Diversity Backpressure Scheduling and Routing with Mutual Information Accumulation in Wireless Ad-hoc Networks

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***Abstract*—We suggest and analyze algorithms for routing in multi-hop wireless ad-hoc networks that exploit mutual information accumulation as the physical layer transmission scheme, and are capable of routing multiple packet streams (commodities) when only the *average* channel state information is present, and that only locally. The proposed algorithms are modifications of the *Diversity Backpressure* (DIVBAR) algorithm, under which the packet whose commodity has the largest "backpressure metric" is chosen to be transmitted and is forwarded through the link with the largest differential backlog (queue length). In contrast to traditional DIVBAR, each receiving node stores and accumulates the partially received packet in a separate "partial packet queue", thus increasing the probability of successful reception during a later possible retransmission. We present two variants of the algorithm: DIVBAR-RMIA, under which all the receiving nodes clear the received partial information of a packet once one or more receiving nodes firstly decode the packet; and DIVBAR-FMIA, under which all the receiving nodes retain the partial information of a packet until the packet has reached its destination. We characterize the network capacity region with the Renewal Mutual Information Accumulation (RMIA) transmission scheme and prove that (under certain mild conditions) it is strictly larger than the network capacity region with the Repetition (REP) transmission scheme that is used by the traditional DIVBAR. We also prove that DIVBAR-RMIA is throughput-optimal among the polices with RMIA, i.e., it achieves the network capacity region with RMIA, which in turn demonstrates that DIVBAR-RMIA outperforms traditional DIVBAR with respect to the achievable throughput. Moreover, we prove that DIVBAR-FMIA performs at least as well as DIVBAR-RMIA with respect to throughput. Simulations also confirm these results.**

***Index Terms*—Stochastic Network Optimization, Wireless Ad-hoc Networks, Backpressure Algorithm, Renewal Mutual Information Accumulation (RMIA), Full Mutual Information Accumulation (FMIA), Repetition Transmission Scheme (REP), d-timeslot Lyapunov drift**

I. Introduction

Wireless multi-hop ad-hoc networks have drawn significant attention in recent years, due to their flexibility and low cost, and their resulting importance in factory automation, sensor networks, security systems, and many other applications. A fundamental problem in such networks is the routing of data packets, i.e., which nodes should transmit which packets in which sequence. The throughput performance becomes an issue when a single stream or multiple streams of packets intended for a single destination or multiple destinations (i.e., multiple *commodities*) flow through a network. In wired networks, the single packet stream case has been well-explored by several approaches, such as Ford-Fulkerson algorithm and Preflow-Push algorithm (see Ref. [1], Chapter 7 and references therein), and Goldberg-Rao algorithm (see [2] and references therein); simultaneous routing of multiple packet streams have also been extensively explored (see Ref. [3], [4] and references therein). However, this problem becomes more challenging to solve in wireless scenarios, where the links are neither reliable or precisely predictable.

To deal with this issue, several studies focus on the routing in the wireless network with unreliable channels and possible multiple commodities. The ExOR algorithm [5] takes advantage of the *broadcast effect*, i.e., the packet being transmitted by a node can be overheard by multiple receiving nodes. After confirming the successful receivers among all the potential receiving nodes after each attempt of transmission, the transmitting node determines the best node among the successful receivers to forward the packet in the future according to the *Expected Transmission Count Metric* (ETX) [6], which indicates the proximity from each receiving node to the destination node in terms of *forward delivery probability*. As a further improvement, the proactive SOAR algorithm [7] also uses ETX as the underling routing metric but leverages the path diversity by certain *adaptive forwarding path selections*. Both ExOR and SOAR have shown better throughput performance than the traditional routing methods, but neither theoretically provides a *throughput-optimal* routing approach for multi-hop, multi-commodity wireless ad-hoc networks.

Throughput maximization can be tackled by *stochastic network optimization*, which involves routing, scheduling and resource allocation in networks without reliable or precisely predictable links but with certain stochastic features. Refs. [8], [9] systematically analyze this kind of problems by using *Lyapunov drift* analysis originating from control theory, which follows and generalizes the *Backpressure* algorithm proposed in [10] [11]. The backpressure algorithm establishes a *Max-weight-matching* metric for each commodity on each available link that takes into account the local differential *backlogs* (queue lengths or the number of packets of the particular commodity at a node) as well as the channel state of the corresponding link observed in time. The packet of the commodity with the largest metric will be transmitted from each node. Thus, the backpressure algorithm achieves routing without ever designing an explicit route and without requiring centralized information, and therefore, is considered as a very promising approach to stochastic network

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optimization problems with multiple commodities. The idea of Backpressure routing was later extended to many other communication applications, e.g., power and server allocation in satellite downlink [12], routing and power allocation in time-varying wireless networks [13], and throughput optimal routing in cooperative two hop parallel relay networks [14].

Based on the principle of Backpressure, [15] developed the *Diversity Backpressure* (DIVBAR) algorithm for routing in multi-hop, multi-commodity wireless ad-hoc networks. Similar to ExOR and SOAR, DIVBAR assumes a network with no reliable or precisely predictable channel states and exploits the broadcast nature of the wireless medium. In essence, each node under DIVBAR locally uses the backpressure concept to route packets in the direction of maximum differential backlog. Specifically, each transmitting node under DIVBAR chooses the packet with the commodity optimal to transmit by computing the Max-weight-matching metric, whose factors include the observed differential backlogs and the link success probabilities resulting from the fading channels; after getting the feedbacks indicating the successful receptions from all the receiving nodes, the transmitting node lets the successful recipient with the largest positive differential backlog get the forwarding responsibility. DIVBAR has been theoretically proved to be throughput-optimal in wireless ad-hoc networks under a set of assumptions, notably that any packet not correctly received by any receiving node needs to be completely retransmitted in future transmission attempts. Here we call the scheme of complete retransmission the *Repetition* (REP) transmission scheme.

The efficiency of transmissions can be greatly enhanced by *Mutual Information Accumulation* (MIA), where the receiving nodes store *partial information* of the packets that cannot be decoded at the previous transmission attempts. MIA can be implemented, e.g., by using Fountain Codes (or rateless codes) [16] [17] [18]. The transmitter encodes and transmits the source information in code stream of unbounded length, and the receiver can recover the original source information from any portions of the code streams, as long as the amount of total accumulated information exceeds the entropy of the source information. Moreover, Fountain codes can work at any SNR, and therefore, the same code design for MIA can be used for broadcasting from one transmitter to multiple receivers whose links to the transmitter have different channel gains. In the meanwhile, Fountain codes can accumulate the partial information from multiple transmitters. Thus, with the MIA technique, the transmissions of a packet in previous timeslots can facilitate the decoding of the packet in the current timeslot, which is the key difference from REP. Refs. [19], [20] introduce MIA into the routing of multi-hop ad-hoc networks, and have shown that the delay performance can be enhanced with constrained power and bandwidth resources. However, none of above papers touches the throughput performance of ad-hoc networks with MIA.

For multi-hop, multi-commodity wireless ad-hoc networks, the throughput might be increased when implementing MIA instead of REP. An intuitive approach of exploring this problem is to combine MIA with Lyapunov drift analysis, and design a "MIA version" of Backpressure or DIVBAR algorithms. Following this strategy and parallel to our work, Ref. [21] proposed a T-slot routing algorithm and a virtual queue routing algorithm for multi-hop, multi-commodity wireless ad-hoc network with broadcast effect and *single-copy routing* assumption (redundant packet transfers are not used). These two algorithms assume that each link in the network has fixed and reliable transmission rate, and each transmitting node making local decisions can predetermine the local transmitting and forwarding realizations based on the backlog and virtual queue observations.

In this paper, in contrast with Ref. [21], we explore the multi-hop, multi-commodity, single-copy routing in the case of unreliable and non-precisely predictable rates. We assume that the network has stationary channel fading, i.e., the distribution of the channel realizations remain the same, however, the realizations vary with time; although no precise *channel state information* (CSI) at the transmitter is available, the distributions of the channel realizations of each link can be obtained by the transmitter beforehand, i.e., each transmitting node has the average CSI; the transmitting node can obtain the receiving (decoding) results by some simple feedbacks sent by the receiving nodes through certain reliable control channels.

Our contributions of this paper are summarized as follows:

- We analyze the *network capacity region* [8] [9] of ad-hoc networks employing the *Renewal Mutual Information Accumulation* (RMIA) transmission scheme under the single-copy routing assumption. Here "Renewal" stands for a clearing operation; and RMIA is the transmission scheme in which all the receiving nodes accumulate the partial information of a certain packet and try to decode the packet when receiving it, but clear the partial information of a packet every time the corresponding packet is firstly decoded by one or more receiving nodes in the network. We prove that the network capacity region with RMIA is strictly larger than the network capacity region with REP under some mild assumptions, and quantitatively compute a guaranteed extension magnitude of the region boundary for each source-destination pair.
- We propose and analyze two new routing algorithms that combine the concept of DIVBAR with MIA. The first version, DIVBAR-RMIA is implemented with RMIA, and is shown to be throughput-optimum among all routing algorithms with RMIA. Under the second version, DIVBAR-FMIA, all the received partial information of a packet is retained at all the nodes in the network until that packet has reached its destination, which is called the *Full Mutual Information Accumulation* (FMIA) transmission scheme. We prove that DIVBAR-FMIA's throughput performance is at least as good as DIVBAR-RMIA's. In summary, both proposed algorithms can achieve larger throughput limits than the original DIVBAR algorithm with REP.

The remainder of the paper is organized as follows: Section II presents the network model including the implementation of MIA, the timing diagram for one timeslot and queuing dynamics, and the RMIA and FMIA transmission schemes. Section III characterizes the network capacity region with

RMIA and compares it with the network capacity region with REP. Section IV describes the two proposed algorithms: DIVBAR-RMIA and DIVBAR-FMIA. Section V proves the throughput optimality of DIVBAR-RMIA with the RMIA assumption and proves the throughput performance guarantee of DIVBAR-FMIA. Section VI-A presents the simulation results. Section VII concludes the paper. Mathematical details of the proofs are relegated to Appendices.

II. Network Model

Consider a stationary wireless ad-hoc network with N nodes, denoted as node set $\mathcal{N}$. Multiple packet streams indexed by $c \in \{1, \cdots N\}$ are transmitted, possibly via multi-hop. Categorize all packets in the packet stream destined for a particular node c as *commodity c packets* irrespective of their origin. Each directed wireless link in the network is denoted as (n, k), where $n \in \mathcal{N}$ is the transmitting node and k is the receiving node belonging to the receiver set, denoted as $\mathcal{K}_n$, of node n. Data flows through the network in units of packet, all of which have the same fixed (positive) amount of information (entropy) H_0. Packets arriving at each node either exogenously or endogenously are stored in a queue waiting to be forwarded, except at the destination, where they leave the network immediately upon arrival/decoding. The transmission power of each node is constant.

