Blind Detection and Parameter Estimation of Multiuser and Multipath DS-CDMA Signal Using Cyclostationary Statistics

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***Abstract*—Communication signals are appropriately to be modeled as cyclostationary stochastic processes. The most significant advantage of cyclostationarity is its suppression on the stationary noise and interference. Therefore, cyclostationary theory has been a powerful tool in LPI (Low Probability of interception) signal processing. In non-cooperative condition, blind detection and parameter estimation of DS-CDMA (Direct Spread Spectrum code division multiple access) signals is the first step of PN (Pseudo Noise) codes extraction and information recovery, which has been a challenging problem for a long time and has much theoretical and practical values. The main work of this paper is to deal with multiuser DS-CDMA signals in multipath entironment. Method based on second order cyclostationary statistics was adopted to detect whether the modulated signal exists in background noise. The close formula of SCF (spectrum correlation function) for complex DS-CDMA signals was derived, chip rate can be achieved from the section of SCF where $\alpha=f_0$ (α is the cyclic frequency and f_0 is the carrier frequency). Simulation results containing the $\alpha=f_0$ section and the ROC (Receiver Operating Characteristics) curve have been shown, which proves that the proposed method can achieve good performance even when signals are buried in strong background noise.**

Keywords; detection; estimation; cyclostationary;CDMA;

I. INTRODUCTION

As the advantage of low power density spectrum, DSSS (Direct Spread Spectrum Signal) has been used as main technology of LPI (Low Probability of Interception) for a long time. Combined with the merit of CDMA (Code Division Multiple Access), DS-CDMA (Direct Spread Spectrum code division multiple access) has attracted much attention in military and commerce fields. Under non-cooperative conditions, the receiver has none prior knowledge of the signals, thus, blind detection and parameter estimation of DS-CDMA signals become the first step of PN (Pseudo Noise) codes extraction and information recovery which plays an important role in electronic warfare or in the civil field of wireless communication ward.

In common communication theory, signals have been modeled as stationary stochastic processes. However, in communication systems, various periodic operations such as sampling, scanning, modulating, multiplexing, and coding, give rise to underlying periodicities in the signals[1]. Signals whose certain order statistic parameter have periodicities are more appropriately modeled as cyclostationary stochastic processes[2]. In stationary stochastic processes, autocorrelation function and PSD (Power Spectrum Density) function are used to analyze the characteristics of the processes. Similarly, cyclic autocorrelation function and cyclic spectrum function (which is also called SCF (Spectrum Correlation Function)) are used in cyclostationary processes. The spectral correlation theory of cyclostationary processes was advanced by Gardner[3] and explicit formulas for SCF of various types of analog and digitally modulated signals were first derived by him[4、5]. Compared with PSD, predominantly, SCF has significantly practical value: when cyclic frequency is not identically zero, stationary noise and interference exhibit no spectral correlation (the spectral correlation function is identically zero). Thus, SCF has been a powerful tool to analyze and process modulated signals which are burried in strong stationary background noises. In a theoretical framework, Gardner proved that the energy detector, delay and multiply detector and cyclic detector were similar in nature but the cyclic detector had the best performance[6、7]. A method of detecting the presence of cyclostationary signal using k th order cyclic statistics was proposed by Dandawaté[8] and the performance of this detector was analyzed by Rostaing [9]. But the detection of DSSS signals has not been considered in those papers. JIN yan extended this statistical test method in a simple DS-CDMA model of a single user with a single path[10]. Using cyclostationary statistics, Mazzenga proposed a simple algorithm to jointly parameter estimation of each individual signal in the multiuser CDMA model[11]. But, up to now, researches on detection and parameter estimation of the complex situation of multiuser and multipath have rarely been seen in public paper.

In this paper, model of complex DS-CDMA was given in next part. And then, the close formula for the SCF of the model was deduced in the third part. To illustrate, the curve of ROC and section of SCF were displayed, simulation results were discussed and analyzed. Conclusion that the cyclostationary characteristic of multiuser and multipath DS-CDMA signal can be utilized in signal detection and parameter estimation was drawn in last part.

