Receive Antenna Selection with Imperfect Channel Knowledge From Training

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***Abstract*—Receive antenna selection (AS), in which a subset of a receiver's antennas, is selected based on the instantaneous channel condition, enables the receiver to benefit from having multiple antenna elements despite using significantly less hardware. However, the limited hardware resources also impose unique constraints on how channel state information, which is necessary to select the antennas and decode data, can be acquired for AS. We derive novel closed-form expressions for the symbol error probability of MPSK and MQAM over time-varying Rayleigh fading channels using AS. Our analysis explicitly considers a practical and general training model that accounts for noisy channel estimates and training delays that affect both selection and data decoding. Our results show that training plays a crucial role in determining the overall performance, and can even limit the number of receive antennas that can gainfully be employed in the system.**

I. Introduction

Many next generation wireless communication systems employ multiple receive antennas to exploit the benefits of spatial diversity. However, issues such as increased power consumption and multiple radio frequency (RF) chains required to process signals from multiple antenna elements, make it infeasible to support many receive antennas.

Receive antenna selection (AS) is a popular technique that addresses this problem by only processing the signals from a dynamically selected subset of available antennas. It greatly reduces the cost of a transceiver since antenna elements are typically cheap and easy to implement, while RF chains are expensive. Despite its lower hardware complexity, AS provably achieves the maximum diversity order. Considerable work has also appeared in the literature on its capacity benefits and on antenna selection criteria (see [1], [2] for a detailed set of references). However, most of these papers assume perfect channel state information (CSI) at the receiver.

In practice, the CSI needs to be acquired using a pilot-based training scheme. The low hardware complexity, which is a key motivator for AS, also imposes unique constraints on how training gets done for AS. This is because, given the limited number of RF chains, only a subset of antennas can be activated at any instant. This implies that the transmitter needs to transmit pilots multiple times to enable the receiver to receive the signals with different antennas and estimate their corresponding links to the transmitter. The receive antenna is then selected based on these estimates. For example, in the AS training scheme used in IEEE 802.11n wireless local area network standard, pilots (and packets) for reception by different receive antennas are transmitted at different time instants, which can be several tens of milliseconds apart [3]. Thus, the CSI at the receiver is imperfect because of noise in the channel estimates and also because of training delays, which are different for different antennas. Imperfect CSI leads to inaccurate selection and imperfect data decoding, both of which effect the symbol error probability (SEP).

In this paper, we analyze the performance of single receive antenna selection in time-varying Rayleigh fading channels with imperfect channel estimation. Even single antenna selection is relevant because it achieves the full diversity order under perfect CSI. Under this general and practical set up, we develop closed-form expressions for the SEP of AS for MPSK and MQAM constellations. Our results show that the training delays lead to error floors, which do not occur for perfect CSI or even noisy channel estimates. Interestingly, our results show that the training delays make it futile to increase the number of receive antennas beyond a point. While [4], [9], and [10] considered channel estimation errors, they assumed either a time-invariant channel model or did not account for the different training delays of different antennas.

The paper is organized as follows. The system model is developed in Sec. II, followed by analysis in Sec. III. The results are verified using simulations in Sec. IV. Our conclusions follow in Sec. V.

II. Model

Consider a system with one transmit antenna, N_r receive antennas, and one RF chain at the receiver. Let $h_k(t)$ denote the frequency-flat channel between the transmitter and the k^{th} receive antenna at time t. It is modeled as a circularly symmetric complex Gaussian random variable (RV) with unit variance. Furthermore, the channel gains for different receive antennas are assumed to be independent and identically distributed (i. i. d.), which is the case when the receive antennas are spaced sufficiently apart.