Time is slotted and normalized into integer units $\tau = 0, 1, 2, 3, \cdots$. The timeslot length is assumed to be equal to the coherence time of the wireless medium in the network, so that we can adopt the common block-fading model: for each link, the (instantaneous) channel gain is constant within a timeslot duration, while it is i.i.d. (independent and identically distributed) across timeslots. Correspondingly, the amount of information transmitted over link (n, k) in timeslot τ, denoted as $R_{nk}(\tau)$, is i.i.d. across timeslots. Let $F_{R_{nk}}(x)$ represent the cdf (cumulative distribution function) of $R_{nk}(\tau)$. We make the following assumption on $R_{nk}(\tau)$ and $F_{R_{nk}}(x)$ for all $n \in \mathcal{N}$ and $k \in \mathcal{K}_n$:

Assumption 1. *$R_{nk}(\tau)$ is continuously distributed on $[0, \infty)$, and $0 < F_{R_{nk}}(H_0) < 1$.*[1]

Statistics of CSI of each link are known locally, i.e., at the node from which the link is emanating; however, instantaneous CSI (i.e., channel gains in a timeslot) is never known at any transmitting node. The exogenous packet arrival rate $a_n^{(c)}(\tau)$ (packet/timeslot) at node n for commodity c is i.i.d. across timeslots, and define a constant $A_{\max}$ satisfying $\sum_c a_n^{(c)}(\tau) \leq A_{\max}$. When a packet is transmitted by a node n in each timeslot, it can be simultaneously overheard by all the nodes in $\mathcal{K}_n$ ("multi-cast effect"). In this network model, a transmission of a packet over a link (n, k) can be interpreted as a process, in which a new copy of the packet is being created at the receiving node k while the original copy of packet is retained at the transmitting node n, and correspondingly, the multi-cast effect indicates that multiple copies of the same packet can be created at multiple receiving nodes simultaneously. However, after each *forwarding decision* is made among the nodes that have fully decoded the packet (including the transmitting node and successful receiving nodes), only the one node that gets the forwarding responsibility can keep the packet, while the others discard their copies. Here the *forwarding decision result can be either choosing a successful receiver to forward the packet to or retaining the packet by the transmitting node itself*. This packet-transferring procedure is consistent with the following single-copy routing assumption for all the policies discussed in this paper:

Assumption 2. *In each timeslot, the complete copy of each packet in the network can only be kept by one node.*[2]

Thus, if defining $b_{nk}^{(c)}(\tau)$ as the number of packets of commodity c that are forwarded from node n to node $k \in \mathcal{K}_n$ in timeslot τ, then based on Assumption 2, $b_{nk}^{(c)}(\tau) \in \{0, 1\}$ and $\sum_{c:c\in\mathcal{N}} \sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(\tau) \leq 1$, for all $n \in \mathcal{N}$.

A. Mutual Information Accumulation Technique

Ref. [15] analyzes the routing algorithms implemented based on REP, i.e., for each transmission, the packet either is successfully received at another node, or has to be completely re-transmitted in a later timeslot. As has been described in Section I, we suggest to avoid the inefficiencies of complete retransmission by enabling the MIA technique into the transmission scheme by using, e.g., Fountain codes.

In our scenario, we assume that each link uses a capacity-achieving coding scheme, so that a packet is received correctly in timeslot τ if the amount of partial information of the packet received by the end of timeslot τ exceeds the entropy of the packet H_0, i.e., a successful transmission from node n to node k in timeslot τ occurs when $\log_2(1 + \gamma_{nk}(\tau)) + I_k(\tau) \geq H_0$, where $\gamma_{nk}(\tau)$ is the SNR over link (n, k) in timeslot τ, whose distribution depends on the average channel state of link (n, k); $I_k(\tau)$ is the *pre-accumulated partial information* before timeslot τ, i.e., amount of partial information of the corresponding packet already accumulated in the receiving node k by timeslot $\tau - 1$. Moreover, although each receiving node may simultaneously overhear the signals transmitted from multiple neighbor nodes, we assume that there is no inter-channel interference among these signals and the successful reception of each signal is independent of the signals transmitted through other links.[3]

To implement routing with MIA technique, each node sets up two kinds of queues: the *compact packet queue* (CPQ)

[1]This assumption on $R_{nk}(\tau)$ and $F_{R_{nk}}(H_0)$ is mild and reasonable, since it is consistent with many practical wireless scenarios. For example, Rayleigh fading channels and Rice fading channels (see Ref. [22]) satisfy this assumption.

[2]In a wireless transmission scenario, due to the cooperative nature of the MIA technique, the network capacity regions under single-copy packet routing and multiple-copy routing (allowing transferring redundant copies of packets) can be different. This fact is different from the situation of using the REP transmissions scheme, where the network capacity regions under multiple-copy routing and single-copy routing are the same [15]. An efficient multiple-copy routing with the MIA technique involves designing efficient cooperation schemes for transmissions from different nodes, which has been beyond the policy space discussed in this paper.

[3]While this assumption is not practically realizable in wireless scenarios unless we use orthogonal channels, it is a standard assumption in the literature of stochastic network optimizations for wireless networks [13] [15].

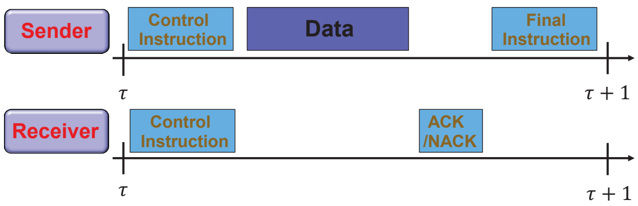


Fig. 1. Timing diagram of the working protocol within one timeslot

and *partial packet queue* (PPQ). CPQs are FIFO (*first in, first out*) buffers storing the packets that have already been decoded and are categorized by packets' commodities; while the pieces of partial information stored in PPQ are distinguished by the packets they belong to. As soon as the partial information of a specific packet accumulated in the PPQ of a receiving node exceeds the entropy of that packet, the packet is decoded and moved out of the PPQ, and then put into the CPQ if this node gets the forwarding responsibility, or discarded otherwise.

B. Timing Diagram in One Timeslot and Queuing Dynamics

The timing diagram of the communication protocol between each pair of sending node (sender) and receiving node (receiver) within one timeslot is illustrated in Fig. 1, which resembles the protocol in [15]. As is shown in this figure, at the beginning of each timeslot τ, the transmitting node and receiving node exchange their control instructions which include the backlog information of CPQs. Then the transmitting node makes the *transmitting decision* on which commodity to transmit or to keep silent in timeslot τ. After the decision is made, the data transmission starts and lasts for a fixed time period,[4] during which the coded bits of a packet with entropy H_0 are being transmitted and overheard by the receiving node(s). After the data transmission period ends, each receiving node sends an ACK/NACK signal back to the transmitting node through a stable control channel indicating whether the packet is successfully decoded by it during the transmission period (as will be shown in Section IV, each receiving node under the proposed DIVBAR-FMIA algorithm sends two kinds of ACK/NACK signals back to the transmitting node). Based on the ACK/NACK signals gathered from all the receiving nodes, node n may make the forwarding decision on which successful receiver to transfer the forwarding responsibility to or whether to retain the forwarding responsibility. This decision is made and is related to the transmission schemes being used (described in Subsection II-C). If a forwarding decision is made, a final instruction carrying the forwarding decision will be sent to the receiving nodes through the control channel at the end of the timeslot; otherwise, no final instruction will be sent to the receiving nodes.

The queuing dynamics over each timeslot is based on the above timing diagram. Let $Q_n^{(c)}(\tau)$ represent the backlog of the CPQ of commodity c at node n in timeslot τ. The backlog of commodity c at node n is updated over timeslot τ as follows:

$$Q_n^{(c)}(\tau+1) \leq \max\left\{Q_n^{(c)}(\tau) - \sum\nolimits_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(\tau), 0\right\} + \sum\nolimits_{k:k\in\mathcal{K}_n} b_{kn}^{(c)}(\tau) + a_n^{(c)}(\tau), \quad (1)$$

where the term $\sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(\tau)$ is the total output rate; $\sum_{k:k\in\mathcal{K}_n} b_{kn}^{(c)}(\tau)$ is the total endogenous input rate flowing from the neighbor nodes; $a_n^{(c)}(\tau)$ is the exogenous input rate. The expression in (1) is an inequality instead of an equality because the endogenous input rate of the data-carried packets may be less than $\sum_{k:k\in\mathcal{K}_n} b_{kn}^{(c)}(\tau)$. This occurs if a neighbor node $k \in \mathcal{K}_n$ has no data of commodity c to send (its CPQ of commodity c is empty), while the decision made by node k under an algorithm (policy) is to send a commodity c packet, so node k sends a *fake commodity* c *packet* that is counted into $\sum_{k:k\in\mathcal{K}_n} b_{kn}^{(c)}(\tau)$ while is not counted into $Q_n^{(c)}(\tau+1)$.

C. The RMIA and FMIA transmission schemes

For each transmitting node $n \in \mathcal{N}$ in the network under an arbitrary algorithm using the MIA technique, define an *epoch* for a node n as the sequence of timeslots that node n uses to transmit a copy of a packet: *in the sequence of timeslots when node* n *transmits packets of a commodity* c*, an epoch of commodity* c *starts from the first timeslot after a forwarding decision is made for a previous copy of commodity* c *packet, and ends at the timeslot when the forwarding decision is made for the current copy of commodity* c *packet*.

In this paper, we propose two versions of transmission schemes based on the MIA technique: the Renewal Mutual Information Accumulation (RMIA) transmission scheme and the Full Mutual Information Accumulation (FMIA) transmission scheme. Under both of the transmission schemes, each receiving node accumulates the received partial information to decode the corresponding packet. A key difference of the two schemes is whether they have the *renewal operation*:

- RMIA: once a transmitting node n confirms that one or more receiving nodes in $\mathcal{K}_n$ first successfully decode the copy of a packet being transmitted, node n makes the forwarding decision, which indicates the end of an epoch;[5] immediately after the forwarding decision is made, all the partial information of this packet accumulated at each receiving node is cleared, which is the renewal operation; the timeslot when the first successful reception occurs is called the *first-decoding timeslot*, and the set of successful receiving nodes is called the *first successful receiver set*. With RMIA, for each transmitting node in a network that satisfies Assumption 1, each of the receiving nodes becomes a member in the first successful receiver set with a positive probability in each first-decoding timeslot.

