Linearization-based Cross-Layer Design for Throughput Maximization in OFDMA Wireless Ad-hoc Networks

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***Abstract*—In this paper, we consider joint routing, scheduling, and resource allocation to maximize the throughput of OFDMA based wireless ad-hoc networks subject to MAC layer and network layer constraints. Comparing with previous work (e.g., *Rashtchi et al., ICC 2012*) that assumes each subchannel to be orthogonally accessed by all network links through time sharing, we schedule the orthogonal multiple access (e.g., CDMA) among the outgoing links of each node to each subchannel and treat the interferences caused by other nodes as noise. We propose an iterative heuristic approach to decompose the original problem into subproblems, each of which can be solved approximately through linearization. Simulations demonstrate that, particularly in large networks, our proposed cross-layer design significantly outperforms the previously proposed "orthogonal-only" access.**

I. Introduction

Throughput maximization is a fundamental goal of wireless ad-hoc networks. However, it is very challenging due to interference. Promising solutions require joint consideration of routing, scheduling and power allocations (JRSPA) across different communication layers.

Cross-layer design for throughput maximization has been investigated extensively over the past years. Based on the assumption of queueing incoming packets and considering different physical layer transmission schemes, Refs. [1], [2], [3] propose throughput optimal algorithms based on backpressure-type routing. Without queueing, some other works characterize the achievable rates of multi-hop wireless network with interference avoidance, e.g., [4], [5] and [6], where hard decisions are made to determine the interfering links either by comparing the geometric distances or the SIRs to the corresponding thresholds. In other literature, e.g., [7] and [8], the JRSPA problem is formulated as a convex optimization problem. Ref. [7] models the link capacity as a linear function of the SINR, which is only valid for the case of low SINRs on all the activated links. Ref. [8] models the link capacity as a convex function of power and bandwidth allocation variables or of time allocation variables. However, the model in [8] is based on the assumption that the link capacity depends only on the locally allocated resources while ignoring issues of interference, which is typically not fulfilled in practice.

In this paper, we explore cross-layer design for throughput maximization in Orthogonal Frequency Division Multiple Access (OFDMA) wireless ad-hoc networks. OFDMA allows the efficient use of spectrum in frequency-selective fading environments, and therefore is promising for wireless ad-hoc networking. Some papers have explored the cross-layer design of OFDMA-based wireless networks. For instance, resource allocation has been studied in [9] for multi-source, single destination, two-hop OFDMA relay networks subject to the restriction of exclusive (orthogonal) subchannel access, i.e., each subchannel can only be used once in the whole network. With the same subchannel access restriction, a heuristic approach has been proposed in [10] to maximize the throughput of OFDMA wireless mesh backhaul networks. Ref. [11] extends the framework to OFDMA *ad-hoc networks*; it allows all links in the network to access each subchannel through Time Division Multiple Access (TDMA). However, the model still assumes that each time-frequency resource can be used only once in the whole network. The original problem is finally transformed into a convex optimization problem.

Similar to [11], we focus on throughput maximization in OFDMA wireless ad-hoc network, using a similar network model and assuming that the outgoing links of each node are scheduled orthogonally. However, in contrast to [11], the outgoing links from *different* nodes need not to be orthogonal, and thus may cause interference to one another. The motivation for the change in setting is to expand the policy space for choosing the scheduling variables. Correspondingly, we can then trade off spatial frequency reuse with interference. The main contribution of this paper is to propose a linearization-based heuristic approach to tackle the new JRSPA problem. Simulation results demonstrate a significant throughput performance enhancement compared to the results of [11] in particular for large networks.

The remainder of the paper is organized as follows. Section II describes the system model. Section III formulates the optimization problem and describes the linearization-based heuristic approach. Simulation results are provided in Section IV. The paper is concluded in Section V.

II. System Model

A. Network Settings

We consider an OFDMA wireless ad-hoc network with N nodes denoted as set $\mathcal{N}$. The network operates over a frequency band equally divided into K subchannels, denoted as set $\mathcal{K}$. Each subchannel has bandwidth W. Let P_i represent the power budget of the ith node. We assume that the signal transmitted from any node i can be overheard by all other nodes in the network at various strengths, depending on the channel gains. The network is thus fully connected. Let node pair (i, j) represent the link from node i to node j; g_{ij}^k is the channel (power) gain of link (i, j) on subchannel k.