As shown in Figure 1, the transmitter transmits pilot symbol p_k (of duration T_s) to enable the receiver to estimate the channel between the transmitter and k^{th} receive antenna at time T_k. Two consecutive pilot symbols are separated in time by a duration T_p. Note that the order in which antennas are trained does not matter since the channel gains of different antennas are i. i. d. The signal received by antenna k is therefore

$$r_k(T_k) = p_k h_k(T_k) + n_k(T_k),$$

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where $n_k(t)$ is an ergodic stationary Gaussian process with zero mean and power N_0 that is independent of $h_k(t)$. Therefore, the channel estimate for the k^{th} receive antenna is

$$\hat{h}_k(T_k) = \frac{p_k^* r_k(T_k)}{|p_k|^2} = h_k(T_k) + e_k, \tag{1}$$

where, the estimation error $e_k = \frac{n_k(T_k)}{|p_k|^2}$ has a variance $\sigma_e^2 = \frac{N_0}{E_p}$. Here E_p denotes the pilot symbol energy.

Rearranging the estimated channel gains for the N_r antennas in decreasing order of their absolute values, we get $\left|\hat{h}_{[\hat{1}]}\left(T_{[\hat{1}]}\right)\right| > \left|\hat{h}_{[\hat{2}]}\left(T_{[\hat{2}]}\right)\right| > \cdots > \left|\hat{h}_{[\hat{N_r}]}\left(T_{[\hat{N_r}]}\right)\right|$, where $[\hat{i}]$ denotes the index of the antenna with the i^{th} largest gain. Antenna $[\hat{1}]$ is used for receiving all the data symbols.

The pilots are followed by D data symbols, each of duration T_s and average energy E_s. When the i^{th} data symbol, s_i, is transmitted, the signal received by antenna $[\hat{1}]$ at time t_i, after matched filtering, is given by

$$y_{[\hat{1}]}(t_i) = h_{[\hat{1}]}(t_i)s_i + n_{[\hat{1}]}(t_i). \tag{2}$$

The data symbols belong to a finite-sized constellation. (We shall assume that all the constellation symbols are equi-probable.) For example, for MPSK, s_i belongs to the set $\left\{\sqrt{E_s}\exp(\frac{j2\pi m}{M}),\ m = 0, 1, \ldots, M-1\right\}$. For MQAM, the data symbol, $s_i = \sqrt{\frac{3E_s}{2(M-1)}}(a_I + ja_Q)$, where a_I, a_Q belongs to the set $\{2i - 1 - \sqrt{M}, i = 1, 2, \ldots, \sqrt{M}\}$.

Due to the time-varying nature of the wireless links, the N_r channels will have changed by the time data transmission starts. Specifically, the channel for receive antenna i at time $t + \delta$ can be written in terms of the channel at time t as [8]

$$h_i(t + \delta) = \rho_i(\delta)h_i(t) + \sqrt{1 - |\rho_i(\delta)|^2}\, n_i'(t + \delta), \tag{3}$$

where $\rho_i(\delta)$ is the channel correlation coefficient, which depends only on the time difference δ. The variation $n_i'(t+\delta)$, $i = 1, 2, \ldots, N_r$, is a circularly symmetric complex Gaussian RV with unit variance that is independent of $h_i(t)$. From the Jakes' model [5], which we use for illustrative purposes in this paper, we have

$$\rho_i(\delta) = J_0(2\pi f_d \delta), \tag{4}$$

where $J_0(.)$ is the zeroth order Bessel function of the first kind [6] and f_d is the Doppler spread.

III. SEP of MPSK and MQAM

We now analyze the SEP for MPSK and MQAM for receive antenna selection with imperfect and outdated CSI. Henceforth, we simplify our notation as follows: we denote $\hat{h}_k(T_k)$ by $\hat{h}_k$, $n_k'(t)$ by n_k', and $n_k(t)$ by n_k. Let $\mathbf{E}[A]$ and $\mathbf{var}[A]$ denote the expectation and variance of event A, respectively. Similarly, $\mathbf{E}[A|B]$ and $\mathbf{var}[A|B]$ shall denote the conditional expectation and variance of A given B, respectively.