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II. SYSTEM MODEL

An asynchronous BPSK DS-CDMA system with K users operating over a Rayleigh fading mupltipath channel is considered here. The signal received can be written as:

$$r(t) = s(t) + n(t) = \sum_{k=1}^{K} s_k(t) + n(t) \qquad (1)$$

$s_k(t)$ is the k th user's transmitted signal, its complex band-pass bandpass representation can be modeled as:

$$s_k(t) = \sqrt{P_k} \sum_{l=1}^{L_k} \sum_{n=1}^{N} a_{kl} I_k(n) C_k(t - nT_b - d_{kl}T_c) \cdot \cos\left[2\pi j f_0 \left(t - d_{kl}T_c\right)\right] \qquad (2)$$

P_k	k th user's signal power;
a_{kl}	attenuation factor for the l th path of the k th user ;
$I_k(n)$	k th user's data symbol in the n th symbol interval $I_k(n) \in \{-1, +1\}$;
$C_k(t)$	k th user's PN codes whose periodicity is P $C_k(t) \in \{-1, +1\}$;
T_b	symbol interval;
T_c	chip interval;
d_{kl}	delay factor for the l th path of the k th user;
f_0	carrier frequency;
K	number of total users;
N	number of data symbols
L_k	path number of the k th user;

The k th user's delay, d_{kl} is uniformly distributed; while the k th user's attenuation factor, a_{kl} , obey the Rayleigh distribution. Assuming each user has the same power, formula (2) can be simplified as:

$$s_k(t) = \sum_{l=1}^{L_k} \sum_{n=1}^{N} a_{kl} s_{k0}\left(t - d_{kl}T_c\right) \qquad (3)$$

$s_{k0}(t)$ is the single path signal of the k th user with no attenuation and delay which can be represented as:

$$s_{k0}(t) = \sum_{n=1}^{N} I_k(n) C_k(t - nT_b) \cos\left(2\pi j f_0 t\right) \qquad (4)$$

III. CYCLOSTATIONARY SIGNAL ANALYSIS

A signal x(t) is defined to be second order cyclostatinary if its autocorrelation function

$$R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = E\left\{x(t + \tau/2) x^*(t - \tau/2)\right\} \qquad (5)$$

is periodic in time t for each lag τ . It is assumed that the Fourier series expansion for this periodic function converges, and therefore $R_x(t, \tau)$ can be expressed as

$$R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t} \qquad (6)$$

for which $\left\{R_x^{\alpha}\right\}$ are the Fourier coefficients and α ranges over all integer multiples of the fundamental frequency 1/T. Consequently, R_x^{α} is defined as the CAF (Cyclic Autocorrelation Function) and α is called the cycle frequency which is represented as follows [12].

$$R_x^{\alpha}(\tau) \triangleq \lim_{Z\to\infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt \qquad (7)$$

The SCF of can be achieved by Fourier-transform of CAF.

$$\mathrm{S}_{\mathrm{x}}^{\alpha}(f) = \int_{-\infty}^{\infty} R_{\mathrm{x}}^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau \qquad (8)$$

To calculate the SCF of time series practically, the method of frequency field smooth was commonly adopted[13].

$$\hat{S}_x^{\alpha}(f) = \lim_{\Delta f\to 0} \lim_{\Delta t\to\infty} \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} \frac{1}{\Delta t} X(t, v + \alpha/2) \cdot X^*(t, v - \alpha/2) dv \qquad (9)$$

Because of the independence of PN codes which are used by each user, Seidl[14] thought the SCF of multiuser DS-CDMA signals was just the sum of the SCF of each user. If the signal of each user only contains a single path and the signals are synchronous without delay, this assumption might be close to the fact. But when there were more than one path, situations might be quite different. Reciprocity of the signals of different paths must be considered carefully. A result that the SCF for $s'(t) = s(t - t_0)$ is $S_{s'}^{\alpha}(f) = S_s^{\alpha}(f) \cdot e^{-2\pi j\alpha t_0}$ was mentioned by Gardner[6], but the proof was not given out. And that is correct only for time-invariant systems. Therefore, SCF of multiuser and multipath DS-CDMA signals must be deduced carefully.

