Device-to-Device Collaboration through Distributed Storage

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***Abstract*—Video is the main driver for the inexorable increase in wireless data traffic. In this paper we analyze a new architecture in which device-to-device (D2D) communications is used to drastically increase the capacity of cellular networks for video transmission. Users cache popular video files and - after receiving requests from other users - serve these requests via D2D localized transmissions; the short range of the D2D transmission enables frequency reuse within the cell. We analyze the scaling behavior of the throughput with the number of devices per cell. The user content request statistics, as well as the caching distribution, are modeled by a Zipf distribution with parameters $\gamma_{\rm r}$ and $\gamma_{\rm c}$, respectively. For the practically important case $\gamma_{\rm r} < 1$ and $\gamma_{\rm c} > 1$, we derive a closed form expression for the scaling behavior of the number of D2D links that coexist without interference. Our analysis relies on a novel Poisson approximation result for wireless networks obtained through the Chen-Stein Method.**

I. INTRODUCTION

Wireless data traffic has risen sharply over the past years, and is expected to increase by a further factor of 40 over the next five years [1]. While the initial increase was mainly driven by web-browsing and associated applications, emphasis is now moving to wireless delivery of video content. This threatens to overwhelm the already-strained cellular networks. Since increasing cellular data capacity by "traditional" means (more spectrum, larger number of antennas, smaller cells) is either reaching its natural limits or becoming very costly, there is a need for fundamentally new cellular architectures that can exploit the special properties of video requests (*e.g.* from YouTube), as well as of modern smartphone applications. Our architecture relies on two key observations: (i) Video has a high degree of content reuse, *i.e.*, a few popular files are requested by a large number of users. (ii) Smartphones and tablets have significant storage capacity that is rapidly growing and typically underutilized.

Our proposed distributed storage scheme is based on the fact that if there is enough content reuse, *i.e.*, many users are requesting the same video content, the storage space on cellular devices can be used as a distributed cache [2], [3].

We allow users to store popular content and use local device-to-device (D2D) communication for collaborative dissemination when a user in the vicinity requests a popular file. Since D2D communication occurs over a short range, this enables frequency reuse, *i.e.*, the same time-frequency resource can be exploited by multiple D2D links within one macro-cell. This dramatically increases the spectral efficiency and is similar to the constant trend of shrinking cell sizes. What is fundamentally different between D2D communications and small cells, however, is that femto and pico-cell base stations require high-bandwidth backhaul links. The D2D collaborative video content dissemination scheme we are proposing *essentially replaces backhaul with storage*, by caching files that have been already requested by users, without relying on additional infrastructure.

As the number of devices in a cell increases, the distance between devices becomes smaller, which allows to use less power, and increase the frequency reuse. Yet, on the other hand, more users will request a greater variety of files, which decreases the probability that a device is within communication range of another device that has the desired content. Thus, while it seems intuitive that a greater density of devices allows a larger number of *active* and non-interfering links within a cell, the actual functional scaling behavior cannot be obtained simply by inspection.

In our recent paper [4], we analyzed the scaling behavior of the average number of D2D links that can coexist without interference when users choose which file to store randomly. Throughout this work, the geometry of our networks follows the standard Random Geometric Graph protocol model [5], [6]. We showed that the scaling behavior depends on both the content request statistics and content cache statistics which are modeled by Zipf distributions. In particular, our prior work demonstrated that for particular types of content popularity and cache statistics, a linear scaling becomes feasible. However, for a different range of parameters for content statistics, we could only determine loose upper bounds.

Our contributions: In this paper we close this gap in our theoretical understanding of the scaling behavior. Specifically, we analyze the previously-unknown case where the Zipf distribution for the requests is characterized by a parameter $\gamma_{\rm r} < 1$ and the caching distribution by a parameter $\gamma_{\rm c} > 1$, and provide a unique scaling law for the number of active D2D links as a function of the number of devices in the cell. Our analysis relies on a novel Poisson approximation result for

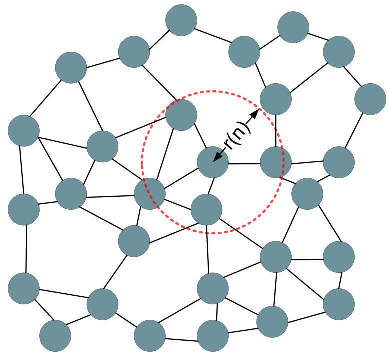

Fig. 1. Random geometric graph example. The connectivity radius is $r(n)$.

wireless networks obtained through the Chen-Stein Method. We note that the case of $\gamma_{\rm r} < 1$ is the most practically relevant one, since prior empirical Youtube studies [7] have estimated this exponent to be in the range of $0.5 - 0.6$.