The selected antenna $[\hat{1}]$ is used to receive the entire data packet. Therefore, the decision variable, $\mathcal{D}$, for decoding s_i is

$$\mathcal{D} = \hat{h}_{[\hat{1}]}^* y_{[\hat{1}]}(t_i).$$

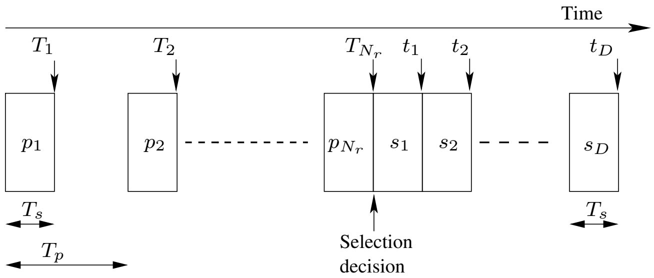

Fig. 1. Training for antenna selection

Note that the same imperfect channel estimate used for antenna selection is also used for data decoding. Using (1) and (3), we can write the channel at time t_i in terms of its estimate. Hence,

$$\mathcal{D} = \hat{h}_{[\hat{1}]}^*\left(\rho_{[\hat{1}]}^{(i)}\left(\hat{h}_{[\hat{1}]} - e_{[\hat{1}]}\right)s_i + \sqrt{1 - \left|\rho_{[\hat{1}]}^{(i)}\right|^2}\, n_{[\hat{1}]}' s_i + n_{[\hat{1}]}\right), \tag{5}$$

where $\rho_i^{(j)}$ denotes $\rho_i(t_j - T_i)$. This expression also shows that due to the time-varying nature of the channel, symbols transmitted at different times experience different error rates.

We first state the following Lemma about the statistics of $\mathcal{D}$, which shall be useful in the rest of the paper.

Lemma *1:* Conditioned on $\{\hat{h}_i\}_{i=1}^{N_r}$ and s_i, $\mathcal{D}$ is a complex Gaussian RV with conditional mean and variance given by

$$\mathbf{E}\left[\mathcal{D}\,\middle|\,\{\hat{h}_i\}_{i=1}^{N_r}, s_i\right] = \left|\hat{h}_{[\hat{1}]}\right|^2 \rho_{[\hat{1}]}^{(i)} s_i q^2, \tag{6}$$

$$\begin{aligned}\mathbf{var}\left[\mathcal{D}\,\middle|\,\{\hat{h}_i\}_{i=1}^{N_r}, s_i\right] &= \left(1 - \left|\rho_{[\hat{1}]}^{(i)}\right|^2\right)|s_i|^2\left|\hat{h}_{[\hat{1}]}\right|^2 \\ &+ \left|\hat{h}_{[\hat{1}]}\right|^2\left|\rho_{[\hat{1}]}^{(i)}\right|^2|s_i|^2\sigma_e^2 q^2 + \left|\hat{h}_{[\hat{1}]}\right|^2 N_0,\end{aligned} \tag{7}$$

where $q^2 \triangleq 1/(1 + \sigma_e^2)$.

Proof: The proof is given in the Appendix. ■

We are now ready to derive the SEP for MPSK and MQAM in the following two theorems.

Theorem *1:* With training delays and imperfect channel estimation, the SEP for an MPSK symbol s_i received at time t_i is given by

$$\begin{aligned}P_i^{\text{MPSK}}(\gamma) &= \frac{M-1}{M} + \frac{1}{N_r\pi}\sum_{l=0}^{N_r-1}(-1)^{l+1}\binom{N_r}{l+1} \\ &\times\left[\sum_{k=1}^{N_r}\frac{1}{\sqrt{\alpha_{k,l}^{(i)}(\gamma)}}\tan^{-1}\left(\sqrt{\alpha_{k,l}^{(i)}(\gamma)}\tan\left(\frac{M-1}{M}\pi\right)\right)\right],\end{aligned} \tag{8}$$

where $\gamma \triangleq \frac{E_s}{N_0}$ is the average SNR per branch, $\varepsilon \triangleq \frac{E_p}{E_s}$, and

$$\alpha_{k,l}^{(i)}(\gamma) \triangleq 1 + \frac{(l+1)/\varepsilon}{\left|\rho_k^{(i)}\right|^2\sin^2(\frac{\pi}{M})}\left(\varepsilon\left(1 - \left|\rho_k^{(i)}\right|^2\right) + \frac{1+\varepsilon}{\gamma} + \frac{1}{\gamma^2}\right). \tag{9}$$