In our model, noise n(t) is independent to signal s(t), then the cross-correlation function between n(t) and s(t) can be ignored. When calculating the autocorrelation function of r(t), attention should be focused on the autocorrelation function of s(t) which is represented as:

$$R_{ss}\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = E\left\{s(t + \tau/2) s^*(t - \tau/2)\right\} \qquad (10)$$

Substitution of formula (2) into formula (10) yields:

$$R_{ss}\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = E\left\{\sum_{u=1}^{K} s_u\left(t + \frac{\tau}{2}\right) \sum_{v=1}^{K} s_v^*\left(t - \frac{\tau}{2}\right)\right\}$$

$$= E\left\{ \underbrace{\sum_{u=1}^{K} s_u\left(t+\frac{\tau}{2}\right) s_u^{*}\left(t-\frac{\tau}{2}\right)}_{(a)} + \underbrace{\sum_{\substack{u=1\\ v=1\\ u\neq v}}^{K} s_u\left(t+\frac{\tau}{2}\right) s_v^{*}\left(t-\frac{\tau}{2}\right)}_{(b)} \right\} \tag{11}$$

where(a) is the autocorrelation function of $s_u(t)$ and (b) is the cross-correlation function of $s_u(t)$ and $s_v(t)$. Because of the independence of PN codes of different users, the value of (b) is close to zero. When (b) is ignored and (3) is imported, formula (11) turns into:

$$R_{ss}\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right)$$

$$= E\left\{ \sum_{k=1}^{K} \left[\begin{array}{l} \sum_{l=1}^{L_k}\sum_{n=1}^{N} a_{kl} s_{k0}\left(t-d_{kl}T_c+\frac{\tau}{2}\right) \\ \cdot \sum_{l=1}^{L_k}\sum_{n=1}^{N} a_{kl} s_{k0}^{*}\left(t-d_{kl}T_c-\frac{\tau}{2}\right) \end{array} \right] \right\} \tag{12}$$

$$= E\left\{ \begin{array}{l} \underbrace{\sum_{k=1}^{K}\left[\sum_{l=1}^{L_k}\sum_{n=1}^{N} a_{kl} s_{k0}\left(t-d_{kl}T_c+\frac{\tau}{2}\right) a_{kl} s_{k0}^{*}\left(t-d_{kl}T_c-\frac{\tau}{2}\right)\right]}_{(c)} \\ \underbrace{+\sum_{\substack{i=1\\ j=1\\ i\neq j}}^{L_k}\sum_{n=1}^{N} a_{ki} s_{k0}\left(t-d_{ki}T_c+\frac{\tau}{2}\right) a_{kj} s_{k0}^{*}\left(t-d_{kj}T_c-\frac{\tau}{2}\right)}_{(d)} \end{array} \right\}$$

where (c) is the autocorrelation of the l th path signal of the k th user, (d) is the correlation of the ith path and jth path signal of the k th user. Here two new variable (t',τ') need to be defined.

$$\begin{cases} t_{ij}' = t-\left(d_{ki}+d_{kj}\right)T_c/2 = t-\Delta_{ijc}/2 \\ \tau_{ij}' = \tau+\left(d_{ki}-d_{kj}\right)T_c = \tau+\nabla i_j \end{cases} \tag{13}$$

Substituted by (13), formula (12) come into:

$$R_{ss}\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right)$$

$$= \sum_{k=1}^{K}\left\{ R_{s_{k0}s_{k0}}\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right) + \sum_{\substack{i=1\\ j=1\\ i\neq j}}^{L_k} R_{s_{k0}s_{k0}}\left(t_{ij}'+\frac{\tau_{ij}'}{2}, t_{ij}'-\frac{\tau_{ij}'}{2}\right) \right\} \tag{14}$$