Assume the g_{ij}^k to be constant during multiple frames of transmission. This can be approximately true if the duration of communication is less than the coherence time of the wireless medium. We assume full channel state information (CSI) known by a centralized coordinator. Each node in the network can serve as source, relay and/or destination simultaneously; we allow each node to work in full-duplex mode[1], i.e., transmissions and receptions of a node can overlap in time without causing self-interference. Multiple data streams ("commodities"), indexed by $d \in \mathcal{D}$ and intended for different destination nodes, may flow through the network concurrently.

B. System Constraints

In order to maximize the throughput of the network, we jointly schedule subchannel usage for each link, route the data streams, and allocate the power to each transmission, all subject to the constraints of different layers.

1) Scheduling Constraints: The scheduling of subchannels is characterized by the set of variables $\{c_{ij}^k\}$, where i and j are the transmitting and receiving node, respectively, and k is the subchannel index.

First consider the case where the c_{ij}^k are restricted to take binary values: c_{ij}^k takes value 1 to indicate the exclusive usage of subchannel k by link (i, j), and 0 otherwise. In the OFDMA ad-hoc network, the constraints on the c_{ij}^k are

$$\sum_{j:j\neq i} c_{ij}^k \leq 1, \ \forall\, i \in \mathcal{N},\ k \in \mathcal{K}, \tag{1}$$

$$c_{ij}^k \in \{0, 1\},\ \forall\, i, j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}. \tag{2}$$

Eq. (1) restricts each subchannel to be used by at most one of the outgoing links of each node.

With the binary c_{ij}^k, the JRSPA problem for a network with a general topology is an integer programming problem that falls into the category of NP-hard problems [9]. To make the problem tractable, we relax the c_{ij}^k to take values in the continuous interval $[0, 1]$. Technologically, this models the application of multiple access techniques among the outgoing links of each transmitting node on each subchannel. Retaining (1), constraints (2) can then be replaced by

$$0 \leq c_{ij}^k \leq 1,\ \forall\, i, j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}. \tag{3}$$

[1]Note that recent researches [12] [13] have made full-duplex implementation practically feasible as well as theoretically interesting. The treatment of the half-duplex mode will be subject of future work.

Then (1) and (3) imply that the outgoing links of each node access each subchannel orthogonally. Correspondingly, c_{ij}^k represents the fraction that the amount of resource used by link (i, j) on subchannel k is divided by the total amount of resource under the multiple access scheme, e.g., the fraction of the time used under TDMA or the fraction of the spreading codes used under Code Division Multiple Access (CDMA).

In later parts of the paper, we adopt CDMA as the multiple access scheme to analyze, and briefly call the use of CDMA with the constraints (1), (3) the "local CDMA" scheme. The motivation is two fold:

a) As will be shown later, the interference can be characterized by the c_{ij}^k and the power allocation variables in a closed form, c.f., (9). This makes our problem tractable.
b) For the transmission over each link (i, j), all the signals overheard by node j from nodes other than node i on the same subchannel are whitened as noise. In contrast, under other local multiple access schemes, some interferers are possibly avoided because of being allocated with the resource, e.g., timeslots under "local TDMA", orthogonal to the one allocated to link (i, j). Therefore, applying local CDMA on each subchannel achieves a lower bound of the throughput of the OFDMA network.

We assume that the total number of orthogonal spreading codes generated to be sufficiently large such that the c_{ij}^k can approximately take any values in interval $[0, 1]$.

2) Power Allocation Constraints: Under the local CDMA scheme, let p_{ij}^k represent the amount of power allocated to link (i, j) for the transmission on subchannel k if link (i, j) used all the spreading codes. Then

$$\sum_k \sum_{j:j\neq i} c_{ij}^k p_{ij}^k \leq P_i,\ \forall\, i \in \mathcal{N}, \tag{4}$$

$$p_{ij}^k \geq 0,\ \forall\, i, j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}, \tag{5}$$

where $c_{ij}^k p_{ij}^k$ is the actual amount of power allocated to link (i, j) for the transmission on subchannel k. Eq. (4) describes the constraint of the power consumed by node i, summed over all outgoing links and all subchannels.