Proof: Using Lemma 1 and [7, eq.(40)], the SEP for

MPSK, conditioned on $\{\hat{h}_i\}_{i=1}^{N_r}$ is given by

$$P_i\left(\text{Err}\,\Big|\,\{\hat{h}_i\}_{i=1}^{N_r}\right)=\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\exp\left(\frac{-\left|\hat{h}_{[\hat{1}]}\right|^2 b_{[\hat{1}]}^{(i)}}{\sin^2\theta}\right)d\theta, \qquad (10)$$

where $b_k^{(i)} \triangleq \frac{E_s\left|\rho_k^{(i)}\right|^2 q^4 \sin^2\left(\frac{\pi}{M}\right)}{\left|\rho_k^{(i)}\right|^2 E_s\sigma_e^2 q^2+N_0+\left(1-\left|\rho_k^{(i)}\right|^2\right)E_s}$.

From (1), the probability density function, $f(x)$, and cumulative distribution function, $F(x)$, of $\left|\hat{h}_k\right|^2$ are given by $f(x)=\frac{1}{1+\sigma_e^2}\exp\left(\frac{-x}{1+\sigma_e^2}\right)$ and $F(x)=1-\exp\left(\frac{-x}{1+\sigma_e^2}\right)$.

Therefore, the SEP averaged over $\{\hat{h}_i\}_{i=1}^{N_r}$ equals

$$P_i\left(\text{Err}\right)=\sum_{k=1}^{N_r}\frac{1}{\pi}\int_0^{\infty}\int_0^{\frac{M-1}{M}\pi}\exp\left(\frac{-x b_k^{(i)}}{\sin^2\theta}\right) \times f(x)\left(F(x)\right)^{N_r-1}d\theta dx. \qquad (11)$$

Now, $\left(F(x)\right)^{N_r-1}=\sum_{l=0}^{N_r-1}(-1)^l\binom{N_r-1}{l}\exp\left(\frac{-lx}{1+\sigma_e^2}\right)$.

Substituting the above expansion in (11) and simplifying further we get,

$$P_i\left(\text{Err}\right)=\frac{1}{\pi(1+\sigma_e^2)}\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}(-1)^l\binom{N_r-1}{l} \times\int_0^{\frac{M-1}{M}\pi}\left(\frac{b_k^{(i)}}{\sin^2\theta}+\frac{l+1}{1+\sigma_e^2}\right)^{-1}d\theta.$$

This result can be further simplified by using the following identity, which follows from [6, eq. (2.562)]:

$$\int_0^{\frac{M-1}{M}\pi}\left(\frac{b_k^{(i)}}{\sin^2\theta}+\frac{l+1}{1+\sigma_e^2}\right)^{-1}d\theta\equiv\frac{1}{c_l}\left(\frac{M-1}{M}\pi\right) -\frac{1}{c_l}\sqrt{\frac{b_k^{(i)}}{b_k^{(i)}+c_l}}\tan^{-1}\left(\sqrt{\frac{b_k^{(i)}+c_l}{b_k^{(i)}}}\tan\left(\frac{M-1}{M}\pi\right)\right),$$

where $c_l \triangleq \frac{l+1}{1+\sigma_e^2}$. This leads to

$$P_i\left(\text{Err}\right)=\frac{1}{N_r\pi}\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}(-1)^l\binom{N_r}{l+1}\left[\frac{M-1}{M}\pi- \sqrt{\frac{b_k^{(i)}}{b_k^{(i)}+c_l}}\tan^{-1}\left(\sqrt{\frac{b_k^{(i)}+c_l}{b_k^{(i)}}}\tan\left(\frac{M-1}{M}\pi\right)\right)\right].$$