When (14) substituted into(7) and the Wiener-Khinchin relation (8) , the SCF of multiuser and multipath DS-CDMA signal can be represented as:

$$S_s^{\alpha}(f) = \sum_{k=1}^{K}\left\{ S_{s_{k0}}^{\alpha}(f) + \sum_{\substack{i=1\\ j=1\\ i\neq j}}^{L_k} S_{s_{k0}}^{\alpha}(f) e^{-j\pi\Delta_{ij}} e^{j2\pi f\nabla_{ij}} \right\} \tag{15}$$

$S_{s_{k0}}^{\alpha}(f)$ is the SCF of single user BPSK DS-CDMA signal with a single path. $S_{s_{k0}}^{\alpha}(f)$ could be deducd by substituded the pulse shape function of DS signal into the SCF of normal BPSK modulated signals. Considering the same chip rate, symbol rate and carrier frequence, the SCF of different user's is just the same, whose expression is:

$$S_{s_{k0}}^{\alpha}(f) = \begin{cases} \frac{1}{4T_c}\left\{ Q\left(f+f_c+\frac{\alpha}{2}\right)Q^{*}\left(f+f_c-\frac{\alpha}{2}\right) \right. \\ \left. +Q\left(f-f_c+\frac{\alpha}{2}\right)Q^{*}\left(f-f_c-\frac{\alpha}{2}\right)\right\}, \alpha=\frac{k}{T_c} \\ \frac{1}{4T_c}\left\{ Q\left(f+f_c+\frac{\alpha}{2}\right)Q^{*}\left(f-f_c-\frac{\alpha}{2}\right) \right. \\ \left. +Q\left(f-f_c+\frac{\alpha}{2}\right)Q^{*}\left(f+f_c-\frac{\alpha}{2}\right)\right\}, \alpha=\pm 2f_0+\frac{k}{T_c} \end{cases} \tag{16}$$

In which $Q(f)$ is the frequency spectrum of pulse shape function of DS signal..

$$Q(f) = \frac{\sin\left(\pi f T_c\right)}{\pi f} \tag{17}$$

From formula (15), delay of different path cause frequency shift in α and f, but the module of $S_s^{\alpha}(f)$ has the same shape as $\left|S_{s_{k0}}^{\alpha}(f)\right|$ in the bi-frequency plane, so method based on cyclostationary to implement detection or parameter estimation can also be adopted in the multiuser and multipath situations.

IV. DETECTION AND PARAMETER ESTIMATION

A. *Detection of multiuser and multipath DS-CDMA signal*

From the discrete form of (7), the CAF ($\hat{R}_x^{\alpha}(\tau)$) of a real signal can be calculated out. Through formula (18), a cyclic vector $\Re$ can be got by arranging the real part and imaginary part of $\hat{R}_x^{\alpha}(\tau)$ in series.

$$\Re \triangleq \left[\mathrm{Re}\left(\hat{R}_x^{\alpha}(\tau)\right), \mathrm{Im}\left(\hat{R}_x^{\alpha}(\tau)\right)\right] \tag{18}$$

where Re( ) and Im( ) denotes the real and imaginary part respectively. Then a second order test statistics is given as follows, which is derived from the general maximum likelihood ratio[8]:

$$\xi_R = T\Re\Sigma^{-1}\Re' \tag{19}$$

Where Σ is the asymptotic covariance matrix of the cyclic vector $\Re$, $\Re'$ reprsents the complex transpose of and Σ is the asymptotic covariance matrix of $\Re$, which can be expressed as:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \mathrm{Re}\left\{\dfrac{Q+Q^*}{2}\right\} & \mathrm{Im}\left\{\dfrac{Q-Q^*}{2}\right\} \\ \mathrm{Im}\left\{\dfrac{Q+Q^*}{2}\right\} & \mathrm{Re}\left\{\dfrac{Q^*-Q}{2}\right\} \end{bmatrix} \qquad (20)$$

with:

$$Q = S_{2f}\left(2\alpha,\alpha\right),\ \ Q^* = S^*_{2f}\left(0,-\alpha\right) \qquad (21)$$

$$\begin{aligned} \hat{S}_{2f}\left(2\alpha,\alpha\right) = \frac{1}{TL}\sum_{s=-(L-1)/2}^{(L-1)/2} W\left(s\right) \\ \cdot F\left(\alpha-\frac{2\pi s}{T}\right)F\left(\alpha+\frac{2\pi s}{T}\right) \end{aligned} \qquad (22)$$