3) Routing Constraints: Let $s_i^{(d)}$ represent the exogenous input rate for commodity d arriving at node i; let $x_{ij}^{k(d)}$ represent the data rate for commodity d that flows through link (i, j) on subchannel k. Then the $s_i^{(d)}$ and the $x_{ij}^{k(d)}$ satisfy:

$$x_{ij}^{k(d)} \geq 0,\ \forall\, i, j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K},\ d \in \mathcal{D}, \tag{6}$$

$$s_i^{(d)} \geq 0,\ \forall\, i \in \mathcal{N},\ d \in \mathcal{D},\ i \neq d. \tag{7}$$

Eq. (7) does *not* hold for $i = d$ since $s_d^{(d)}$ can be negative, which represents that data flows out of the network. Moreover, flow conservation requires that

$$\sum_{j:j\neq i} \sum_k x_{ij}^{k(d)} - \sum_{j:j\neq i} \sum_k x_{ji}^{k(d)} - s_i^{(d)} = 0,\ \forall\, i \in \mathcal{N},\ d \in \mathcal{D}. \tag{8}$$

4) Link Capacity Constraints: Consistent with the scheduling constraints shown in (1) and (3), interference may exist among the transmissions over the links on the same subchannel but emanating from different nodes. Under the local CDMA scheme, different transmitting nodes use different sets of orthogonal spreading codes created, e.g., by multiplying a set of Walsh-Hadamard codes by m-sequence "scrambling" codes [14]. The result is that orthogonality among the spreading codes used on the outgoing links of each node can be guaranteed, while the spreading codes used to transmit by different nodes are non-orthogonal but distinguishable. The aggregate interference to each link (i,j) on each subchannel k is

$$\sum_{n:n\neq i}\sum_{z:z\neq n} c_{nz}^k p_{nz}^k g_{nj}^k, \quad \forall\ i,j\in\mathcal{N},\ k\in\mathcal{K}. \tag{9}$$

Treating interferences as noise, it follows that the link capacities satisfy the following non-convex constraints:

$$\sum_d x_{ij}^{k(d)} \leq W c_{ij}^k \log_2\left(1+\frac{p_{ij}^k g_{ij}^k}{N_0W+\sum\limits_{n:n\neq i}\sum\limits_{z:z\neq n} c_{nz}^k p_{nz}^k g_{nj}^k}\right), \quad \forall\ i,j\in\mathcal{N},\ j\neq i,\ k\in\mathcal{K},\ d\in\mathcal{D}, \tag{10}$$

where N_0W is the power of the additive white Gaussian noise.

III. Linearization-based JRSPA Optimization

The goal of this section is to determine the routing, scheduling and resource allocation variables, i.e., $x_{ij}^{k(d)}$, c_{ij}^k and p_{ij}^k, respectively, to maximize the throughput of the network by formulating and approximately solving the JRSPA problem.

A. Problem Formulation

Throughput performance can be expressed as maximizing the weighted summation of the supportable exogenous input rates $s_i^{(d)}$ subject to the constraints (1), (3)-(8) and (10). The resulting optimization problem can be formulated as

$$\max_{\left\{s_i^{(d)}\right\},\left\{c_{ij}^k\right\},\left\{x_{ij}^{k(d)}\right\},\left\{p_{ij}^k\right\}} \sum_d \sum_{i:i\neq d} \omega_i^{(d)} s_i^{(d)} \tag{11a}$$

subject to:

$$\text{Eq. (1), Eq. (3)-(8) and Eq. (10)}, \tag{11b}$$

where the $\omega_i^{(d)}$ are non-negative constant weights.

The constraints (4) and (10) are non-convex, and therefore theoretically achieving the global optimum of the resulting problem (11) is not guaranteed. In this paper, we instead develop a heuristic approach to (11).