Finally, using two additional identities $\frac{b_k^{(i)}+c_l}{b_k^{(i)}}\equiv\alpha_{k,l}^{(i)}$ and $\frac{1}{N_r}\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}(-1)^l\binom{N_r}{l+1}\equiv 1$, proves the theorem. ■

Theorem *2:* With training delays and imperfect channel estimation, the SEP for an MQAM symbol s_i received at time t_i is given by

$$P_i^{\text{MQAM}}(\gamma)=\frac{M-1}{M}-\frac{4}{N_r\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2 \times\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}\frac{(-1)^{l+1}}{\sqrt{\beta_{k,l}^{(i)}(\gamma)}}\binom{N_r}{l+1}\tan^{-1}\left(\sqrt{\beta_{k,l}^{(i)}(\gamma)}\right) +\frac{2}{N_r}\left(1-\frac{1}{\sqrt{M}}\right)\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}\frac{(-1)^{l+1}}{\sqrt{\beta_{k,l}^{(i)}(\gamma)}}\binom{N_r}{l+1}, \qquad (12)$$

where $\gamma\triangleq\frac{E_s}{N_0}$ is the average SNR per branch, $\varepsilon\triangleq\frac{E_p}{E_s}$, and

$$\beta_{k,l}^{(i)}(\gamma)\triangleq 1+\frac{(l+1)}{\varepsilon\left|\rho_k^{(i)}\right|^2\left(\frac{3}{2(M-1)}\right)}\left(\varepsilon\left(1-\left|\rho_k^{(i)}\right|^2\right)+\frac{1+\varepsilon}{\gamma}+\frac{1}{\gamma^2}\right). \qquad (13)$$

Proof: Using Lemma 1 and [7, eq. (48)], the SEP for MQAM, conditioned on $\{\hat{h}_i\}_{i=1}^{N_r}$, is given by

$$P_i\left(\text{Err}|\{\hat{h}_i\}_{i=1}^{N_r}\right)=\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\exp\left(\frac{-\left|\hat{h}_{[\hat{1}]}\right|^2 c_{[\hat{1}]}^{(i)}}{\sin^2\theta}\right)d\theta -\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}}\exp\left(\frac{-\left|\hat{h}_{[\hat{1}]}\right|^2 c_{[\hat{1}]}^{(i)}}{\sin^2\theta}\right)d\theta,$$

where $c_k^{(i)} \triangleq \frac{E_s\left|\rho_k^{(i)}\right|^2 q^4\left(\frac{3}{2(M-1)}\right)}{\left|\rho_k^{(i)}\right|^2 E_s\sigma_e^2 q^2+N_0+\left(1-\left|\rho_k^{(i)}\right|^2\right)E_s}$. Using steps similar to Theorem 1, we can derive $P_i^{\text{MQAM}}(\gamma)$. ■

A. Asymptotic Behavior of SEP

Let us now consider the asymptotic behavior of our SEP expressions in (8) and (12) as the average SNR per branch, γ, increases. Let $P_{i,\text{asym}}^{\text{MPSK}}\triangleq\lim_{\gamma\to\infty}P_i^{\text{MPSK}}(\gamma)$ and $P_{i,\text{asym}}^{\text{MQAM}}\triangleq\lim_{\gamma\to\infty}P_i^{\text{MQAM}}(\gamma)$.

We first consider the case where training delays are absent, *i.e.*, $\rho_k^{(i)}=1$, for all k,i. From (9) and (13), we can see that $\lim_{\gamma\to\infty}\alpha_{k,l}^{(i)}(\gamma)=1$ and $\lim_{\gamma\to\infty}\beta_{k,l}^{(i)}(\gamma)=1$, for all k,l, and i. For MPSK, using a binomial series identity in (8), we get

$$P_{i,\text{asym}}^{\text{MPSK}}=\frac{M-1}{M}\left(1+\frac{1}{N_r}\sum_{k=1}^{N_r}\sum_{l=0}^{N_r-1}(-1)^{l+1}\binom{N_r}{l+1}\right)\equiv 0.$$

Similarly for MQAM, we can also show from (12) that $P_{i,\text{asym}}^{\text{MQAM}}\equiv 0$. The above results are consistent with those in [4].