[4]The data transmission period is shorter than a timeslot, due to the time needed for control information. However, to simplify notation, we will henceforth neglect this overhead, which can be trivially taken into account for the throughput and delay results.

[5]In general, when to make the forwarding decision is part of the scheduling. But the RMIA transmission scheme analyzed in this paper has a fixed rule on when to make forwarding decision, which is equivalently to form a restricted policy space. The case of a general policy space with controlling the time of making forwarding decision is one of the future research directions.

- FMIA: after the forwarding decision is made (end of an epoch), instead of clearing the partial information of the copy of packet being transmitted, each receiving node $k \in \mathcal{K}_n$ that does not decode the packet retains the partial information of the packet and possibly uses it in the future decoding if node k overhears another copy of the same packet in later transmissions; here the forwarding decisions made by each node n are in the first-decoding timeslots with RMIA, i.e., once one or more receiving nodes firstly accumulate enough partial information to decode the packet without using the partial information retained before the beginning of transmitting the current copy of packet, a forwarding decision is made. In contrast to RMIA, FMIA allows the partial information of a packet to be accumulated from different transmitting nodes since the retained partial information can be from different transmitting nodes.

III. Network Capacity Region with Renewal Mutual Information Accumulation

In this section, we characterize the throughput potential of a stationary wireless ad-hoc network. For a multi-hop, multi-commodity network, let $(\lambda_n^{(c)})$ represent the matrix of exogenous time average input rates, where each entry $\lambda_n^{(c)} = \mathbb{E}\{a_n^{(c)}(\tau)\}$. Let $Y_n^{(c)}(t)$ represent the number of packets with source node n that have been successfully delivered to destination node c within the first t timeslots. Then a routing algorithm is *rate stable* if

$$\lim_{t\to\infty} \frac{Y_n^{(c)}(t)}{t} = \lambda_n^{(c)}, \text{ with prob. 1, } \forall n, c \in \mathcal{N}. \tag{2}$$

With the above definitions, Ref. [15] defines the *network capacity region* as the set of all exogenous input rate matrices $(\lambda_n^{(c)})$ that can be stably supported by the network using certain rate stable routing algorithms. However, considering the effect of the transmission scheme on the throughput performance, in this paper, we specify the network capacity region with different transmission schemes. For example, all the algorithms discussed in Ref. [15] are based on the Repetition (REP) transmission scheme, and therefore, we specify the network capacity region defined in Ref. [15] as *REP network capacity region*, denoted as Λ_{REP}. In our work, we define the *RMIA network capacity region*, denoted as Λ_{RMIA}, as the set of all exogenous input rate matrices that can be stably supported by the network using certain rate stable routing algorithms with the RMIA transmission scheme.

Throughout this paper, most of the analysis is based on analyzing the network's *d-timeslot Lyapunov drift*, which is defined as

$$\Delta_d(\mathbf{Q}(t_0)) \triangleq \frac{1}{d}\sum_{n,c}\mathbb{E}_\omega\left\{\left(Q_n^{(c)}(t_0+d)\right)^2 - \left(Q_n^{(c)}(t_0)\right)^2 \middle| \mathbf{Q}(t_0)\right\}, \tag{3}$$

where $d \geq 1$ is a certain positive interval length (in units of timeslots); t_0 is an arbitrary timeslot; the vector $\mathbf{Q}(t_0)$ represents the CPQ backlog state of the network in timeslot t_0; $\mathbb{E}_\omega$ is the expectation operator taken over ω, which represents any realization of the ensemble of channel states, exogenous packet arrivals and policy decisions of the whole network over the whole time horizon. In the rest part of the paper, the expectation operator $\mathbb{E}$ represents $\mathbb{E}_\omega$ for notational simplification. With the definition of d-timeslot Lyapunov drift, the following lemma will be used in the proofs of later theorems:

Lemma 1. *If there exists a constant $\varepsilon > 0$ and integer $d > 0$, such that for each timeslot t_0 and the backlog state $\mathbf{Q}(t_0)$ of the network in timeslot t_0, the d-timeslot Lyapunov drift satisfies:*

$$\Delta_d(\mathbf{Q}(t_0)) \leq B_0(d) - \varepsilon \sum_{n,c} Q_n^{(c)}(t_0), \tag{4}$$

where $B_0(d)$ is a constant depending on d, then the mean time average backlog of the whole network satisfies:

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n,c}\mathbb{E}\left\{Q_n^{(c)}(\tau)\right\} \leq \frac{B_0(d)}{\varepsilon}. \tag{5}$$

The proof of Lemma 1 is shown in Appendix A. A multi-hop, multi-commodity network is *strongly stable* when the mean time average total backlog is finite, as is shown in (5).

Let $F_{R_{nk}}^{(m)}(x)$ represent the cdf of $\sum_{\tau=1}^{m} R_{nk}(\tau)$, where $F_{R_{nk}}^{(1)}(x) = F_{R_{nk}}(x)$; let $F_{R_{nk}}^{(0)}(x) = 1$. To facilitate the theoretical analysis later, we then have the following lemma to characterize the basic statistical properties of the flow rates:

Lemma 2. *Based on Assumption 1, we have the following relations:*

$$F_{R_{nk}}^{(m)}(H_0) < F_{R_{nk}}^{(m-1)}(H_0)\, F_{R_{nk}}(H_0), \quad \text{for } m \geq 2; \tag{6}$$

$$F_{R_{nk}}^{(m)}(H_0) < \left[F_{R_{nk}}(H_0)\right]^m, \quad \text{for } m \geq 2; \tag{7}$$

$$F_{R_{nk}}^{(m)}(H_0) < F_{R_{nk}}^{(m')}(H_0), \quad \text{for } m > m' \geq 0. \tag{8}$$

The proof of Lemma 2 is in Appendix B. Eq. (6)-(8) demonstrate the fact that the success probability of the transmissions of a packet over a link in a time interval (i) increases by applying MIA instead of applying REP and (ii) increases with the length of the considered time interval.

A. The network capacity region with RMIA

To begin with, we re-state the characterization of the REP network capacity region derived in Ref. [15]:

Theorem 1. *The (REP) network capacity region Λ_{REP} consists of all the exogenous time average input rate matrices $(\lambda_n^{(c)})$, for each of which there exists a stationary randomized policy (with REP and single-copy routing), denoted as $Policy^{**}$, that chooses probabilities $\alpha_n^{**(c)}$, $\theta_{nk}^{**(c)}(\Psi_n)$, and forms the time average flow rate taking value $b_{nk}^{**(c)}$ with prob. 1, for all nodes $n, c \in \mathcal{N}$, $k \in \mathcal{K}_n$ and all the subsets $\Psi_n \subseteq \mathcal{K}_n$, such that:*

$$b_{nk}^{**(c)} \geq 0, b_{cn}^{**(c)} = 0, b_{nn}^{**(c)} = 0, \text{ for } n \neq c, \tag{9}$$

$$\sum_{k:k\in\mathcal{K}_n} b_{kn}^{**(c)} + \lambda_n^{(c)} \leq \sum_{k:k\in\mathcal{K}_n} b_{nk}^{**(c)}, \text{ for } n \neq c, \tag{10}$$

$$b_{nk}^{**(c)} = \alpha_n^{**(c)} \sum_{\Psi_n:\Psi_n\subseteq\mathcal{K}_n} q_{n,\Psi_n}^{\text{rep}}\, \theta_{nk}^{**(c)}(\Psi_n), \tag{11}$$

where $\alpha_n^{**(c)}$ is the probability that node n decides to transmit a packet of commodity c in each timeslot; $q_{n,\Psi_n}^{\mathrm{rep}}$ is the probability that Ψ_n is the successful receiver set for a packet transmitted by node n with REP; $\theta_{nk}^{**(c)}(\Psi_n)$ is the conditional probability that node n forwards a packet of commodity c to node k, given that the successful receiver set is Ψ_n.

In Theorem 1, the REP transmission scheme is used in $Policy^{**}$ belonging to the class of *stationary randomized policy*, under which each node n uses a fixed probability to choose each commodity to transmit in each timeslot (the transmission decision) and a fixed probability to forward the decoded packet to each successful receiver (the forwarding decision). The superscript $**$ of a variable indicates that the value of the variable is determined under $Policy^{**}$; in the later part of the paper, we use a similar notational convention for the variables specified by other specific policies. The detailed proof of Theorem 1 is given in Ref. [15].

Theorem 1 demonstrates the fact that throughput optimality among all possible policies with REP can be achieved within the class of stationary randomized policies with single-copy routing. Although directly obtaining the parameters for the stationary randomized policy is generally difficult due to the geometric complexity of the network, the characterization shown in Theorem 1 is useful in the performance analysis of DIVBAR.

In our work, an analogous statement can be made to characterize the RMIA network capacity region:

Theorem 2. *For a network satisfying Assumption 1, the RMIA network capacity region Λ_{RMIA} under Assumption 2 consists of all the exogenous time average input rate matrices $(\lambda_n^{(c)})$, for each of which there exists a stationary randomized policy (with RMIA), denoted as $Policy^{*}$, that chooses probability $\alpha_n^{*(c)}$, $\theta_{nk}^{*(c)}(\Omega_n)$ and forms the time average flow rate taking value $b_{nk}^{*(c)}$ with prob. 1, for all nodes $n, c \in \mathcal{N}$, $k \in \mathcal{K}_n$ and all the nonempty subsets $\Omega_n \subseteq \mathcal{K}_n$, such that:*

$$b_{nk}^{*(c)} \geq 0, b_{cn}^{*(c)} = 0, b_{nn}^{*(c)} = 0, \text{ for } n \neq c, \tag{12}$$

$$\sum_{k:k\in\mathcal{K}_n} b_{kn}^{*(c)} + \lambda_n^{(c)} \leq \sum_{k:k\in\mathcal{K}_n} b_{nk}^{*(c)}, \text{for } n \neq c, \tag{13}$$

$$b_{nk}^{*(c)} = \alpha_n^{*(c)} \beta_n^{\mathrm{rmia}} \sum_{\Omega_n:\Omega_n\subseteq\mathcal{K}_n,\Omega_n\neq\emptyset} q_{n,\Omega_n}^{\mathrm{rmia}} \theta_{nk}^{*(c)}(\Omega_n), \tag{14}$$

where $\alpha_n^{*(c)}$ is the probability that node n decides to transmit a packet of commodity c in each timeslot; β_n^{rmia} is the inverse value of the expected epoch length for node n;[6] $q_{n,\Omega_n}^{\mathrm{rmia}}$ is the probability that Ω_n is the first successful receiver set in each epoch for node n; $\theta_{nk}^{*(c)}(\Omega_n)$ is the conditional probability that node n forwards a packet of commodity c to node k, given that the first successful receiver set is Ω_n.