$$\begin{aligned} \hat{S}^*_{2f}\left(0,-\alpha\right) = \frac{1}{TL}\sum_{s=-(L-1)/2}^{(L-1)/2} W\left(s\right) \\ \cdot F^*\left(\alpha+\frac{2\pi s}{T}\right)F\left(\alpha+\frac{2\pi s}{T}\right) \end{aligned} \qquad (23)$$

where W is a normalized spectral window of odd length L and $F\left(\omega\right) = \sum_{t=0}^{N-1} x\left(t\right) x^{(*)}\left(t+\tau\right) e^{-j\omega t}$.

Then comes the binary hypothesis for signal detection:

$$\begin{cases} H_0 : \xi_R \sim \chi^2_{2N} \\ H_1 : \xi_R \sim N\left(T\Re\Sigma^{-1}\Re', 4T\Re\Sigma^{-1}\Re'\right) \end{cases} \qquad (24)$$

When a probability of false alarm, P_{fa}, is given, the decision threshold could be found through the central χ^2 distribution for 2N degrees of freedom. If the value of the cyclic cumulant is bigger than the threshold ($\xi_R > \Gamma$) at certain α for some lag τ (τ1, τ2,…, τn) , we can declare the presence of the signals.

B. Parameter estimation of multiuser and multipath DS-CDMA signal

From formula (15), the SCF of multiuser with multipath DS-CDMA signals has the same shape as the SCF of single user with single path. Parameters such as carrier frequency and chip rate can be revealed from the SCF of (16). In the section of SCF where f=f0, the maximum value appears at α=0 and the second largest value comes at α=±1/Tc.From the distance between the maximum value and the second largest value along α axis, the chip rate can be estimated accurately.

V. SIMULATION

In this section, several experiments were made to test the performance of detector and estimator proposed above. All the experiments were accomplished by MATLAB®7. The simulation time $T=10^{-4}$ second, symbol rate f_b=1MHz, carrier frequency f_0= 90 MHz, period of PN codes is 63, that is, the chip rate f_c=63MHz. Each chip is sampled 8 times, thus the sample frequency fs=504MHz.