Now consider the case of non-zero training delays, where $0<\rho_k^{(i)}<1$, for all k and i. From (9) and (13), we see that

$$\lim_{\gamma\to\infty}\alpha_{k,l}^{(i)}(\gamma)\triangleq\alpha_{k,l}^{(i)}=1+\frac{(l+1)\left(1-\left|\rho_k^{(i)}\right|^2\right)}{\left|\rho_k^{(i)}\right|^2\sin^2\left(\frac{\pi}{M}\right)},$$

$$\lim_{\gamma\to\infty}\beta_{k,l}^{(i)}(\gamma)\triangleq\beta_{k,l}^{(i)}=1+\frac{(l+1)\left(1-\left|\rho_k^{(i)}\right|^2\right)}{\left|\rho_k^{(i)}\right|^2\left(\frac{3}{2(M-1)}\right)}.$$

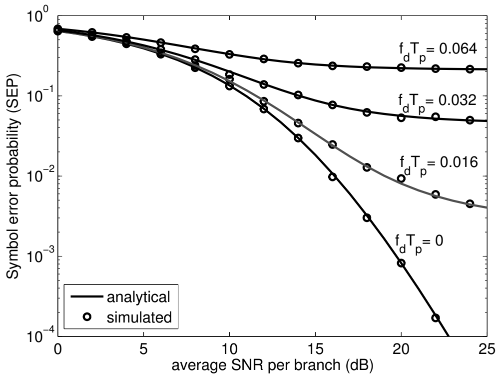

Fig. 2. Effect of normalized doppler spread (8PSK and $N_r = 4$).

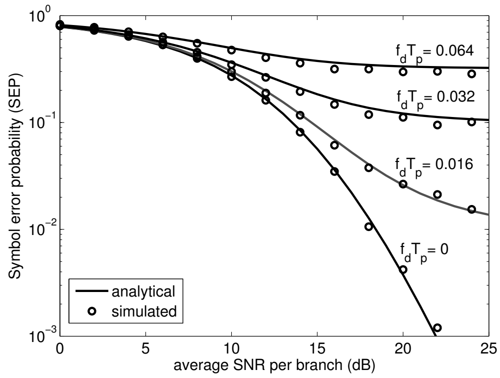

Fig. 3. Effect of normalized doppler spread (16QAM and $N_r = 4$).

Replacing $\alpha_{k,l}^{(i)}(\gamma)$ and $\beta_{k,l}^{(i)}(\gamma)$ with $\alpha_{k,l}^{(i)}$ and $\beta_{k,l}^{(i)}$ respectively, it can be seen that $P_{i,\text{asym}}^{\text{MPSK}}$ and $P_{i,\text{asym}}^{\text{MQAM}}$ are not identically 0. Hence, an irreducible error floor exists at high SNR, with its value depending on the correlations $\{\rho_k^{(i)}\}_{k=1}^{N_r}$.

Thus far, we have analyzed the fading-averaged SEP. Note that the uncoded fading-averaged packet error rate (PER) cannot be directly determined from the fading-averaged SEPs of the D different symbols. This is because the same antenna is used for decoding all the D data symbols of a packet. However, when conditioned on $\{\hat{h}_i\}_{i=1}^{N_r}$, the symbol errors are independent and are each given by (10), albeit with different correlations ($\rho_k^{(i)}$), which depend on the time instants when the data symbols are transmitted. The resulting expression for the PER can be unconditioned on the channel gains and simplified along the lines presented in this paper. However, due to space constraints, we do not analyze PER in this paper.

