n\}_{n=1}^N$ make little contribution to making the test statistic $T_r(\alpha)$ display normal distribution, but rather add the length of $\hat{\mathbf{r}}_x(\alpha)$, and therefore increase the detection threshold and amount of computation. In contrast, the proposed method uses only one lag that makes the most contribution to making the test statistic display normal distribution, without any negative influences from other lags.

IV. SIMULATION

In this section, the proposed method is tested through simulation. The primary user signal to be tested is a QPSK signal, which can be expressed as

$$x(t)=\sum_{n=-\infty}^{\infty} q(t-nT_c-t_0)\cos(2\pi f_0 t+\theta_n+\phi_0) \quad (26)$$

where f_0 is the carrier frequency, $q(t)$ is the unit rectangle pulse with T_c as its symbol period, t_0 and ϕ_0 are the initial time and initial phase respectively, and $\{\theta_n\}$ is an independent sequence of information variables whose values satisfy: $\theta_n = \frac{\pi m}{2}, m=1,2,3,4$. The spectral correlation of QPSK signal is given in [5] [10]. Through inverse Fourier transformation, we get the cyclic autocorrelation of QPSK signal shown as

$$R_x^\alpha(\tau)=\begin{cases}\dfrac{\sin(\pi\alpha(T_c-|\tau|))}{2\pi\alpha T_c}\cos(2\pi f_0\tau)e^{-j2\pi\alpha t_0}, & \alpha=\dfrac{k}{T_c}\\ 0, & \text{else}\end{cases}. \quad (27)$$

Equation (27) demonstrates that QPSK signal exhibits cyclostationarity with cyclic frequencies are: $\alpha = k/T_c, k=0,\pm1,\pm2,\cdots$.

The following simulation is carried out in an additive white Gaussian noise (AWGN) channel. Denote f_s to be the sampling frequency and $f_0 = f_s/8$, $T_c = 55/f_s$. A Kaiser window of parameter 10 was used to smooth the covariance estimator given in (10) (11) with length: $L=41$. In addition, to be convenient, assume t_0 and ϕ_0 are both zeros. The experiment treats $\alpha = 1/T_c$ as the cyclic frequency to implement the detection.

Fig. 1 depicts the detecting performance of the proposed method vs. SNR ranging between $-15dB$ and $5dB$ at a constant false alarm rate: $P_f = 0.005$ for various data lengths: $T=1100$, $T=2200$, $T=4400$. The experiment is carried out over 300 Monte Carlo runs. From the simulation results, we find that the proposed method generally works well under low *SNR* conditions, and the performance becomes better as the received data length increases. When $T=1100$, the obtained P_d can achieve 1 at $\text{SNR}=-4dB$; while when the data length increases to 4400, the obtained P_d can achieve 1 even at $\text{SNR}=-7dB$.

Fig. 2 depicts the receiver operation characteristic curves (ROC) of the proposed method under $\text{SNR}=-7dB$ with the three data lengths as in Fig. 1. The horizontal coordinate represents the probability of false alarm P_f , while the longitudinal coordinate represents the probability of detection P_d . The experiment is carried out over 1000 Monte Carlo runs. The figure shows that the method has desirable receiver operating characteristics. That is, the probability of detection increases as the probability of false alarm increases. Moreover, we still find that the larger the data length is, the better the performance is.

Fig. 3 depicts the ROC curves of the proposed method and Dandawate's method for comparison. Both detectors are set to work under $\text{SNR} = -4dB$ with the received data length $T = 1100$. Specially, in plotting the ROC curve of Dandawate's method, we use 5 lags whose values compose a sequence: $26/f_s, \cdots, 30/f_s$, which are near the half of a symbol period. In fact, the optimum lag found in this experiment is $28/f_s$, which belongs to the above lag sequence. In this experiment, we only compare the ROC performances of the two methods, without considering their time consuming, although the time needed by the proposed detector is only $1/25$ of that of Dandawate's method. Results in Fig. 3 shows that the proposed method outperforms Dandawate's method. The reason for this is that $\left|R_x^{\alpha}(\tau)\right|$ is actually zero at $\tau = 26/f_s$ and $30/f_s$, as is shown in Fig. 4, which means that these two lags make no contribution to detecting the presence of the primary signal but rather increase the required detecting threshold due to $P_f = P\left(\chi_{10}^2 > \Gamma\right)$.