The detailed proof of Theorem 2 consists of a necessity part and a sufficiency part, which are shown in Appendix C and Appendix D, respectively. The necessity part is proven by showing that the given constraints (12)-(14) are necessary for network stability; the sufficiency part is proven by showing that strong stability is achieved under $Policy^{*}$ with the input rate matrix $(\lambda_n^{(c)})$ *interior* to Λ_{RMIA}.

Similar to Theorem 1 characterizing the REP network capacity region, Theorem 2 demonstrates the fact that, assuming single-packet-copy routing, throughput optimality among all possible policies with RMIA can be achieved within the class of stationary randomized policies, which can be used for the theoretical analysis of DIVBAR-RMIA and DIVBAR-FMIA.

B. Network capacity region: RMIA versus REP

In contrast to REP, RMIA potentially increases the success probability of a transmission attempt over each link in the network by using the pre-accumulated information. Therefore, for each transmitting node, the set of successful receivers can be enlarged by applying RMIA instead of REP (possibly enlarged from an empty set to a non-empty set), and correspondingly, a positive supportable average flow increase can be obtained over each outgoing link. Specifically, when a node n transmits a copy of a packet, if using RMIA, the first successful receiver set is $\Omega_n \subseteq \mathcal{K}_n$ in the first-decoding timeslot τ, while if using REP with the same channel realizations, the successful receiver set in timeslot τ is $\Psi_n \subseteq \mathcal{K}_n$ instead, and we have $\Psi_n \subseteq \Omega_n$. Based on these facts, we define $q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}}$ as the probability that the first successful receiver set for node n is Ω_n in the first-decoding timeslot of an epoch with RMIA, while the successful receiver set for node n in the same timeslot is Ψ_n with REP. $q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}}$ is used in the proof of the following theorem showing that the RMIA network capacity region covers the REP network capacity region:

Theorem 3. *For a network satisfying Assumption 1 and any input rate matrix $(\lambda_n^{(c)}) \in \Lambda_{\mathrm{REP}}$, there exists a stationary randomized policy with RMIA satisfying Assumption 2 that can stably support $(\lambda_n^{(c)})$, which indicates that $\Lambda_{\mathrm{REP}} \subseteq \Lambda_{\mathrm{RMIA}}$.*

The detailed proof is in Appendix E.

With Theorem 3, given a non-zero input rate matrix $(\lambda_n^{(c)}) \in \Lambda_{\mathrm{REP}}$ stably supported by $Policy^{**}$ with REP, we can further quantitatively characterize the (time average) input rate increase that can be guaranteed to support by using RMIA instead of REP over a *simple path* l_{n_0,c_0} from a source node n_0 to a destination node c_0, along which a positive time average flow has been formed under $Policy^{**}$.[7] This characterization is summarized as the following theorem:

Theorem 4. *For a network satisfying Assumption 1, if an input rate matrix $(\lambda_n^{(c)}) \in \Lambda_{\mathrm{REP}}$ has a positive entry $\lambda_{n_0}^{(c_0)}$, where (n_0, c_0) is a source-destination pair, then there exists an input rate matrix $({\lambda'}_n^{(c)})(l_{n_0,c_0}) \in \Lambda_{\mathrm{RMIA}}$, such that its (n,c)th entry satisfies*

$$\lambda'_n(l_{n_0,c_0}) = \begin{cases} \lambda_{n_0}^{(c_0)} + \delta_{l_{n_0,c_0}}^{(c_0)}, & \text{if } n = n_0, c = c_0 \\ \lambda_n^{(c)}, & \text{otherwise} \end{cases}. \tag{15}$$

[6]With Assumption 1, the expectation of the epoch length for each node exists.

[7]A path having a positive time average flow under a stationary randomized policy is defined as the path, on which each link has a positive time average flow rate (with prob. 1) under the policy.

In (15), we have $\delta_{l_{n_0,c_0}}^{(c_0)} \triangleq \min_{(n,k)\in l_{n_0,c_0}}\{\delta_{nk}^{(c_0)}\}$*, in which*

$$\delta_{nk}^{(c_0)} = \begin{cases} \alpha_n^{**(c_0)}\beta_n^{\mathrm{rmia}}\left(\sum_{\Omega_n:k\in\Omega_n} q_{n,\emptyset,\Omega_n}^{\mathrm{rep,rmia}} \right. \\ \left. + \sum_{\Omega_n:\{k,p_{l_{n_0,c_0}}(n)\}\subseteq\Omega_n} q_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}^{\mathrm{rep,rmia}}\right), & \text{if } n\neq n_0, \\ \alpha_{n_0}^{**(c_0)}\beta_{n_0}^{\mathrm{rmia}}\sum_{\Omega_{n_0}:k\in\Omega_{n_0}} q_{n_0,\emptyset,\Omega_{n_0}}^{\mathrm{rep,rmia}}, & \text{if } n=n_0, \end{cases} \tag{16}$$

where $p_{l_{n_0,c_0}}(n)$ *is the predecessor node of node* n *on path* l_{n_0,c_0}.

The intuition of Theorem 4 is based on characterizing the flow increase over a link (n,k) on path l_{n_0,c_0}: in the first-decoding timeslots (with RMIA) when $\Psi_n=\emptyset$ or $\Psi_n=\{p_{l_{n_0,c_0}}(n)\}$ with REP, while Ω_n is nonempty and $k\in\Omega_n$ or $\{k,p_{l_{n_0,c_0}}(n)\}\subseteq\Omega_n$ with RMIA, then the transmitting node n retains the packet in these timeslots with REP,[8] while it can forward the packet to node k with RMIA. Therefore, a time average flow increase can be obtained on link (n,k). The detailed proof is shown in Appendix F.

With the result of Theorem 4, it immediately follows that the RMIA network capacity region is strictly larger than the REP network capacity region under certain mild assumptions:

Corollary 1. *For a network satisfying Assumption 1, the RMIA network capacity region* Λ_{RMIA} *under Assumption 2 is strictly larger than the REP network capacity region* Λ_{REP}*, i.e.,* $\Lambda_{\mathrm{RMIA}}\supset\Lambda_{\mathrm{REP}}$.

Corollary 1 demonstrates that the network has the potential of supporting larger input data rates by using RMIA instead of using REP, and its proof is given in Appendix G.

IV. Diversity Backpressure Routing Algorithms with Mutual Information Accumulation

In this section, we propose two routing algorithms using the MIA technique: DIVBAR-RMIA and DIVBAR-FMIA, which respectively use the RMIA and FMIA transmission schemes, in order to further enhance the throughput performance in comparison with the traditional DIVBAR algorithm with the REP transmission scheme in [15].

A. Diversity Backpressure Routing with Renewal Mutual Information Accumulation (DIVBAR-RMIA)

We summarize the implementation of the DIVBAR-RMIA algorithm at each node n for its ith epoch in the following steps, where the variables with the notation form $\hat{x}$ are specified by DIVBAR-RMIA:

1) In the starting timeslot $\hat{u}_{n,i}$ of each epoch i for the transmitting node n, node n observes the CPQ backlog $\hat{Q}_k^{(c)}(t)$ of each commodity $c\in\mathcal{N}$ at each of its potential receivers $k\in\mathcal{K}_n$. With its own CPQ backlogs $\hat{Q}_n^{(c)}(t)$, node n computes the *differential backlog coefficient* $\hat{W}_{nk}^{(c)}(\hat{u}_{n,i})$ as follows:

$$\hat{W}_{nk}^{(c)}(\hat{u}_{n,i}) = \max\left\{\hat{Q}_n^{(c)}(\hat{u}_{n,i}) - \hat{Q}_k^{(c)}(\hat{u}_{n,i}),0\right\}. \tag{17}$$

2) For each commodity c, the (receiving) nodes in $\mathcal{K}_n$ are ranked according to their differential backlog coefficients sorted in descending order. Define $\hat{\mathcal{R}}_{nk}^{\mathrm{high},(c)}(\hat{u}_{n,i})$ and $\hat{\mathcal{R}}_{nk}^{\mathrm{low},(c)}(\hat{u}_{n,i})$ respectively as the set of the nodes in $\mathcal{K}_n$ with higher and lower ranks than node $k\in\mathcal{K}_n$ in timeslot $\hat{u}_{n,i}$. Define $\hat{\varphi}_{nk}^{(c)}(i)$ as the probability that, in a first decoding timeslot, node $k\in\mathcal{K}_n$ belongs to the first successful receiver set, while the nodes in $\hat{\mathcal{R}}_{nk}^{\mathrm{high},(c)}(\hat{u}_{n,i})$ do not successfully decode, i.e, node k has the highest priority among the successful receivers in the first successful receiver set.
3) Define $\hat{c}_n(i)$ as the optimal commodity that maximizes the following backpressure metric:
$$\sum_{k:k\in\mathcal{K}_n}\hat{W}_{nk}^{(c)}(\hat{u}_{n,i})\hat{\varphi}_{nk}^{(c)}(i). \tag{18}$$
Define $\hat{\Xi}_n(i)$ as the corresponding maximum value:
$$\hat{\Xi}_n(i) = \sum_{k:k\in\mathcal{K}_n}\hat{W}_{nk}^{(\hat{c}_n(i))}(\hat{u}_{n,i})\hat{\varphi}_{nk}^{(\hat{c}_n(i))}(i). \tag{19}$$
4) If $\hat{\Xi}_n(i)>0$, node n chooses a packet stored at the head of its CPQ of commodity $\hat{c}_n(i)$ to transmit for the epoch i: node n keeps transmitting a copy of packet of commodity $\hat{c}_n(i)$ with the MIA technique in a contiguous sequence of timeslots starting from timeslot $\hat{u}_{n,i}$ to a timeslot (the first-decoding timeslot), in which one or more nodes in $\mathcal{K}_n$ firstly accumulate enough partial information and decode the packet (this can be detected by checking the ACK/NACK feedbacks in each timeslot); else, node n transmits a *null packet* for epoch i with the same procedure representing a silent epoch.
5) In the first-decoding timeslot (after the data transmission period in this timeslot), the forwarding decision is made:
   - If the transmitted packet is not null, node n finds the successful receiver $\hat{k}(i)$ with the largest differential backlog coefficient $\hat{W}_{n\hat{k}(i)}^{(\hat{c}_n(i))}(\hat{u}_{n,i})$ and checks the coefficient's value. If $\hat{W}_{n\hat{k}(i)}^{(\hat{c}_n(i))}(\hat{u}_{n,i})>0$, node n shifts the forwarding responsibility to node $\hat{k}(i)$, while other successful receivers discard their copies of the packet; else, node n retains the forwarding responsibility, while all the successful receivers discard their copies of the packets.
   - If the transmitted packet is null, each successful receiver discards its copy of the null packet.
6) By the end of the first-decoding timeslot, all the partial information accumulated at the nodes in $\mathcal{K}_n$ is cleared.