IV. Simulations

We now present graphically the results derived in Sec. III and study the effect of N_r and f_dT_p on the SEP. We also compare our analytical results with Monte Carlo simulations (with 10^4 samples being generated for each SNR), which use the simulator of [11] to generate the time-varying Rayleigh channels. From (4), the correlation values for $k = 1, 2, \ldots, N_r$, equal $\rho_k^{(i)} = J_0\left(2\pi f_d((N_r - k)T_p + iT_s)\right)$. The figures are plotted for $T_p = 10\ T_s$ and $E_p = E_s$. Unless mentioned otherwise, the SEP of the first data symbol is plotted.

Figures 2 and 3 plot the SEP as a function of the SNR for MPSK and MQAM, respectively, for $N_r = 4$ and different values of the normalized Doppler spread f_dT_p. In both figures, one can see that the SEP decreases to 0 as the SNR increases when $f_dT_p = 0$, as discussed in Sec. III-A. On the other hand, for both MSPK and MQAM, an error floor exists when $f_dT_p > 0$; it increases as f_dT_p increases. Notice the excellent match between analytical and simulation results. Therefore, we only plot the analytical results henceforth.

We now consider the effect of training delays on the SEP as the number of receive antennas increases. Figure 4 compares performance of $N_r = 2$, 4, 8, and 16 receive antennas as a function of SNR at $f_dT_p = 0.016$. We can see that more receive antennas is no longer always better. In fact, for larger f_dT_p, $N_r = 2$ turns out to be the best choice. This is because, at a fixed T_p, the greater the number of receive antennas, the longer is the training delay. Hence, the correlation between the channels seen at the time of channel estimation and data transmission is lower. This figure can be better understood by considering an idealistic scenario, plotted in Figure 5, in which the total training period (N_rT_p) is kept constant regardless of N_r. (This can be achieved in theory by adjusting T_p according to N_r.) Not surprisingly, the SEP does decrease as N_r increases.

An alternate view is presented in Figure 6, which compares the performance of $N_r = 2$, 4, 8, and 16 antennas at an SNR of 10 dB as a function of the normalized Doppler spread. (As in Figure 4, T_p is fixed.) It can be seen that for $0 \leq f_dT_p \leq 0.007$, $N_r = 16$ outperforms others. However, for $0.007 \leq f_dT_p \leq 0.017$, $0.017 \leq f_dT_p \leq 0.044$, and $f_dT_p \geq 0.044$, $N_r = 8$, $N_r = 4$, and $N_r = 2$, respectively, are the best choices. (Similar behavior was observed at other values of SNR too.)

The effect of different training delays for different antennas is studied in Figure 7. $\rho_k = \rho_{min}$ corresponds to all antennas experiencing a maximum training delay of $(N_r - 1)T_p + T_s$ and $\rho_k = \rho_{max}$ corresponds to all antennas experiencing a minimum training delay of T_s. The effect of different training delays is more significant at higher values of SNR.

V. Conclusions

For practical training scenarios encountered for single receive AS, we developed closed-form expressions for the SEP of MPSK and MQAM that accounted for both noisy channel estimates and AS training delays. Our analysis showed that noisy channel estimates impact the performance differently from imperfections due to training delays. When both these imperfections are considered together, the SEP exhibits an error floor, which increases as the normalized Doppler spread increases. As the number of receive antennas increases, the training delays also increase, which negatively impacts performance. Consequently, for higher Doppler spreads, fewer antennas to train over and select from may actually be better.

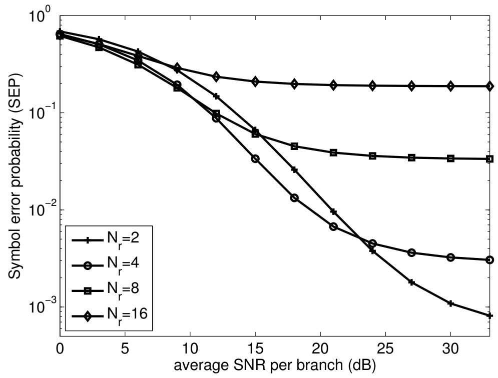

Fig. 4. Effect of number of receive antennas for fixed Doppler spread and T_p (8PSK and $f_d T_p = 0.016$).