B. Linearization-based Approach

Our approach is to iterate a sequence of subproblems, each of which has a simpler form. We keep iterating, updating the parameters of each subproblem according to the solutions of the previous one, until they converge to a local optimum.

We aim to transform (4) and (10) into linear forms. To do this, we set $\{c_{ij}^k\}$ and $\{p_{ij}^k\}$ as two alternatively updating sets of parameters of the subproblems. We introduce the Δc_{ij}^k and Δp_{ij}^k as new variables, defined respectively as the changes in the c_{ij}^k and p_{ij}^k between two successive iterations, i.e.,

$$c_{ij}^k[l+1] = c_{ij}^k[l] + \Delta c_{ij}^k[l], \quad \forall\ i,j\in\mathcal{N},\ i\neq j,\ k\in\mathcal{K},\ l\geq 0, \tag{12}$$

$$p_{ij}^k[l+1] = p_{ij}^k[l] + \Delta p_{ij}^k[l], \quad \forall\ i,j\in\mathcal{N},\ i\neq j,\ k\in\mathcal{K},\ l\geq 0, \tag{13}$$

where l is the index of the iteration.

Updating the two sets of parameters $\{c_{ij}^k[l]\}$ and $\{p_{ij}^k[l]\}$ alternatively forms two consecutive sub-steps in each iteration l: the "scheduling update" and the "power allocation update", denoted as sub-step l_c and l_p, respectively. The update sequence of each iteration followed in this paper is: starting with $\{c_{ij}^k[l]\}$ and $\{p_{ij}^k[l]\}$, sub-step l_c updates $\{c_{ij}^k[l]\}$ to $\{c_{ij}^k[l+1]\}$; then, continuing with $\{c_{ij}^k[l+1]\}$ and $\{p_{ij}^k[l]\}$, sub-step l_p updates $\{p_{ij}^k[l]\}$ to $\{p_{ij}^k[l+1]\}$. Having clarified the update sequence, in the remainder of this subsection and for notational convenience, we omit the iteration indices in reformulating certain constraints.

By replacing the c_{ij}^k in (4) by $c_{ij}^k+\Delta c_{ij}^k$, the power constraint in sub-step l_c becomes linear:

$$\sum_k \sum_{j:j\neq i} \left(c_{ij}^k+\Delta c_{ij}^k\right) p_{ij}^k \leq P_i,\ \forall\ i\in\mathcal{N}. \tag{14}$$

Define $I_{ij}^k \triangleq \sum_{n:n\neq i}\sum_{z:z\neq n} c_{nz}^k p_{nz}^k g_{nj}^k$. We perform Taylor expansion on (10) with the c_{ij}^k replaced by the $c_{ij}^k+\Delta c_{ij}^k$, and keep the constant and linear terms. The link capacity constraints in sub-step l_c are approximately

$$\begin{aligned}\sum_d x_{ij}^{k(d)} \leq & W\Delta c_{ij}^k \log_2\left(1+\frac{p_{ij}^k g_{ij}^k}{N_0W+I_{ij}^k}\right) \\ & - \frac{W c_{ij}^k p_{ij}^k g_{ij}^k \sum\limits_{n:n\neq i}\sum\limits_{z:z\neq n} \Delta c_{nz}^k p_{nz}^k g_{nj}^k}{(\ln 2)\left(N_0W+I_{ij}^k+p_{ij}^k g_{ij}^k\right)\left(N_0W+I_{ij}^k\right)} \\ & + W c_{ij}^k \log_2\left(1+\frac{p_{ij}^k g_{ij}^k}{N_0W+I_{ij}^k}\right), \\ & \forall\ i,j\in\mathcal{N},\ j\neq i,\ k\in\mathcal{K}.\end{aligned} \tag{15}$$