The remainder of this paper is organized as follows: In Section II we setup the problem formulation, defining the popularity distributions and active links. Section III contains our main theorem, the scaling behavior of the average number of D2D interference-free links. After some simulation results, in Section V we discuss future directions, open problems and conclusions.

II. Model and Setup

Consider a cellular network where each cell/base station (BS) serves n users. For simplicity we assume that the cells are square, and neglect inter-cell interference, so that we can consider one cell in isolation. Users are distributed uniformly in the cell. Every user is assumed to have a storage capacity called cache, which is filled up with some video files. The base station (BS) might be aware of the stored files and channel state information of the users and control the D2D communications. We assume that the D2D communication does not interfere with communication between the BS and users. This assumption is justified if the D2D communications occur in a separate frequency band (*e.g.*, WiFi). For the device-to-device throughput, we henceforth do not need to consider explicitly the BS and its associated communications.

Each user in the cell requests a file randomly and independently from a library of size m. The size of the library is a function of the number of users n, since a larger number of users covers a larger set of files they could be interested in. Studies show that there is redundancy in video requests and some popular files are requested more [7]. We assume that the various file popularities are distributed as per the Zipf distribution, which has been established in numerous studies as being a good approximation to the measured popularity of video files [8]. In a Zipf distribution, the frequency of the ith popular file, denoted by f_i, is inversely proportional to its rank:

$$f_i = \frac{\frac{1}{i^{\gamma_r}}}{\sum_{j=1}^{m} \frac{1}{j^{\gamma_r}}}, \quad 1 \le i \le m, \tag{1}$$

The exponent γ_r characterizes the distribution by controlling the relative popularity of files. A large γ_r means a concentrated distribution, *i.e.,* the first few popular files account for the majority of requests. Ref. [7] confirmed the Zipf model for video popularity based on empirical YouTube data and measured the γ_r exponent in the range of $0.5 - 0.6$. In this paper we refer to the range $\gamma_r > 1$ as the *high content reuse* regime and $\gamma_r < 1$ as the *low content reuse* regime.

Each user (peer) has some storage capability that we use to cache video files. For simplicity in our analysis, we assume that all files have the same size, and each user can store one file. This assumption has the advantage of yielding a clean formulation; however, our work can be easily extended to larger cache size.

Our communication network is modeled by a random geometric graph $G(n, r(n))$ where two users (assuming D2D communication is possible) can communicate if their physical distance is smaller than some collaboration distance $r(n)$ [5], [6]. The maximum allowable distance for D2D communication $r(n)$ is determined by the power level for each transmission. Figure 1 illustrates an example of random geometric graph (RGG).

Our model works as follows: if a user requests one of the files stored in neighbors' caches in the RGG, neighbors will handle the request locally through D2D communication; otherwise, the BS should serve the request. Thus, to have D2D communication it is not sufficient that the distance between two users be less than $r(n)$; users should find their desired files locally in caches of their neighbors. Therefore, the probability that D2D communications happen depends on what files are stored by the users.

Caching decisions can be made either in a distributed or centralized way. A central control of the caching by the BS allows very efficient file-assignment to the users [9]. However, if such control is not desired, and/or the users are moving around quickly, then the caching has to be done distributedly and randomly. In other words, each user will cache files according to a probability density function (pdf). In order to avoid the difficulties of optimizing a function, we assumed that the functional form of the caching pdf is also a Zipf distribution, but with a parameter γ_c that can be different from the parameter γ_r. In our previous work, we showed the surprising fact that the optimum γ_c is not equal to γ_r [2].

We assume that all D2D links share the same time-frequency transmission resource within one cell area. This is possible since the distance between requesting user and smartphone with the stored file will be small in most cases. However, there should be no interference of a transmission by others on an active D2D link. We assume that - given that node u wants to transmit to node v - any transmission within range $r(n)$ from v (the receiver) can introduce interference for the $u - v$ transmission. Thus, such two links cannot be activated simultaneously. This model is known as *protocol model*; while it neglects important wireless propagation effects such as fading [10], it can provide fundamental insights and has been widely used in the literature [5].