In step 3) of the above summary of the DIVBAR-RMIA algorithm, $\hat{\varphi}_{nk}^{(c)}(i)$ can be computed with the knowledge of

[8] Because the time average packet flow over path l_{n_0,c_0} is positive under the policy with REP, the policy with REP can be assumed to assign zero forwarding probability on the reverse link $(n,p_{l_{n_0,c_0}}(n))$; otherwise, a time average flow loop will be formed between node n and node $p_{l_{n_0,c_0}}(n)$, which can be eliminated without affecting the net flow value.

the average CSI:

$$\hat{\varphi}_{nk}^{(c_n)}(i) = \sum_{m=1}^{\infty} \left[F_{R_{nk}}^{(m-1)}(H_0) - F_{R_{nk}}^{(m)}(H_0) \right] \times \prod_{j:j\in\hat{\mathcal{R}}_{nk}^{\text{high},(c)}(u_{n,i})} F_{R_{nj}}^{(m)}(H_0) \prod_{j:j\in\hat{\mathcal{R}}_{nk}^{\text{low},(c)}(u_{n,i})} F_{R_{nj}}^{(m-1)}(H_0). \quad (20)$$

Based on the assumption of using orthogonal channels for transmissions to each receiving node, the DIVBAR-RMIA algorithm is a distributed algorithm, where each node n makes local scheduling and routing decisions based on the CPQ backlog information of itself and of its neighbor nodes in $\mathcal{K}_n$ and the average channel state information of its outgoing links. Moreover, as shown in (19), a key feature of DIVBAR-RMIA is that each local transmission decision is made for an epoch, which consists of random number of timeslots determined by the channel realizations of the outgoing links of the transmitting node.

B. Diversity Backpressure Routing with Full Mutual Information Accumulation (DIVBAR-FMIA)

In contrast with DIVBAR-RMIA, DIVBAR-FMIA does not have the "regular" renewal operations by the end of each epoch but retains the partial information of each packet in the network until the packet has been delivered to the destination. Retaining the partial information can further facilitate the decoding of this packet if the retaining receiving node overhears the transmissions of this packet later, possibly from a different transmitting node. In this paper, we propose a version of the DIVBAR-FMIA algorithm that is associated with the DIVBAR-RMIA algorithm, such that it is set to perform in synchronized epochs with DIVBAR-RMIA, i.e., each node makes the forwarding decisions under DVIBAR-FMIA in the same timeslots as when the node makes the forwarding decisions if under DIVBAR-RMIA.

Define $\hat{\hat{p}}_{n,i}$ as the packet being transmitted by node n in its ith epoch under DIVBAR-FMIA. Let $\hat{\hat{I}}_k^{\text{pre}}(\hat{\hat{p}}_{n,i}, \tau)$ represent the amount of pre-accumulated partial information of packet $\hat{\hat{p}}_{n,i}$ before timeslot τ stored at node k. Additionally, define $\hat{\hat{I}}_{nk}^{\text{rmia}}(\hat{\hat{p}}_{n,i}, \hat{\hat{u}}_{n,i}, \tau)$ as the amount of partial information of packet $\hat{\hat{p}}_{n,i}$ purely accumulated by the transmissions in the time interval from timeslot $\hat{\hat{u}}_{n,i}$ to the timeslot τ, where $\hat{\hat{u}}_{n,i}$ is the index of the starting timeslot of epoch i for node n under DIVBAR-FMIA, and where τ is the index of an arbitrary timeslot in epoch i for node n. Then the DIVBAR-FMIA algorithm summarization for each node n and each epoch i is as follows, where we denote its specified variables in the form of $\hat{\hat{x}}$:

1) At the beginning of timeslot $\hat{\hat{u}}_{n,i}$, node n executes the similar steps as Step 1) - 4) of DIVBAR-RMIA based on observations of $\hat{\hat{\mathbf{Q}}}(\hat{\hat{u}}_{n,i})$ and the $\hat{\hat{\varphi}}_{nk}^{(c_n)}(i)$, where $k \in \mathcal{K}_n$, in order to make the transmission decisions on whether to choose a packet $p_{n,i}$ stored at the head of its CPQ of commodity $\hat{\hat{c}}_n(i)$ to transmit for the epoch i.
2) On the receiver side, in each timeslot τ of epoch i, each receiving node sends two feedback signals back to node n after the data transmission period in timeslot τ: $(\text{ACK/NACK})_{\text{FMIA}}$ and $(\text{ACK/NACK})_{\text{RMIA}}$. $(\text{ACK/NACK})_{\text{FMIA}}$ indicates whether node k successfully decodes the packet with the FMIA transmission scheme, which is true if $\hat{\hat{I}}_k^{\text{pre}}(\hat{\hat{p}}_{n,i}, \hat{u}_{n,i}) + \hat{\hat{I}}_{nk}^{\text{rmia}}(\hat{\hat{p}}_{n,i}, \hat{\hat{u}}_{n,i}, \tau) \geq H_0$; $(\text{ACK/NACK})_{\text{RMIA}}$ indicates whether the partial information accumulated at node k purely during the current epoch had been enough to decode if using the RMIA transmission scheme, which would be true if $\hat{\hat{I}}_{nk}^{\text{rmia}}(\hat{\hat{p}}_{n,i}, \hat{\hat{u}}_{n,i}, \tau) \geq H_0$.
3) After gathering all the $(\text{ACK/NACK})_{\text{FMIA}}$ and $(\text{ACK/NACK})_{\text{RMIA}}$ feedbacks from the receiving nodes in timeslot τ, node n firstly checks $(\text{ACK/NACK})_{\text{RMIA}}$ feedbacks to confirm if there is any receiving node whose accumulated information $\hat{\hat{I}}_{nk}^{\text{rmia}}(\hat{\hat{p}}_{n,i}, \hat{\hat{u}}_{n,i}, \tau)$ exceeds H_0. If not, node n will keep transmitting in timeslot $\tau + 1$; otherwise, timeslot τ is the ending timeslot of epoch i for node n (also the first-decoding timeslot with RMIA), and the forwarding decision will be made:
   - If the transmitted packet is not null, node n further checks the gathered $(\text{ACK/NACK})_{\text{FMIA}}$ feedbacks, based on which node n finds the successful receiver $\hat{\hat{k}}(i)$ with the largest differential backlog coefficient $\hat{\hat{W}}_{n\hat{\hat{k}}(i)}^{(\hat{\hat{c}}_n(i))}(\hat{\hat{u}}_{n,i})$ and checks the coefficient's value. If $\hat{\hat{W}}_{n\hat{\hat{k}}(i)}^{(\hat{\hat{c}}_n(i))}(\hat{\hat{u}}_{n,i}) > 0$, node n shifts the forwarding responsibility to node $\hat{\hat{k}}(i)$, while other successful receivers discard their copies of the packet; else, node n retains the forwarding responsibility, while all the successful receivers discard their copies of the packets.
   - If the transmitted packet is null, each successful receiver discards its copy of the null packet, and each unsuccessful receiver also discards the partial information of the null packet they have received.

Additional to the steps in each epoch shown above, after the packet $\hat{\hat{p}}_{n,i}$ is delivered to its destination, all the partial information of packet $\hat{\hat{p}}_{n,i}$ stored in the network is cleared in order to free up the memory.

According to above algorithm summary, DIVBAR-FMIA can also be a distributed algorithm, because each node n makes local scheduling and routing decisions based on the CPQ backlog information of itself and of its neighbor nodes in $\mathcal{K}_n$ and the average channel state information of its outgoing links. However, the practical implementation for DIVBAR-FMIA is more challenging than that for DIVBAR-RMIA, for instances, efficiently accumulating the partial information of the same packet from two different transmitting nodes requires extra coordination in the implementation of the rateless codes; a notification signal has to be broadcast to inform the nodes in the network to clear the partial information of a delivered packet; extra storage and efficient retrieving mechanism for the retained partial information in the PPQs have to be properly set up. In the later part of the paper, we assume these implementation issues have been properly done for DIVBAR-FMIA and analyze its throughput performance.

V. Performance Analysis

In this section, the performances of DIVBAR-RMIA and DIVBAR-FMIA are evaluated. As will be shown in Theorem 5, we first prove the throughput optimality of DIVBAR-RMIA among all possible algorithms with RMIA. Secondly, as will be shown in Theorem 6 and Corollary 3, we prove that DIVBAR-FMIA's throughput performance is at least as good as DIVBAR-RMIA's.

A. Throughput optimality of DIVBAR-RMIA among all possible policies with RMIA

In this subsection, our goal is to analyze the throughput performance of the DIVBAR-RMIA algorithm and show that it is throughput optimal among all possible policies with RMIA.

To begin with, we give an initial analysis of the backpressure metric under DIVBAR-RMIA over a single epoch. Consider a policy under which each epoch consists of contiguous timeslots. Define $Z_n\left(i, \mathbf{Q}\left(\tau\right)\right)$ as the following metric over the ith epoch under such a policy:

$$Z_n\Big(i, \hat{\mathbf{Q}}\left(\tau\right)\Big) = \sum_c \sum_{\tau'=u_{n,i}}^{u_{n,i+1}-1} \sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}\left(\tau'\right)\Big[Q_n^{(c)}\left(\tau\right) - Q_k^{(c)}\left(\tau\right)\Big], \tag{21}$$

where $u_{n,i}$ is the starting timeslot of the ith epoch. Let $\mathcal{P}$ represent the set of policies with RMIA consisting of DIVBAR-RMIA and all the policies having synchronous epochs with DIVBAR-RMIA. With the definitions of $Z_n\left(i, \mathbf{Q}\left(\tau\right)\right)$ and $\mathcal{P}$, we propose Lemma 3 as follows to demonstrate the origin of the backpressure metric formulation (18) in step 3) of the DIVBAR-RMIA algorithm summary.