Additionally, we use a parameter η to control the approximation accuracy by restricting the ranges of the Δc_{ij}^k relative to the c_{ij}^k according to the following relations:

$$\left|\sum_{n:n\neq i}\sum_{z:z\neq n} \Delta c_{nz}^k p_{nz}^k g_{nj}^k\right| \leq \eta\left(N_0W+I_{ij}^k\right), \quad \forall\ i,j\in\mathcal{N}, j\neq i, k\in\mathcal{K}, \tag{16a}$$

$$\left|\Delta c_{ij}^k p_{ij}^k g_{ij}^k\right| \leq \eta\left(N_0W+I_{ij}^k+p_{ij}^k g_{ij}^k\right), \quad \forall\ i,j\in\mathcal{N}, j\neq i, k\in\mathcal{K}, \tag{16b}$$

Similarly, the power budget constraints in sub-step l_p are

$$\sum_k \sum_{j:j\neq i} c_{ij}^k \left(p_{ij}^k+\Delta p_{ij}^k\right) \leq P_i,\ \forall\ i\in\mathcal{N}. \tag{17}$$

The link capacity constraints in sub-step l_p are approximately

$$\begin{aligned}\sum_d x_{ij}^{k(d)} \leq & \frac{W c_{ij}^k \Delta p_{ij}^k g_{ij}^k}{(\ln 2)\left(N_0 W + I_{ij}^k + p_{ij}^k g_{ij}^k\right)} \\ & - \frac{W c_{ij}^k p_{ij}^k g_{ij}^k \sum\limits_{n:n\neq i} \sum\limits_{z:z\neq n} c_{nz}^k \Delta p_{nz}^k g_{nj}^k}{(\ln 2)\left(N_0 W + I_{ij}^k + p_{ij}^k g_{ij}^k\right)\left(N_0 W + I_{ij}^k\right)} \\ & + W c_{ij}^k \log_2\left(1 + \frac{p_{ij}^k g_{ij}^k}{N_0 W + I_{ij}^k}\right), \\ & \forall\, i,j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}, \end{aligned} \tag{18}$$

and the corresponding range restrictions on the Δp_{ij}^k are

$$\left| \sum_{n:n\neq i} \sum_{z:z\neq n} c_{nz}^k \Delta p_{nz}^k g_{nj}^k \right| \leq \eta \left(N_0 W + I_{ij}^k\right), \quad \forall\, i,j \in \mathcal{N}, j \neq i,\ k \in \mathcal{K}, \tag{19a}$$

$$\left| \Delta p_{ij}^k g_{ij}^k + \sum_{n:n\neq i} \sum_{z:z\neq n} c_{nz}^k \Delta p_{nz}^k g_{nj}^k \right| \leq \eta \left(N_0 W + I_{ij}^k + p_{ij}^k g_{ij}^k\right), \ \forall\, i,j \in \mathcal{N}, j \neq i,\ k \in \mathcal{K}. \tag{19b}$$

Based on (14)-(19), the two subproblems corresponding to sub-steps l_c and l_p, respectively, are formulated as follows:

1) Optimization Subproblem of Sub-step l_c:

$$\max_{\left\{s_i^{(d)}\right\}, \left\{x_{ij}^{k(d)}\right\}, \left\{\Delta c_{ij}^k\right\}} \sum_d \sum_{i:i\neq d} \omega_i^{(d)} s_i^{(d)} \tag{20a}$$

subject to:

$$\text{Eq. (6)-(8) and Eq. (14)-(16),} \tag{20b}$$

$$\sum_{j:j\neq i} \left(c_{ij}^k + \Delta c_{ij}^k\right) \leq 1,\ \forall\, i \in \mathcal{N},\ k \in \mathcal{K} \tag{20c}$$

$$c_{ij}^k + \Delta c_{ij}^k \geq 0,\ \forall\, i,j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}. \tag{20d}$$

2) Optimization Problem of Sub-step l_p:

$$\max_{\left\{s_i^{(d)}\right\}, \left\{x_{ij}^{k(d)}\right\}, \left\{\Delta p_{ij}^k\right\}} \sum_d \sum_{i:i\neq d} \omega_i^{(d)} s_i^{(d)} \tag{21a}$$

subject to:

$$\text{Eq. (6)-(8) and Eq. (17)-(19),} \tag{21b}$$

$$p_{ij}^k + \Delta p_{ij}^k \geq 0,\ \forall\, i,j \in \mathcal{N},\ j \neq i,\ k \in \mathcal{K}. \tag{21c}$$

It can be seen that both (20) and (21) are linear programs. We conclude that the original problem (11) can be tackled by iteratively solving these linear programming subproblems with controllable approximations. Since the original problem is non-convex, convergence to a global optimum cannot be guaranteed. However, the simulations in Sec. IV show that for some special cases where an optimum solution can be found analytically, the solution of our iterative approach is close to the global optimum.