The problem we investigate is: how does the number of active D2D links scale given that users request files based on a Zipf distribution with exponent γ_r and cache files with exponent γ_c.

III. Analysis

In this section, we investigate the asymptotic behavior of the average number of D2D links that can exist without introducing interference to other D2D links. We consider a dense scenario in which the number of users in the cell n goes to infinity. Recall big O and Ω notations : $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ respectively denote that $|f(n)| \leq cg(n)$ and $|f(n)| \geq cg(n)$ where c is a constant. Besides, according to Knuth's notations if $f(n) = \Theta(g(n))$, it means that $f(n) = O(g(n))$ as well as $f(n) = \Omega(g(n))$. Little-o notation, *i.e.,* $f(x) = o(g(x))$ is equivalent to $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$. Throughout this paper, c is used to denote deterministic positive constants not depending on n.

The scaling law for the high content reuse regime was derived in our recent prior work [4]. We showed that if $\gamma_r > 1$, the expected number of active D2D links scales like

$$E[L] = \Theta(n).$$

However, for the case low content reuse regime, $\gamma_r < 1$, the upper and lower bound scaling laws of [4] have different exponents.

The main result of this paper is a precise characterization of $E[L]$ for the low content reuse regime.

Theorem 1: If $\gamma_r < 1$, $\gamma_c > 1$, and $r(n) = \Theta(\sqrt{\frac{m^b}{n}})$,

$$E[L] = \Theta(\frac{n}{m^{(1-a)(1-\gamma_r)}}),$$

where $a = \frac{b}{\gamma_c}$ and $0 < a < \frac{(1-\gamma_r)}{2\gamma_c+1-\gamma_r}$. This is achieved for any value of the storage exponent $\gamma_c > 1$.
We can see that as a user sees on average more users in its neighborhood (larger a), the number of active D2D links that can coexist increases with n.

Proof: First, we establish the lower bound for $E[L]$. We divide the cell into $\frac{2}{r(n)^2}$ virtual square clusters. Figure 2(a) shows the virtual clusters in the cell. The cell side is normalized to 1 and the side of each cluster is equal to $\frac{r(n)}{\sqrt{2}}$. Thus, all users within clusters can communicate with each other. Besides, based on the protocol model, in each cluster only one link can be activated simultaneously. When there is an active link within a cluster, we call the cluster *good*. But, not all good clusters can be activated simultaneously. One good cluster can at most block 16 clusters (see Figure 2(b)). The maximum interference happens when a user in the corner of a cluster receives a file from another user. Thus, we have

$$E[L] \geq \frac{E[G]}{17}.$$

where $E[G]$ is the expected number of good clusters. Since we want to find the lower bound for $E[L]$, we can limit users

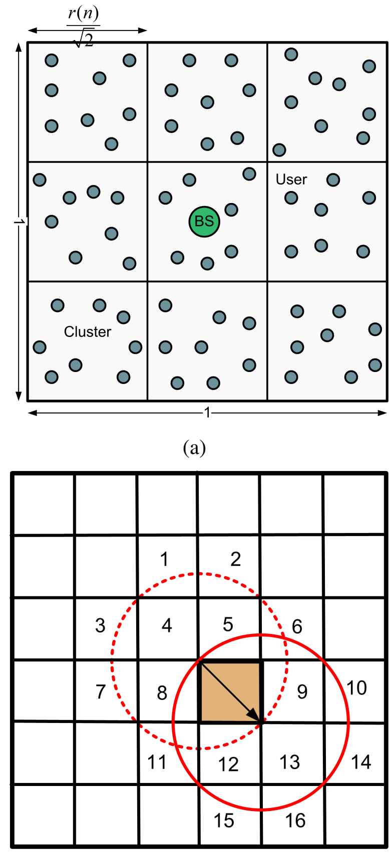

Fig. 2. a) Dividing cell into virtual clusters. b) In the worst case, a good cluster can block at most 16 clusters. In the dashed circle, receiving is not possible and in the solid circle, transmission is not allowed.

to communicate with users in virtual clusters they belong to. Thus, we have:

$$\begin{aligned} E[G] &= \frac{2}{r(n)^2}\sum_{k=0}^{n} \Pr[\text{good}|k]\Pr[K=k] \\ &\geq \frac{2}{r(n)^2}\sum_{k\in A} \Pr[\text{good}|k]\Pr[K=k], \end{aligned} \quad (2)$$

where $A = [nr(n)^2(1-\delta)/2, nr(n)^2(1+\delta)/2]$ and $0 < \delta < 1$. $\frac{2}{r(n)^2}$ is the total number of virtual clusters. K is the number of users in the cluster, which is a binomial random variable with n trials and probability of $\frac{r(n)^2}{2}$, *i.e.,* $K = B(n, \frac{r(n)^2}{2})$. $\Pr[K = k]$ is the probability that there are k users in the cluster and $\Pr[\text{good}|k]$ is the probability that the cluster is good conditioned on k.