For further comparison we plot ROC curves of proposed detector and the conventional energy detector developed in [4], as is shown in Fig. 5. Both detectors work at $\text{SNR} = -7dB$ condition with data length: $T = 2200$. From this figure, we find that the energy detector have lost its detecting ability under such a low SNR condition, because its ROC curve approaches to 45 degree chance line. On the other hand, the proposed detector still has satisfying performance.

Next the anti-interference ability of the proposed detector is performed in comparison with matched filter [11]. Generally speaking, if knowledge of primary user signal is provided, matched filter is usually assumed as a better detector than energy detector for cognitive radio under low SNR levels. However, its performance is susceptible to interference. In addition of interference, the waveforms of the primary user signal may be distorted and, therefore, may not match the matched filter. Fig. 6 shows that the proposed detector outperforms matched filter under the same interference and noise condition. In this experiment, the primary user QPSK signal is suffered from an interfering BPSK signal. The two signals' spectra are overlapped. This experiment takes the same simulation parameters as those for depicting Fig. 5 except for $\text{SNR} = -4dB$. The interfering BPSK signal with symbol period $T_c = 100/f_s$ has the same power as the QPSK signal. Fig. 6 shows the extent to which the interference and noise affect the ROC performances of the two methods. From this figure, we find that in the presence of interference and noise, the proposed detector can still perform a high probability of detection, while the matched filter's performance deteriorates significantly.

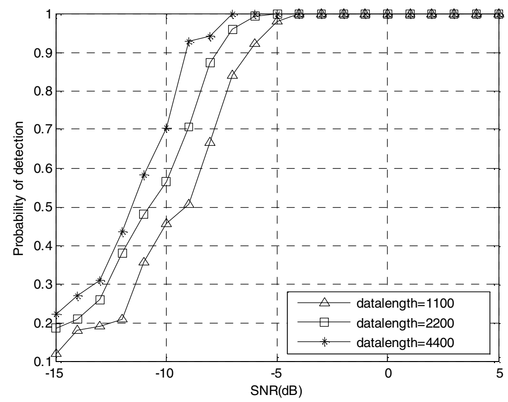

Figure 1. Probability of detection vs. SNR for different data lengths: $T = 1100$, $T = 2200$ and $T = 4400$ with $P_f = 0.005$

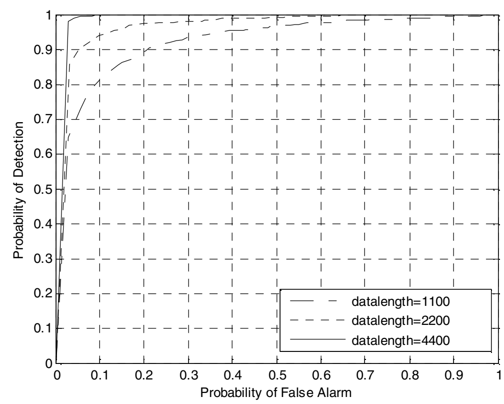

Figure 2. ROC curves for different data lengths: $T = 1100$, $T = 2200$ and $T = 4400$ under $\text{SNR} = -7dB$