Lemma 3. *The metric $\mathbb{E}\{ Z_n(i, \hat{\mathbf{Q}}\left(u_{n,i}\right)) \big| \hat{\mathbf{Q}}\left(u_{n,i}\right)\}$ for each node n under an arbitrary policy within policy set $\mathcal{P}$ is upper bounded by $\hat{\Xi}_n\left(i\right)$, and this upper bound is achieved under the DIVBAR-RMIA algorithm.*

Lemma 3 characterizes the key feature of DIVBAR-RMIA over a single epoch, and its proof is given in Appendix H.

Based on Lemma 3, the following theorem shows that strong stability can be achieved by DIVBAR-RMIA for any input rate matrix in the interior of Λ_{RMIA}, which demonstrates that DIVBAR-RMIA is a throughput optimal algorithm among all the algorithms with the RMIA transmission scheme.

Theorem 5. *For a network satisfying Assumption 1, DIVBAR-RMIA is throughput optimal among the algorithms with RMIA: for an exogenous input rate matrix $(\lambda_n^{(c)})$, if there exists an $\varepsilon > 0$ satisfying $(\lambda_n^{(c)} + \varepsilon) \in \Lambda_{\text{RMIA}}$, then there exists an integer $D > 0$, such that the mean time average CPQ backlog of the whole network can be upper bounded as follows:*

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} \mathbb{E}\left\{\hat{Q}_n^{(c)}\left(\tau\right)\right\} \leq \frac{2\left[B\left(D\right) + C\left(D\right)\right]}{\varepsilon}, \tag{22}$$

when implementing the DIVBAR-RMIA algorithm, where

$$B\left(D\right) \triangleq ND\left[1 + \left(N + A_{\max}\right)^2\right];\ C\left(D\right) \triangleq 4ND\left(N + A_{\max} + 1\right).$$

The proof of Theorem 5 is given in Appendix I. The proof follows the strategy of comparing the upper bounds (containing the backpressure metrics) formulated for the D-timeslot Lyapunov drift under the DIVBAR-RMIA algorithm and under the stationary randomized policy with RMIA ($Policy^*$) that stably supports $(\lambda_n^{(c)})$. Note that the success of transmitting a packet with RMIA depends on the total amount of partial information transmitted over possible multiple timeslots, which motivates the necessity of analyzing the queue length evolvements under DIVBAR-RMIA over multi-timeslot intervals. The proof of Theorem 5 shows the existence of a finite interval length D, such that, by doing Lyapunov drift analysis over every D-timeslot interval, the mean time average CPQ backlog under DIVBAR-RMIA can be upper bounded within the interior of Λ_{RMIA}.[9]

With Theorem 5, we claim the following corollary:

Corollary 2. *For a network satisfying Assumption 1, the throughput achieved by DIVBAR-RMIA is strictly larger than that achieved by DIVBAR (with REP).*

Proof: According to Theorem 5, Λ_{RMIA} is the set of input rate matrices that can be supported by DIVBAR-RMIA. Moreover, reviewing the result of Corollary 1, Λ_{RMIA} is strictly larger than Λ_{REP} given Assumption 1. Thus, with the same assumption, DIVBAR-RMIA can achieve strictly larger throughput than DIVBAR, since Λ_{REP} is the set of input rate matrices that can be supported by DIVBAR (see [15]). ■

B. Throughput performance of DIVBAR-FMIA

Since the proposed DIVBAR-FMIA algorithm is set to have synchronous epochs with DIVBAR-RMIA, with the help of the pre-accumulated information in the receiving nodes by the beginning of each epoch, the successful receiver set at the end of each epoch under DIVBAR-FMIA should include the first successful receiver set under DIVBAR-RMIA. This intuition indicates that the throughput performance of DIVBAR-FMIA should be at least as good as DIVBAR-RMIA, and yields the following theorem:

Theorem 6. *For a network satisfying Assumption 1, for any $(\lambda_n^{(c)})$ interior to Λ_{RMIA}, DIVBAR-FMIA yields the strong stability of the network: there exists an integer $D > 0$, such that the mean time average CPQ backlog of the whole network can be upper bounded as follows:*

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} \mathbb{E}\left\{\hat{\hat{Q}}_n^{(c)}\left(\tau\right)\right\} \leq \frac{2\left[B\left(D\right) + C\left(D\right)\right]}{\varepsilon}, \tag{23}$$

when implementing the DIVBAR-FMIA algorithm, where the positive ε satisfies: $(\lambda_n^{(c)} + \varepsilon) \in \Lambda_{\text{RMIA}}$.

The proof of Theorem 6 is given in Appendix J, and the proof strategy is similar to that of Theorem 5.

[9] Comparing with the mean time average backlog upper bound expression for DIVBAR with REP, which has a similar structure and can be computed through 1-timeslot Lyapunov drift analysis (see [15]), for a $(\lambda_n^{(c)})$ within Λ_{REP}, the upper bound in (22) could be smaller because of the existence of a possibly larger ε value in the denominator due to $\Lambda_{\text{RMIA}} \supset \Lambda_{\text{REP}}$ (according to Corollary 1), or could be larger because of different expressions in the numerator.

Note that retaining the partial information does not affect the stability of the network under DIVBAR-FMIA either, assuming that the retained partial information of a packet is cleared once the packet is delivered. To explain the reason, consider that in each timeslot τ, a packet stored at a node n must have at most $N-1$ pieces of partial information respectively stored in the PPQs at the other $N-1$ nodes, and the backlog of each piece of partial information is less than 1 (in unit of packet). Therefore, the total PPQ backlog in timeslot τ is no more than $(N-1)\sum_{n,c}\hat{\hat{Q}}_n^{(c)}(\tau)$, and according to (23) in Theorem 6, the mean time average PPQ backlog is also upper bounded.

Moreover, based on Theorem 6, we can further compare DIVBAR-FMIA and DIVBAR-RMIA in the throughput performance by showing the following corollary:

Corollary 3. *For a network satisfying Assumption 1, the throughput performance of DIVBAR-FMIA is at least as good as DIVBAR-RMIA.*

Proof: According to Theorem 6, DIVBAR-FMIA is able to support any input rate matrix within Λ_{RMIA}, which indicates that any input rate matrix that can be stably supported by DIVBAR-RMIA can also be stably supported by DIVBAR-FMIA, i.e., the throughput performance of DIVBAR-FMIA is at least as good as DIVBAR-RMIA. ■

VI. Simulations

Example simulations over 10^6 timeslots are carried out in the ad-hoc wireless network shown in Fig. 2. All the links in the network are independent non-interfering links, each of which is subject to Rayleigh fading (independent among links and timeslots), while the average channel states are static. The number on each link represents the mean SNR value (linear scale) over that link; the time average exogenous input rates $\lambda_1^{(9)}$ and $\lambda_2^{(10)}$ are set to be the same.

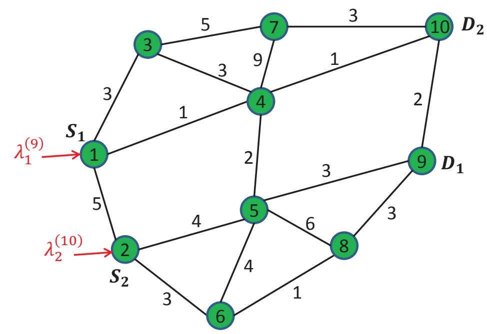


Fig. 2. The ad-hoc network being simulated

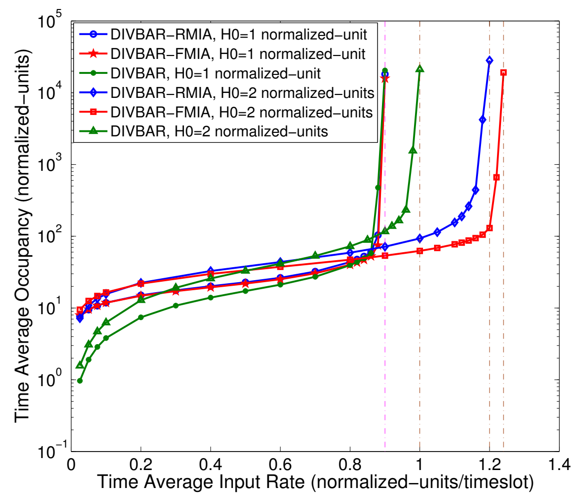


Fig. 3. Throughput performance comparison among DIVBAR-RMIA, DIVBAR-FMIA and DIVBAR algorithms with different packet lengths

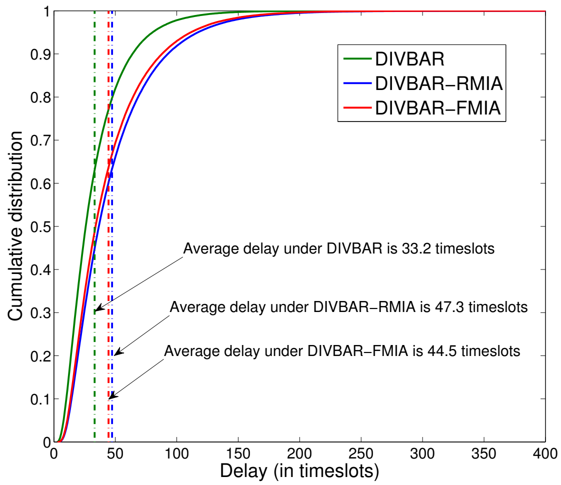
A. Throughput performance

Simulations are conducted comparing throughput performance of the three algorithms: DIVBAR-FMIA, DIVBAR-RMIA, and traditional DIVBAR (with REP). Fig. 3 shows the time average occupancy (total time average backlog in the network measured in *normalized-units*) vs. exogenous time average input rate measured in normalized-units/timeslot. Here a normalized-unit has to be long enough (contain sufficient number of bits) to allow the application of a capacity achieving code. The maximum supportable throughput corresponds to the exogenous time average input rate at which the occupancy goes towards very large values (due to a finite simulation time, it does not approach infinity in our simulations). As is shown in the figure, the throughput under DIVBAR-RMIA algorithm is smaller than that of DIVBAR-FMIA; the throughput under both algorithms are larger than that of the regular DIVBAR algorithm. These observations are in line with the theoretical analysis.