Define

$$k^* \triangleq \arg\min_{k\in A} \Pr[\text{good}|k]\Pr[K=k].$$

Notice that k^* is $\Theta(nr(n)^2)$. From (2), we have:

$$E[G] \geq \frac{2}{r(n)^2}\Pr[\text{good}|k^*]\Pr[K\in A] \quad (3)$$

$$\geq \frac{2}{r(n)^2}\Pr[\text{good}|k^*](1-2e^{-nr(n)^2\delta^2/6}). \quad (4)$$

We apply a Chernoff bound to derive equation (4) [11]. From the range of $r(n)$, the second term in (4), *i.e.,* $(1 - 2e^{-nr(n)^2\delta^2/6})$ is $O(1)$. Thus, to prove the theorem, we should show that $\Pr[\text{good}|k^*] \geq c(\frac{nr(n)^2}{m^{(1-a)(1-\gamma_r)}})$.

Let us define an indicator function $\mathbf{1}_u$ for each user within a cluster. $\mathbf{1}_u$ is 1 if user u can find its requested file within the cluster; otherwise it is 0. Since users cache files randomly, the set of available files through D2D links in the cluster is also random. Thus, $\mathbf{1}_u$ and $\mathbf{1}_{u'}$ for $u \neq u'$ are dependent random variables. In that case, given that user u requests according to the some popularity distribution and it cannot find it locally, it is more likely that files currently within the cluster cache are not popular files. So, if user u requests independently from user u' but according to the same popularity distribution, it is more probable that user u also cannot find its request file locally. The cluster is good if at least one of users within the cluster finds its desired file via D2D communication. Thus,

$$\Pr[\text{good}|k^*] = 1 - \Pr[\sum_{u=1}^{k^*} \mathbf{1}_u = 0].$$

Let us define

$$W \triangleq \sum_{u=1}^{k^*} \mathbf{1}_u. \tag{5}$$

By using the Chen Stein method in lemma 3, we show that W can be approximated by a Poisson random variable with average of $\lambda \triangleq E[\sum_{u=1}^{k^*} \mathbf{1}_u]$. We also show that the probability that W is zero is asymptotically equal to $e^{-\lambda}$ where

$$\lambda = k^* E[\mathbf{1}_u] = k^* \sum_{i=1}^{m} f_i (1 - (1 - p_i)^{k^*-1}). \tag{6}$$

p_i is the probability that a user caches file i based on Zipf distribution with exponent γ_c. In the above equation we consider the possibility of self-requests, *i.e.*, a user might find the file it requests in its own cache; in this case clearly no D2D communication will be activated by this user. Then, we have:

$$\Pr[\text{good}|k^*] = 1 - e^{-\lambda}. \tag{7}$$

In lemma 2, we show that $\lambda = \Theta(\frac{nr(n)^2}{m^{(1-a)(1-\gamma_r)}}) = o(1)$. Thus,

$$\Pr[\text{good}|k^*] = \Theta(\lambda) = \Theta(\frac{nr(n)^2}{m^{(1-a)(1-\gamma_r)}}),$$

from which the lower bound follows.

Now, we prove the upper bound. The probability that a user cannot find its desired file locally is the probability that either he has the file itself or all his neighbours do not have the desired files. Thus we have:

$$\begin{aligned} E[L] &\leq n \Pr[\text{no D2D}] \\ &= n \sum_{j=1}^{m} f_j p_j + n \sum_{k=1}^{n} \Pr[K = k] \sum_{j=1}^{m} f_j (1 - (1 - p_j)^k) \\ &\leq n \sum_{j=1}^{m} f_j p_j + n \sum_{k=1}^{n} \Pr[K = k] [\sum_{j=1}^{m^a} f_j + k \sum_{j=m^a+1}^{m} f_j p_j] \\ &\leq n \sum_{j=1}^{m} f_j p_j + n [\sum_{j=1}^{m^a} f_j + \pi n r(n)^2 \sum_{j=m^a+1}^{m} f_j p_j], \end{aligned}$$

where K is the number of a users that communicate. K is a binomial random variable, *i.e.,* $K = B(n, \pi r(n)^2)$. In the second equation above, the first summation is the probability that a user requests a file that he has already stored. The second summation is the probability that the user requests a file and all users in its neighborhood in RGG have not stored the desired file. Using lemma 1, $E[L] = \Omega(\frac{n}{m^{(1-a)(1-\gamma_r)}})$. ■