C. Implementation of the Iterative Approach

In this section, we describe certain considerations that arise in the algorithmic implementation.

1) Setting the Initial Values of the c_{ij}^k and p_{ij}^k: We must avoid setting inappropriate initial values of the c_{ij}^k and p_{ij}^k, which would increase the likelihood of convergence to a "small value" local maximum. We set the initial values of the c_{ij}^k and p_{ij}^k to zeros so as not to bias the optimization process.

2) Confirming the Stopping Criterion: Because of the approximation error arising from linearization, results achieved by the subproblems can keep fluctuating as the iteration evolves and no theoretical convergence is guaranteed. Therefore, we choose a threshold value T and a positive integer L to parameterize the termination of the iterations. Specifically, as the algorithm iterates, we keep monitoring and updating the maximum peak-to-peak variation of the throughput values achieved in the latest L consecutive iterations, denoted as $V(l, \cdots, l-L+1)$, where l is the index of the latest iteration. Whenever $V(l, \cdots, l-L+1)$ falls below the threshold T, we stop the iterations and output the throughput value of iteration l as the desired one. By setting $L > 2$, we avoids falsely terminating the iteration when the two latest achieved throughput values are accidentally similar.

3) Reducing the Iteration Count: A small value of η, in (16) and (19), guarantees a high linear approximation accuracy in each iteration but increases the iteration count, i.e., the number of required iterations. Therefore, we use a set of monotonically decreasing parameters $\{\eta_m\}_{m=1}^M$ matched with a set of monotonically decreasing threshold values $\{T_m\}_{m=1}^M$. Then the η_m and T_m are traversed as the iterations evolve. Together, the use of the η_m and T_m reduces the overall iteration count and controls the final approximation accuracy.

The pseudo code of the linearization-based approach, denoted as L-JRSPA, can be summarized as follows:

Set $l = 0$;
Set $\{c_{ij}^k[0]\}$ and $\{p_{ij}^k[0]\}$ to zeros;
Set the values of $\{\eta_m\}_{m=1}^M$, $\{T_m\}_{m=1}^M$ and L;
for $1 \leq m \leq M$ **do**
  while $V(l, \cdots, l-L+1) \geq T_m$ **do**
    Optimize (20) with η_m, $\{c_{ij}^k[l]\}$ and $\{p_{ij}^k[l]\}$;
    Update $\{c_{ij}^k[l]\}$ to $\{c_{ij}^k[l+1]\}$;
    Optimize (21) with η_m, $\{c_{ij}^k[l+1]\}$ and $\{p_{ij}^k[l]\}$;
    Update $\{p_{ij}^k[l]\}$ to $\{p_{ij}^k[l+1]\}$;
    $l \Leftarrow l + 1$;
  end while
end for
Output $\{s_i^{(d)}[l]\}, \{c_{ij}^k[l]\}, \{x_{ij}^{k(d)}[l]\}, \{p_{ij}^k[l]\}$.

IV. Simulation Results

The JRSPA proposed in [11] assumes each subchannel to be accessed at a given time by only a single link in the network, and sharing of this channel among links through TDMA (we call this setting as the global TDMA scheme). In contrast, our proposed L-JRSPA assumes the use of the local CDMA

Fig. 1. Line network topology, $K = 8$, $d = N$

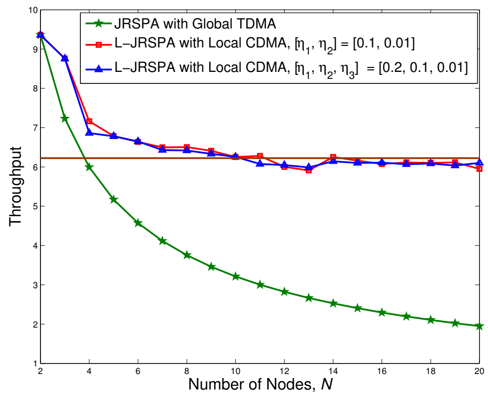

Fig. 2. Throughput of a line network as function of number of nodes, N

scheme and treats interference as noise. In this section, we compare the two approaches through numerical simulations.