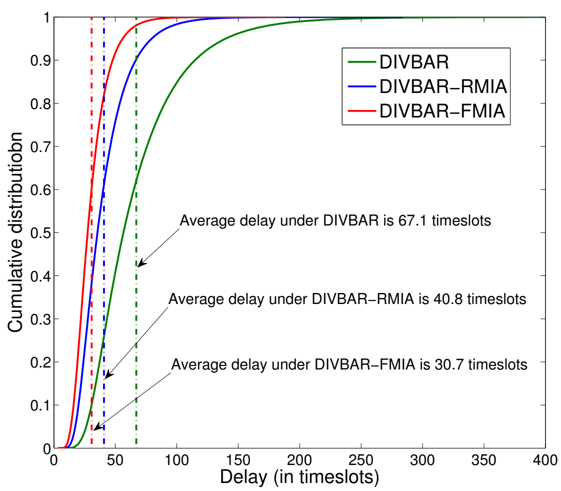
The simulation of the throughput comparison is carried out under different packet entropy conditions. The entropy contained in each packet is denoted by H_0 as is shown in the figure. When $H_0 = 1$ normalized-unit, Fig. 3 shows that the throughput under the three algorithms are nearly identical. This phenomenon is caused by the fact that the packet length is generally small compared to the transmission ability of the links in the network. Therefore nodes in the network can usually achieve a successful transmission over a link at the first attempt, which results in that using the MIA technique in the transmissions has little benefit. However, as H_0 increases to 2 normalized-units, the success probability in a single attempt decreases. Nodes under regular DIVBAR increase the chance of successful transmission just through trying more times, while DIVBAR-FMIA and DIVBAR-RMIA accumulate information in each attempt, which will facilitate the future transmissions. Thus the throughput difference between DIVBAR and DIVBAR-(R)FMIA becomes obvious.

B. Delay performance

Although the DIVBAR-RMIA and DIVBAR-FMIA algorithms are designed to maximize throughput, we here also provided simulations comparing the delay performance. Specifically, DIVBAR, DIVBAR-RMIA and DIVBAR-FMIA are simulated with $H_0 = 2$ normalized-units respectively with $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.3$ normalized-unit/slot and $\lambda_1^{(9)} = \lambda_2^{(10)} =$



(a)



(b)

Fig. 4. The delay performance DIVBAR, DIVBAR-RMIA and DIVBAR-FMIA: a) Cumulative distribution of delay with $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.3$ normalized-unit/slot; b) Cumulative distribution of delay with $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.9$ normalized-unit/slot

0.9 normalized-unit/slot (both average input rates are within the REP network capacity region). Curves showing the empirical cumulative distribution of the packets' delay over 10^6 timeslots are plotted in Fig. 4.

In the case of $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.3$, which is a low traffic case for the simulation network, it can be seen from Fig. 4 a) that the packets' delay under DIVBAR is statistically smaller than the packets' delay under DIVBAR-RMIA and DIVBAR-FMIA. This is caused by the fact that, under DIVBAR-RMIA or DIVBAR-FMIA, choosing the packet for the persistent transmissions in an epoch and making the forwarding decision for the packet are based on the backlog state observed at the beginning timeslot of the epoch, which may become outdated during the epoch. The outdated backlog observation can affect the delay performance on average and can be a dominant factor in affecting delay performance in the low traffic case if the epochs in the network are statistically long enough. In contrast, under DIVBAR, the transmitting decisions and the forwarding decisions are made in every timeslot based on current backlog state observations.

However, when setting average input rates as $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.9$ normalized-unit/slot, as is shown in Fig. 4 b), the packets' delay under DIVBAR-RMIA and DIVBAR-FMIA is statistically smaller than the delay under DIVBAR. The reason is that, in the case of heavier traffic, the transmission enhancement due to implementing the MIA technique becomes the dominant factor of influencing delay, while the outdated backlog observations under DIVBAR-RMIA and DIVBAR-FMIA are less impactful.

Moreover, as is shown in Fig. 4 a) and b), the statistical delay performance difference between DIVBAR-RMIA and DIVBAR-FMIA becomes larger as $\lambda_1^{(9)}$ and $\lambda_2^{(10)}$ increase from 0.3 to 0.9, which is consistent with the intuition that the retained partial information facilitate the transmissions more in heavy traffic case. Furthermore, another interesting phenomenon to notice is that, under DIVBAR-RMIA and DIVBAR-FMIA, the average delay with $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.9$ normalized-unit/slot is smaller than the average delay with $\lambda_1^{(9)} = \lambda_2^{(10)} = 0.3$ normalized-unit/slot. This can be explained based on the facts that backpressure based algorithms route the packets in inappropriate directions before enough packets build up to suggest the efficient paths, and that the building up process of the packets becomes faster when the exogenous input rates increase, which is another significant factor of influencing the delay performance.

VII. Conclusion

In this paper, we proposed two distributed routing algorithms: DIVBAR-RMIA and DIVBAR-FMIA, which exploit the MIA technique for the routing in multi-hop, multi-commodity wireless ad-hoc networks with unreliable and non-precisely predictable links. After setting up a proper network model, including designing the queue structure of each network node to implement the two proposed transmission schemes: RMIA and FMIA, and the working diagram within each timeslot, we analyze the throughput potential of the network with RMIA by characterizing and analyzing the RMIA network capacity region. We prove that, with certain mild assumptions consistent with practical wireless scenarios, it covers and extends the network capacity region with the REP transmission scheme which is traditionally used. Moreover, under the same assumptions, the proposed DIVBAR-RMIA algorithm is proven to be throughput optimal among all the policies with RMIA, and the proposed DIVBAR-FMIA has throughput performance at least as good as DIVBAR-RMIA. Therefore, the proposed two algorithms have superior throughput performance compared to the original DIVBAR with REP. This fact is confirmed by simulations.

Appendix A
Proof of Lemma 1

After taking expectation over $\mathbf{Q}(t_0)$ on both sides of (4) and doing concatenated summations over $t_0 = 0, 1, 2, \cdots, t-1$, it follows that

$$\frac{1}{dt}\sum_{\tau=t}^{t+d-1}\sum_{n,c}\mathbb{E}\left\{\left(Q_n^{(c)}(\tau)\right)^2\right\}-\frac{1}{dt}\sum_{\tau=0}^{d-1}\sum_{n,c}\mathbb{E}\left\{\left(Q_n^{(c)}(\tau)\right)^2\right\}$$
$$\leq B_0(d)-\varepsilon\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n,c}\mathbb{E}\left\{Q_n^{(c)}(\tau)\right\}. \quad (24)$$

Letting $t\to\infty$ yields

$$0=-\limsup_{t\to\infty}\frac{1}{dt}\sum_{\tau=0}^{d-1}\sum_{n,c}\mathbb{E}\left\{\left(Q_n^{(c)}(\tau)\right)^2\right\}$$
$$\leq B_0(d)-\varepsilon\limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n,c}\mathbb{E}\left\{Q_n^{(c)}(\tau)\right\}, \quad (25)$$

and then strong stability is achieved shown as (5).

Appendix B
The Proof of Lemma 2

Let $f_{R_{nk}}(x)$ represent the pdf (probability density function) of $R_{nk}(\tau)$. With Assumption 1, we have

$$F_{R_{nk}}^{(m)}(H_0)=\int_0^{H_0}F_{R_{nk}}^{(m-1)}(H_0-x)\,f_{R_{nk}}(x)\,dx$$
$$<F_{R_{nk}}^{(m-1)}(H_0)\,F_{R_{nk}}(H_0)\,,\ \text{for } m\geq 2, \quad (26)$$

and then recursively applying the above inequality yields (7). By including the case of $F_{R_{nk}}(H_0)<F_{R_{nk}}^{(0)}(H_0)=1$, we further get (8).

Appendix C
Proof of the necessity part of Theorem 2

Consider a network satisfying Assumption 1 with input rate matrix $(\lambda_n^{(c)})$. Suppose there is a single-copy routing policy with RMIA that stably supports $(\lambda_n^{(c)})$.

Define a *unit* as a copy of a packet. Two units are said to be *distinct* if they are copies of different original packets. When a packet is successfully transmitted from one node to another, we say that the original unit is retained in the transmitting node while a copy of the unit is created in the receiving node. After the forwarding decision is made, only one of the non-distinct units is kept, either at the transmitting node or at one of the successful receiving nodes.

Let $A_n^{(c)}(t)$ represent the total number of the distinct units of commodity c that exogenously arrive at node n during the first t timeslots. Define $Y_n^{(c)}(t)$ as the total number of distinct units with source node n and commodity c that are delivered to the destination up to time t. Because of the assumption that the policy is rate stable, for any node n and commodity c, the time average delivery rate is equal to the time average input rate:

$$\lim_{t\to\infty}\frac{Y_n^{(c)}(t)}{t}=\lim_{t\to\infty}\frac{A_n^{(c)}(t)}{t}=\lambda_n^{(c)}\ \text{with prob. 1.} \quad (27)$$

Let $\mathcal{U}_j^{(c)}(t)$ be the set of distinct units that reach their destination c from the source node j during the first t timeslots. Define $G_{nk}^{(c)}(t)$ to be the total number of units of commodity c within the set $\bigcup_{j:j\in\mathcal{N}}\mathcal{U}_j^{(c)}(t)$ that are forwarded from node n to node k within the first t timeslots. Then for node n and commodity c, it follows that

$$Y_n^{(c)}(t)+\sum_{k:k\in\mathcal{K}_n}G_{kn}^{(c)}(t)=\sum_{k:k\in\mathcal{K}_n}G_{nk}^{(c)}(t),\ \text{for } n\neq c. \quad (28)$$

Now define the following variables for all nodes $n\in\mathcal{N}$, $k\in\mathcal{K}_n$ and all commodities $c\in\mathcal{N}$:

- $\alpha_n^{(c)}(t)$: the number of times node n decides to transmit the units of commodity c in the first t timeslots.
- $\beta_n^{(c)}(t)$: the number of epochs for transmitting units of commodity c from node n with RMIA in the first t timeslots.
- $q_{n,\Omega_n}^{(c)}(t)$: the number of times units of commodity c sent by node n with RMIA are *first-decoded* by the set of nodes $\Omega_n\subseteq\mathcal{K}_n$ ($\Omega_n\neq\emptyset$) in the first t timeslots.
- $\theta_{nk}^{(c)}(\Omega_n,t)$: the number of times the units of commodity c in $\bigcup_{j:j\in\mathcal{N}}\mathcal{U}_j^{(c)}(t)$ are forwarded from node n to node k in the first t timeslots, given that the first successful receiver set is Ω_n.