Lemma 1: i) If $\gamma > 1$ and $a = o(b)$,

$$H(\gamma, a, b) = \Theta(\frac{1}{a^{\gamma-1}}).$$

ii) If $\gamma < 1$, $a = o(b)$, and $a = \Theta(1)$.,

$$H(\gamma, a, b) = \Theta(b^{1-\gamma}).$$

iii) If $\gamma_r < 1$, $\gamma_c > 1$,

$$\sum_{j=2}^{m} f_j p_j = \Theta(\frac{1}{m^{1-\gamma_r}}),$$

where the harmonic function $H(\gamma, a, b) = \sum_{j=a}^{b} \frac{1}{i^\gamma}$.

Proof: We first prove the parts i and ii of the lemma. $\frac{1}{x^\gamma}$ is monotonically decreasing. Thus,

$$H(\gamma, a, b) \geq \int_{x=a}^{b} \frac{1}{x^\gamma} = \frac{b^{(-\gamma+1)} - a^{-\gamma+1}}{-\gamma + 1}. \tag{8}$$

We also have the following inequality:

$$\begin{aligned} H(\gamma, a, b) - \frac{1}{a^\gamma} &= \sum_{j=a+1}^{b} \frac{1}{j^\gamma} \\ &\leq \int_{x=a}^{b} \frac{1}{x^\gamma} = \frac{b^{(-\gamma+1)} - a^{(-\gamma+1)}}{-\gamma + 1}. \end{aligned} \tag{9}$$

Thus, $H(\gamma, a, b)$ satisfies:

$$\frac{b^{(-\gamma+1)} - a^{-\gamma+1}}{-\gamma + 1} \leq H(\gamma, a, b) \leq \frac{b^{(-\gamma+1)} - a^{-\gamma+1}}{-\gamma + 1} + \frac{1}{a^\gamma}. \tag{10}$$

Therefore, if $\gamma > 1$, $H(\gamma, a, b) = \Theta(\frac{1}{a^{\gamma-1}})$. Besides, if $\gamma < 1$ and $a = \Theta(1)$, then $H(\gamma, a, b) = \Theta(b^{1-\gamma})$.

For part iii, using (1), we have:

$$\sum_{j=2}^{m} f_j p_j = \frac{\sum_{j=2}^{m} \frac{1}{j^{\gamma_r+\gamma_c}}}{\sum_{j=1}^{m} \frac{1}{j^{\gamma_r}} \sum_{j=1}^{m} \frac{1}{\gamma_c}} = \frac{H(\gamma_c+\gamma_r, 2, m)}{H(\gamma_c, 1, m) H(\gamma_r, 1, m)}, \quad (11)$$

When $\gamma_c > 1$ and $\gamma_r < 1$, $H(\gamma_c+\gamma_r, 2, m)$ and $H(\gamma_c, 1, m)$ are both $\Theta(1)$ and $H(\gamma_r, 1, m) = \Theta(m^{1-\gamma_r})$ which follows the result.

■

Lemma 2: If $\gamma_c > 1$, $\gamma_r < 1$, $k = \Theta(m^b)$,

$$p_u \triangleq E[\mathbf{1}_u] = \Theta(\frac{1}{m^{(1-a)(1-\gamma_r)}}) \quad (12)$$

$$p_{uu'} \triangleq E[\mathbf{1}_u \mathbf{1}_{u'}] = O(\frac{1}{m^{(1-a)(1-\gamma_r)}}), \quad (13)$$

where $a = \frac{b}{\gamma_c}$.