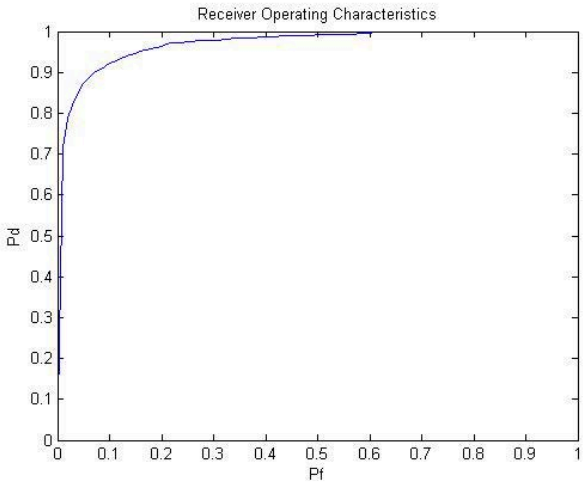

Figure 1. ROC for a single user with a single path

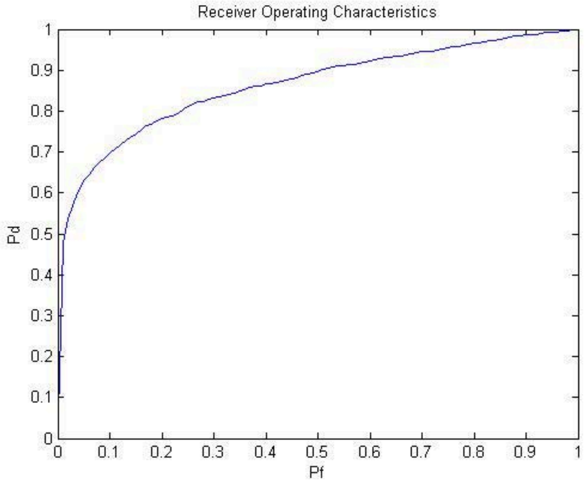

Figure 2. ROC for 6 users with 5 paths each

Figure 1 reveals the ROC of a single user DS-CDMA signal with a single path, where SNR=-10dB. Figure 2 is the ROC curve of 6 users DS-CDMA signal with 5 paths for each user under the noise of the same strength. Both the experiments were repeated with Monte Carlo method for 1000 times. In Figure 1, when the probability of false alarm is 0.1, the probability of detection has reached 0.922. Though the performance suffer a deterioration in some extent in multiuser and multipath entironments, the detector can still work properly. The main reason for the deterioration is when calculating the cyclic-cumulant, the complex value of the CAF (or SCF) is used, thus the frequence shift in f and α deteriorates the performance. This situation may be improved by using the module value to substitute the complex value in formula (17) which is to be investigated in future.

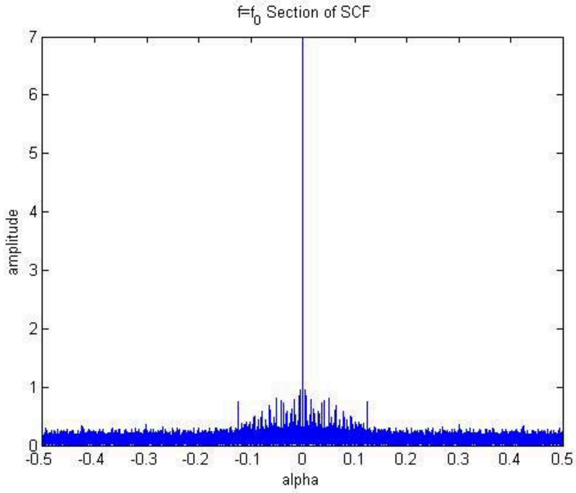

Figure 3. f=f_0 section of SCF for a single user with a single path

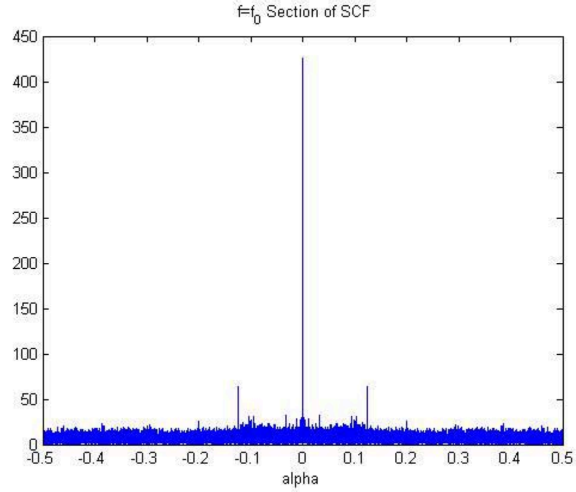

Figure 4. f=f_0 section of SCF for 6 users with 5 paths each

Figure 3 is the f=f_0 section of the SCF for a single user with a single path, and Figure 4 is that of the SCF of multiuser multipath DS-CDMA. X axis of the figure represents the cyclic frequence α and Y axis is the amplitude of $|S^{\alpha}(f=f_0)|$. These two figures were drawn under the noise strength of -10dB. The maximum value appears at α=0 and the second largest value comes at α=±0.125, which denote the chip rate is 1/8 of sample frequency. From these two figures, we can see that when module value is used, the SCF of multiuser and multipath DS-CDMA is improved than that of a single user with a single path.

VI. CONCLUSION

Detection and parameter estimation method of multiuser DS-CDMA signals with multipath was discussed in this paper. The close formula for the SCF of multiuser and multipath DS-CDMA was deduced, from which a conclusion could be made, that the module value of the SCF of multiuser multipath DS-CDMA is just the same as that of DS-CDMA signal with a single user and signal path except for a linear factor in amplititude. Thus, detection and parameter estimation method based on cyclostationarity can also be used in complex situations such as multiuser and multipath. The idea was supported by the simulation result very well.

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