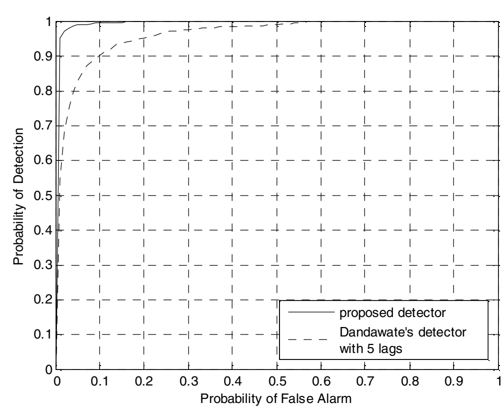

Figure 3. ROC curves of the proposed detector and Dandawate's detector with $T = 1100$ under $\text{SNR} = -4dB$

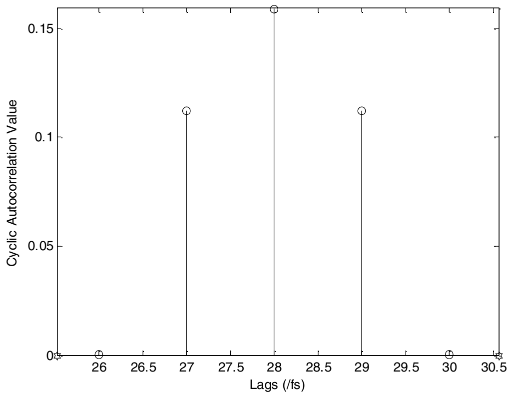

Figure 4. Cyclic autocorrelation values due to 5 lags: $26/f_s,\cdots,30/f_s$

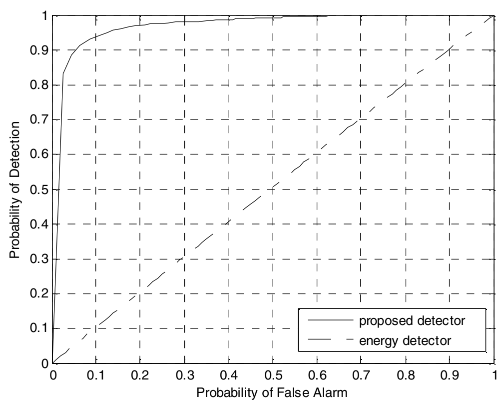

Figure 5. ROC curves of proposed detector and energy detector with $T = 2200$ under $\text{SNR} = -7dB$

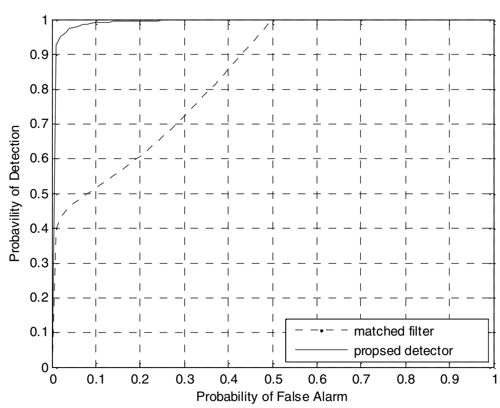

Figure 6. ROC curves of proposed detector and matched filter with $T = 2200$ under $\text{SNR} = -4dB$

V. CONCLUSION

In this paper, in order to detect free bands in cognitive radios, a novel method is proposed based on finding the optimum lag in cyclic autocorrelation of the primary user signals. This method exploits the cyclostationarity of modulated signals and aims at finding a single lag for computing the test statistic in order to reduce the amount of computation and improve the performance. Theoretical analysis demonstrates that the optimum lag is actually the one which maximizes the absolute value of the cyclic autocorrelation of primary user signals. Through calculation, the total amount of computation of the proposed method is estimated, approximately $1/N^2$ of that of Dandawate's method. Simulation demonstrates that this method has satisfying performance under low SNR conditions compared with energy detector, and the performance meliorates as data length increases. In addition, through simulation, the proposed method exhibits better ROC performance than Dandawate's method due to the selecting optimum lag operation. Finally, simulation result also shows that the proposed method has a good anti-interference property compared with matched filter. Hence, the method based on finding optimum lag in cyclic autocorrelation provides an efficient approach for detecting free bands in cognitive radios.

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