Proof: We first prove the first part of the lemma. $E[\mathbf{1}_u]$ is given in (6).

$$\begin{aligned} E[\mathbf{1}_u] &= \sum_{j=1}^{m^a} f_j (1-(1-p_j)^{k-1}) \\ &+ \sum_{j=m^a+1}^{m} f_j (1-(1-p_j)^{k-1}) \\ &\leq \sum_{j=1}^{m^a} f_j + (k-1) \sum_{j=m^a+1}^{m} f_j p_j \\ &\leq \frac{H(\gamma_r, 1, m^a)}{H(\gamma_r, 1, m)} + (k-1) \frac{H(\gamma_c+\gamma_r, m^a+1, m)}{H(\gamma_r, 1, m) H(\gamma_c, 1, m)}, \end{aligned} \quad (14)$$

where the harmonic function H is defined in lemma 1. If we apply the results of lemma 1, we can show that both terms in (14) are $\Theta(\frac{1}{m^{(1-a)(1-\gamma_r)}})$.

For the lower-bound, we have:

$$\begin{aligned} E[1_u] &\geq \sum_{j=1}^{m^a} f_j (1-(1-p_j)^{k-1}) \\ &\geq \sum_{j=1}^{m^a} f_j (1-e^{-(k-1)p_j}). \end{aligned}$$

Using lemma 1, we know $p_j = \Theta(\frac{1}{j^{\gamma}})$. Then, considering that $j = O(m^a)$, the exponent $(k-1)p_j$ is $\Omega(1)$. Therefore,

$$\begin{aligned} E[1_u] &\geq c_1 \sum_{j=1}^{m^a} f_j \\ &= \frac{H(\gamma_r, 1, m^a)}{H(\gamma_r, 1, m)} = \Theta(\frac{1}{m^{(1-a)(1-\gamma_r)}}). \end{aligned} \quad (15)$$

Now, we show the second part of the lemma.

$$\begin{aligned} p_{uu'} &= \Pr[\mathbf{1}_u = 0 \text{ and } \mathbf{1}_{u'} = 0] \\ &= \sum_{i=1}^{m} \sum_{j=1, i\neq j}^{m} f_i f_j (1-(1-p_i-p_j)^{k-1}) \\ &+ \sum_{i=1}^{m} {f_i}^2 (1-(1-p_i)^{k-1}) \\ &\leq \sum_{i=1}^{m} \sum_{j=1}^{m} f_i f_j (1-(1-p_i-p_j)^k) \\ &\leq \sum_{i=1}^{m^a} \sum_{j=1}^{m^a} f_i f_j + 2 \sum_{i=1}^{m^a} \sum_{j=m^a+1}^{m} f_i f_j \\ &+ \sum_{i=m^a+1}^{m} \sum_{j=m^a+1}^{m} f_i f_j k (p_i + p_j) \\ &\leq \big(\sum_{i=1}^{m^a} f_i\big)^2 + 2 \sum_{i=1}^{m^a} \sum_{j=m^a+1}^{m} f_i f_j + 2k \sum_{i=m^a+1}^{m} f_i p_i. \end{aligned} \quad (16)$$

Then, we have:

$$\begin{aligned} p_{uu'} &\leq \frac{H(\gamma_r, 1, m^a)^2}{H(\gamma_r, 1, m)^2} + 2 \frac{H(\gamma_r, 1, m^a) H(\gamma_r, m^a+1, m)}{H(\gamma_r, 1, m)^2} \\ &+ 2k \frac{H(\gamma_c+\gamma_r, m^a+1, m)}{H(\gamma_r, 1, m) H(\gamma_c, 1, m)}. \end{aligned} \quad (17)$$

According to the scaling results of the lemma 1, the first, the second and the third terms are respectively $\Theta(\frac{1}{m^{2(1-a)(1-\gamma_r)}})$, $\Theta(\frac{1}{m^{(1-a)(1-\gamma_r)}})$ and $\Theta(\frac{1}{m^{(1-a)(1-\gamma_r)}})$. Thus, $p_{u,u'} = O(\frac{1}{m^{(1-a)(1-\gamma_r)}})$.

■

Lemma 3: W defined in (5) is asymptotically a Poisson random variable with mean $\lambda = E[W]$ and $\Pr[W = 0]$ asymptotically equals $e^{-\lambda}$ if $k = \Theta(m^{a\gamma_c})$ where $0 < a < \frac{(1-\gamma_r)}{2\gamma_c+1-\gamma_r}$.

Proof: To prove the lemma, we use the Chen Stein method (the following theorem).