A. Throughput of Static Line Networks

This subsection describes the simulations for line networks with a variable number N of nodes, as shown in Fig. 1. The distance between any two adjacent nodes is a constant unit distance D_0; the source is the leftmost node; the destination is the rightmost node. There are 8 subchannels available, each of which has bandwidth W normalized to 1. Assume that there is no small scale fading in the transmissions, but that the channel gain is completely determined by power-law pathloss with coefficient $\alpha = 3.5$ [2]. The transmission power is normalized by the additive white Gaussian noise power on each subchannel. We assume the antenna used in each node is isotropic and require that the SNR on each subchannel at distance D_0 away from the transmitter is no more than 10dB, which determines the power budget of each node.

Although routing in the above line-network is obviously from left to right, scheduling and resource allocation are still challenging issues in maximizing the throughput. With the network ranging in size from $N = 2$ to $N = 20$ nodes, the throughput achieved under L-JRSPA with local CDMA and under JRSPA with global TDMA are compared in Fig. 2. In the figure, the throughput under JRSPA decreases as the network size grows. This is consistent with the following intuition: more nodes sharing the communication time resource means that each node has less time to transmit.

In contrast, in each L-JRSPA curve, the throughput first decreases as network size grows (from 2 nodes to 4 nodes) but then gradually approaches a constant (more than 4 nodes). This can be comprehended as a result of subchannel allocation

[2]This setting implies that all the subchannels over each link have the same power gain, which can happen when the line of sight is available.

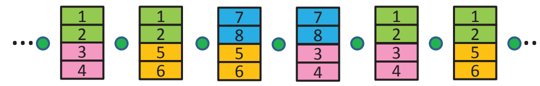

Fig. 3. An optimum set of reuse patterns for 8 subchannels in an infinite-node line network

compensated with spatial reuse. As N increases from 2 to 4, the throughput decreases significantly due to allocating subchannels among more links. After N increases beyond 4, two transmissions far apart can use the same subchannel without causing significant interference to each other, which gradually becomes a dominant factor in preventing further throughput decrease as N increases.

In Fig. 2, we experientially set the parameter $L = 10$ for both of the two curves plotted under L-JRSPA, but they have different sets of controlling parameters $\{\eta_m\}_{m=1}^{M}$ and $\{T_m\}_{m=1}^{M}$. One has $[\eta_1, \eta_2] = [0.1, 0.01]$ and $[T_1, T_2] = [0.02, 0.005]$; the other has $[\eta_1, \eta_2, \eta_3] = [0.2, 0.1, 0.01]$ and $[T_1, T_2, T_3] = [0.2, 0.02, 0.005]$. Note that the two curves do not completely overlap. This can be explained by the fact that iterations with different $\{\eta_m\}_{m=1}^{M}$ evolve along different paths in the policy space and finally converges to distinct local optima. However, the solutions are very close numerically.

Since the throughput achieved under L-JRSPA approaches a constant as the network size grows, it is interesting to explore the difference between our (numerically obtained) stable throughput and the maximum throughput (under the local CDMA assumption) that can be achieved as the number of nodes goes to infinity. The analytically computed throughput of an infinite-node line network with settings in Sec. IV-A is 6.225, which is shown as the height of the horizontal line in Fig. 2; the corresponding optimal set of reuse patterns for the 8 subchannels are shown in Fig. 3, where the numbers represent subchannel indices. For details, see [15]. It can be seen in Fig. 2 that the obtained stable throughput is very close to the maximum throughput of the infinite-node line network.