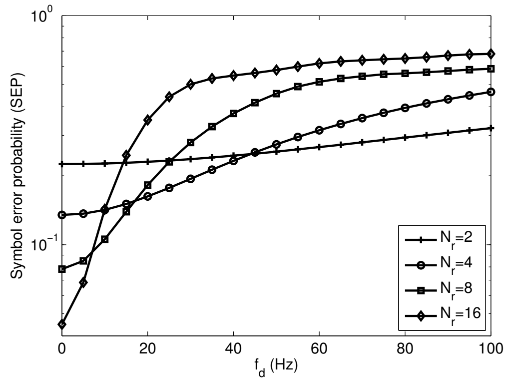

Fig. 6. Effect of Doppler spread and number of receive antennas when sounding interval between adjacent antennas is kept fixed (8PSK, $T_p = 1$ms, and 10 dB SNR).

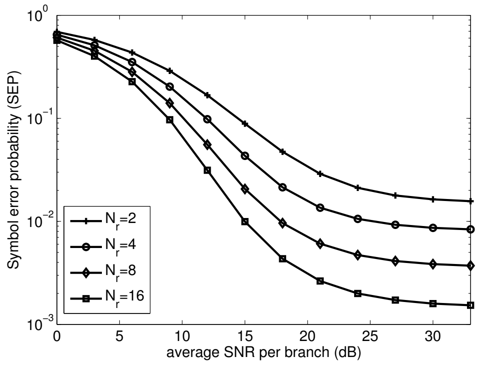

Fig. 5. Effect of number of receive antennas when the total training delay $N_r T_p$ is kept fixed (8PSK and $N_r T_p f_d = 0.08$).

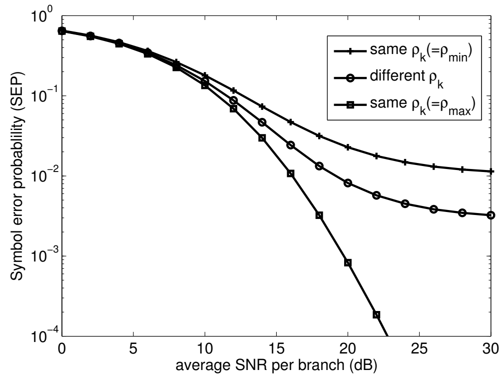

Fig. 7. Effect of different training delays on different antennas (8PSK, $f_d T_p = 0.016$, and $N_r = 4$).

Appendix

A. Proof of Lemma 1

From (1) we can see that $e_{[\hat{1}]}$ conditioned on $\hat{h}_{[\hat{1}]}$, is independent of other variables. Hence,

$$\mathbf{E}\left[e_{[\hat{1}]}|\{\hat{h}_i\}_{i=1}^{N_r}, s_i\right] = \mathbf{E}\left[e_{[\hat{1}]}|\hat{h}_{[\hat{1}]}\right] = \hat{h}_{[\hat{1}]}(1-q^2). \quad (14)$$

The second equality follows from standard results on conditional Gaussians. From (5), we can see that decision variable $\mathcal{D}$ depends only on $\hat{h}_{[\hat{1}]}$ and s_i. Hence,

$$\mathbf{E}\left[\mathcal{D}|\{\hat{h}_i\}_{i=1}^{N_r}, s_i\right] = \mathbf{E}\left[\mathcal{D}|\hat{h}_{[\hat{1}]}, s_i\right]$$
$$= \left|\hat{h}_{[\hat{1}]}\right|^2 \rho_{[\hat{1}]}^{(i)} s_i - \hat{h}_{[\hat{1}]}^* \rho_{[\hat{1}]}^{(i)} s_i \mathbf{E}\left[e_{[\hat{1}]}|\hat{h}_{[\hat{1}]}\right] = \left|\hat{h}_{[\hat{1}]}\right|^2 \rho_{[\hat{1}]}^{(i)} s_i q^2.$$

The first equality follows from the fact that both n'_k and n_k are independent of $\hat{h}_k$. The second equality follows from (14). The conditional variance expressions also follow similarly.

References

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