Theorem 2: [12] Let W be the number of occurrence of dependent events, and Z be a Poisson random variable with $E[Z] = E[W] = \lambda$. Then

$$||\mathcal{D}(W) - \mathcal{D}(Z)|| \leq 2(b_1 + b_2), \quad (18)$$

and

$$|\Pr(W=0) - e^{-\lambda}| < (1 \wedge \lambda^{-1})(b_1 + b_2), \quad (19)$$

where $||\mathcal{D}(W) - \mathcal{D}(Z)||$ is the total variation distance between the distribution of W and Z, $1 \wedge \lambda^{-1}$ is the minimum of 1 and λ^{-1}, and

$$b_1 = \sum_{u=1}^{k} \sum_{u'=1}^{k} p_u p_{u'} \quad (20)$$

$$b_2 = \sum_{u=1}^{k} \sum_{u'=1}^{k} p_{uu'}. \quad (21)$$

p_u and $p_{uu'}$ are respectively defined in (12) and (13). Using the results of lemma 2

$$b_1 = k^2 \Theta(\frac{1}{m^{2(1-a)(1-\gamma_r)}}) \tag{22}$$

$$b_2 = k^2 O(\frac{1}{m^{(1-a)(1-\gamma_r)}}). \tag{23}$$

Then, from (18), W is asymptotically a Poisson random variable if $b_1 + b_2 = o(1)$ which implies that k should be $o(m^{(1-a)(1-\gamma_r)/2})$. Considering that $k = \Theta(m^{a\gamma_c})$, we can conclude that a in the lemma should be less than $\frac{(1-\gamma_r)}{2\gamma_c+1-\gamma_r}$. Beside, from (19), we can see when $b_1 + b_2 = o(1)$, the probability that W is zero asymptotically approaches $e^{-\lambda}$. ■

IV. Simulation results

In this section, we provide some numerical results to verify our theoretical analyses. We consider there are n active users which are randomly distributed across the entire cell. There is a library of size $m = 10^4 \log(n)$. For the simulation, we use Monte Carlo method. In every iteration, we place n users randomly in the cell. Each user stores/requests one file randomly by sampling from the Zipf distribution with exponents γ_c/γ_r. Based on that, we determine all good D2D links and maximum number of D2D links that can be activated simultaneously without introducing interference for each other.

Figure 3 illustrates the average number of active D2D links versus the number users in the cell with $\gamma_c = 1.2$ and for different value of γ_r. The parameter a in the theorem is 0, *i.e.,* $r(n) = 10\sqrt{\frac{1}{n}}$. We can see that the theoretical results match the simulations. As expected, for larger γ_r (more content re-use) there exist more non interfering D2D links.

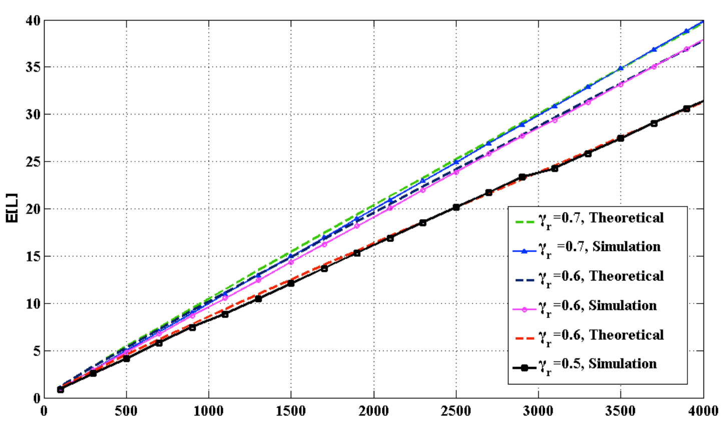

Fig. 3. The average number of active D2D links versus the number users in the cell, $\gamma_c = 1.2$ and $a = 0$.

V. Conclusions

We discussed a novel scheme for increasing the efficiency of video content delivery in cellular communications systems. We found the scaling behavior of number of interference-free D2D links as a function of the statistics of requests, the statistics of the caching, and the collaboration distance.

Simulation results demonstrate that the "scaling law", though defined for large node density, is experimentally verified for 100 mobiles per cell, which is a realistic scale for today's systems. We furthermore find that even though the scaling behavior of the number of non-interfering collaborative links grows sub-linearly with the number of users, the deviation from linearity is small for a wide range of parameters γ_r and γ_c. This theoretically justifies what we would intuitively expect: device-to-device video sharing, enabled by local storage can bring tremendous benefits in future wireless systems.

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