B. Throughput of Networks with Irregular Topologies

In this subsection, we first simulate a network with the topology shown as Fig. 4. The network has 12 nodes with node 1 being the source and node 12 being the destination; the position information ((X, Y) coordinates) of all the nodes is listed by the following set $\mathcal{G}$:

$$\begin{aligned}\mathcal{G} = \{&(1.0, 0.5), (1.0, 2.0), (3.0, 1.0), (2.0, 3.0), \\ &(4.5, 1.0), (3.3, 4.2), (4.7, 4.0), (5.3, 1.8), \\ &(6.0, 2.7), (7.0, 1.7), (8.0, 3.0), (7.0, 4.1)\}. \end{aligned} \tag{22}$$

Additionally, assume the network has 4 subchannels to use, each of which experiences pathloss with coefficient $\alpha = 3.5$ and independent Rayleigh fading. The mean SNR at unit distance on each subchannel is no more than 20dB. We assume that the communication duration is smaller than the coherence time of the wireless medium in the network. For the iterations, we set $L = 10$, $[\eta_1, \eta_2, \eta_3] = [0.2, 0.1, 0.01]$ and $[T_1, T_2, T_3] = [0.2, 0.02, 0.005]$.

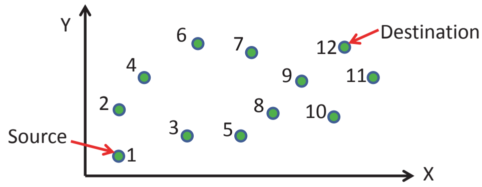

Fig. 4. The network with an irregular topology, $N = 12$, $K = 4$, $d = 12$

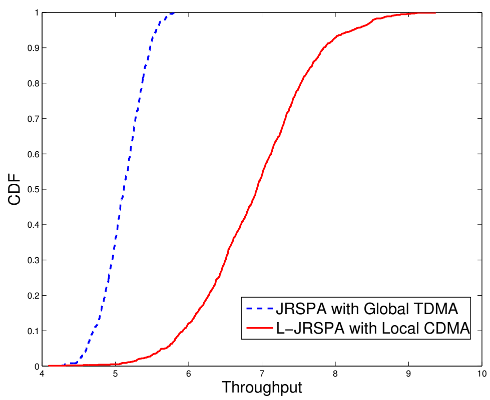

Fig. 5. CDFs of the throughput achieved in the network with the topology shown in Fig. 4

We again compare the throughput performances of L-JRSPA and JRSPA. Fig. 4 plots the Cumulative Distribution Function (CDF) curves of the throughput under L-JRSPA and JRSPA, respectively. Each of the curves is formed by 1000 throughput values, independently generated from 1000 channel realizations. The CDF of L-JRSPA is located to the right of the CDF of JRSPA, and by computing the sample average, the mean throughput of the former is approximately 6.91 and the mean throughput of the latter is approximately 5.10.

However, L-JRSPA does not always outperform JRSPA. In certain small scale networks, where the numbers of hops of the flow paths from the source to the destination are small, simulations show that the throughput under L-JRSPA is not necessarily higher than that under JRSPA. The reason is that interferences among links, rather than spatial reuse, dominate the network throughput under L-JRSPA in such situations; the throughput performance under JRSPA benefits from the orthogonality among the small number of links.

V. Conclusion

We developed the L-JRSPA algorithm for the throughput maximization of OFDMA-based wireless ad-hoc networks. The networks allow the outgoing links of each node to orthogonally access each subchannel, e.g., through CDMA, and spatially reuse the subchannel while treating the arising interference among the transmissions from different nodes as noise. A local optimum is computed based on a novel, iterative approach with linearization in each iteration. After simulating the line networks and the networks with irregular topologies, we find that the throughput performance can be enhanced significantly when compared with the strategy of accessing each subchannel orthogonally among all links, particularly in large networks. Moreover, simulations on line networks with increasing sizes show the throughput convergence to a constant value close to the theoretical maximum throughput of a line network with infinite number of nodes.

L-JRSPA implicitly assumes centralized knowledge of the channel states and centralized control. This can be realized when the channel shows little or no time variance, since then the relative overhead for feedback and control signaling is negligible. Such situations occur, e.g., in fixed-terminal mesh networks and industrial wireless sensor networks. Distributed versions of our algorithm are topics for future research.

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