Then we have

$$\frac{G_{nk}^{(c)}(t)}{t}=\frac{\alpha_n^{(c)}(t)}{t}\frac{\beta_n^{(c)}(t)}{\alpha_n^{(c)}(t)}\sum_{\Omega_n:\Omega_n\subseteq\mathcal{K}_n,\Omega_n\neq\emptyset}\frac{q_{n,\Omega_n}^{(c)}(t)}{\beta_n^{(c)}(t)}\frac{\theta_{nk}^{(c)}(\Omega_n,t)}{q_{n,\Omega_n}^{(c)}(t)}, \quad (29)$$

where we define $0/0\triangleq 0$ for terms in the above equation. Note that for all t, we have:

$$0\leq\frac{\alpha_n^{(c)}(t)}{t}\leq 1;\ 0\leq\frac{\theta_{nk}^{(c)}(\Omega_n,t)}{q_{n,\Omega_n}^{(c)}(t)}\leq 1,\ \Omega_n\neq\emptyset; \quad (30)$$

$$0\leq\frac{G_{nk}^{(c)}(t)}{t}\leq 1;\ G_{cn}^{(c)}(t)=G_{nn}^{(c)}(t)=0. \quad (31)$$

Since the constraints defined in (30) and (31) show closed and bounded regions with finite dimensions, a subsequence of timeslots $\{t_l\}$ must exists, over which the individual terms in (30) and (31) converge to constant values $\alpha_n^{*(c)}$, $\theta_{nk}^{*(c)}(\Omega_n)$ and $b_{nk}^{*(c)}$, respectively.

Moreover, let $T_n^{(c)}(i)$ represent the length of the ith epoch for node n to transmit units of commodity c with RMIA. First note that, with Assumption 1, the expectation of $T_n^{(c)}(i)$ exists because

$$\mathbb{E}\left\{T_n^{(c)}(i)\right\}=\sum_{m=1}^{\infty}\prod_{j:j\in\mathcal{K}_n}F_{nj}^{(m-1)}(H_0)$$
$$<\frac{1}{\prod_{j:j\in\mathcal{K}_n}\left[1-F_{R_{nj}}(H_0)\right]}<\infty, \quad (32)$$

where the inequality holds true due to Lemma 2. Additionally, $\beta_n^{(c)}(t)$ and $\alpha_n^{(c)}(t)$ have the following relation:

$$\sum_{i=1}^{\beta_n^{(c)}(t)}T_n^{(c)}(i)\leq\alpha_n^{(c)}(t)<\sum_{i=1}^{\beta_n^{(c)}(t)+1}T_n^{(c)}(i). \quad (33)$$

With RMIA, $T_n^{(c)}(i)$ is i.i.d. across epochs. If $\alpha_n^{*(c)}>0$, with

(33) and according to the law of large number, we have

$$\lim_{t_l\to\infty}\frac{\beta_n^{(c)}(t_l)}{\alpha_n^{(c)}(t_l)}=\frac{1}{\mathbb{E}\left\{T_n^{(c)}(i)\right\}}\triangleq\beta_n^{\mathrm{rmia}}\ \text{with prob. 1.}\quad(34)$$

Here the notation β_n^{rmia} having a superscript "rmia" but no superscript "c" because its value is only determined by the the RMIA transmission scheme and channel states.

Furthermore, with RMIA, the first successful receiver set for each node n across epochs are i.i.d.. Then we get by the law of large numbers that, if $\alpha_n^{*(c)}>0$,

$$\lim_{t_l\to\infty}\frac{q_{n,\Omega_n}^{(c)}(t_l)}{\beta_n^{(c)}(t_l)}=q_{n,\Omega_n}^{\mathrm{rmia}}\ \text{with prob. 1},\ \Omega_n\neq\emptyset,\quad(35)$$

where $q_{n,\Omega_n}^{\mathrm{rmia}}$ is the probability that Ω_n is the first successful receiver set with RMIA. The value of $q_{n,\Omega_n}^{\mathrm{rmia}}$ is also only determined by the the RMIA transmission scheme and channel states

Suppose under a stationary randomized policy, denoted as $Policy^*$, each node n decides to transmit a unit of commodity c in every timeslot with a fixed probability $\alpha_n^{*(c)}$ and chooses node $k\in\mathcal{K}_n$ to get the forwarding responsibility with a fixed conditional probability $\theta_{nk}^{*(c)}(\Omega_n)$, given that the set of nodes Ω_n firstly decode the unit (if $k\notin\Omega_n$, $\theta_{nk}^{*(c)}(\Omega_n)$ has to be set to 0). According to the law of large numbers and (34), (35), the values $\alpha_n^{*(c)}$, $\theta_{nk}^{*(c)}(\Omega_n)$ and $b_{nk}^{*(c)}$ are the limit values over the whole timeslot sequence $\{t\}$, i.e., the converging subsequence $\{t_l\}$ becomes $\{t\}$, and therefore it follows that

$$\lim_{t\to\infty}\frac{\alpha_n^{*(c)}(t)}{t}=\alpha_n^{*(c)}\ \text{with prob. 1},\quad(36)$$

$$\lim_{t\to\infty}\frac{\theta_{nk}^{*(c)}(\Omega_n,t)}{q_{n,\Omega_n}^{(c)}(t)}=\theta_{nk}^{*(c)}(\Omega_n)\ \text{with prob. 1},\ \Omega_\mathrm{n}\neq\emptyset,\quad(37)$$

$$\lim_{t\to\infty}\frac{G_{nk}^{*(c)}(t)}{t}=b_{nk}^{*(c)}\ \text{with prob. 1.}\quad(38)$$

With (31) and (38), we have

$$b_{nk}^{*(c)}\geq 0, b_{cn}^{*(c)}=0, b_{nn}^{*(c)}=0,\ \text{for } n\neq c.$$

Furthermore, dividing both sides of (28) by t and using the results of (27) and (38) yields:

$$\lambda_n^{(c)}+\sum_{k:k\in\mathcal{K}_n}b_{kn}^{*(c)}=\sum_{k:k\in\mathcal{K}_n}b_{nk}^{*(c)},\ \text{for } n\neq c.$$

Likewise, if $\alpha_n^*(c)>0$, according to (34)-(38), taking the limit $t\to\infty$ in (29) yields:

$$b_{nk}^{*(c)}=\alpha_n^{*(c)}\beta_n^{\mathrm{rmia}}\sum_{\Omega_n:\Omega_n\subseteq\mathcal{K}_n,\Omega_n\neq\emptyset}q_{n,\Omega_n}^{\mathrm{rmia}}\theta_{nk}^{*(c)}(\Omega_n).$$

Note that the above equation also holds true trivially if $\alpha_n^{*(c)}=0$; β_n^{rmia} and $q_{n,\Omega_n}^{\mathrm{rmia}}$ do not have policy-specifying mark because their values only depend on the average channel state.

Thus, for any $\left(\lambda_n^{(c)}\right)\in\Lambda_{\mathrm{RMIA}}$, a stabilizing stationary randomized policy satisfies (12)-(14).

Appendix D
Proof of the sufficiency part of Theorem 2

For a network satisfying Assumption 1 with input rate matrix $(\lambda_n^{(c)})$, suppose there exists a stationary randomized policy, denoted as $Policy^*$, and a constant $\varepsilon>0$, such that $Policy^*$ and $(\lambda_n^{(c)}+\varepsilon)$ satisfy (12)-(14), which yields:

$$\sum_{k:k\in\mathcal{K}_n}b_{nk}^{*(c)}-\sum_{k:k\in\mathcal{K}_n}b_{kn}^{*(c)}-\lambda_n^{(c)}\geq\varepsilon,\ \text{for } n\neq c\quad(39)$$

Start by extending the queueing dynamic (1) to a t-timeslot queueing relation under $Policy^*$:

$$\begin{aligned}Q_n^{*(c)}(t_0+t)\leq\max&\left\{Q_n^{*(c)}(t_0)-\sum_{\tau=t_0}^{t_0+t-1}\sum_{k:k\in\mathcal{K}_n}b_{nk}^{(*c)}(\tau),0\right\}\\&+\sum_{\tau=t_0}^{t_0+t-1}\sum_{k:k\in\mathcal{K}_n}b_{kn}^{*(c)}(\tau)+\sum_{\tau=t_0}^{t_0+t-1}a_n^{(c)}(\tau),\end{aligned}\quad(40)$$

where $t_0\geq 0$; $t\geq 1$. By squaring both sides of (40) and taking expectations on each term given $\mathbf{Q}^*(t_0)$, we upper bound the t-timeslot Lyapunov drift as follows:

$$\begin{aligned}&\Delta_t^*(\mathbf{Q}^*(t_0))\\&\leq B(t)-2\sum_{n,c}Q_n^{*(c)}(t_0)\,\mathbb{E}\left\{\sum_{k:k\in\mathcal{K}_n}\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}b_{nk}^{*(c)}(\tau)\right.\\&\qquad\left.-\sum_{k:k\in\mathcal{K}_n}\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}b_{kn}^{*(c)}(\tau)-\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}a_n^{(c)}(\tau)\right|\mathbf{Q}^*(t_0)\Bigg\}\\&=B(t)-2\sum_{n,c}Q_n^{*(c)}(t_0)\left\{\left[\lambda_n^{(c)}-\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{a_n^{(c)}(\tau)\right\}\right]\right.\\&\qquad+\sum_{k:k\in\mathcal{K}_n}\left[\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{b_{nk}^{*(c)}(\tau)\right\}-b_{nk}^{*(c)}\right]\\&\qquad-\sum_{k:k\in\mathcal{K}_n}\left[\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{b_{kn}^{*(c)}(\tau)\right\}-b_{kn}^{*(c)}\right]\\&\qquad\left.+\left[\sum_{k:k\in\mathcal{K}_n}b_{nk}^{*(c)}-\sum_{k:k\in\mathcal{K}_n}b_{kn}^{*(c)}-\lambda_n^{(c)}\right]\right\},\end{aligned}\quad(41)$$

where the sum of squared terms formed by the flow rate and input rate has been replaced by a constant $B(t)$:

$$\begin{aligned}&\frac{1}{t}\sum_{n,c}\left\{\left[\sum_{\tau=t_0}^{t_0+t-1}\sum_{k:k\in\mathcal{K}_n}b_{nk}^{*(c)}(\tau)\right]^2\right.\\&\qquad\left.+\left[\sum_{\tau=t_0}^{t_0+t-1}\sum_{k:k\in\mathcal{K}_n}b_{kn}^{*(c)}(\tau)+\sum_{\tau=t_0}^{t_0+t-1}a_n^{(c)}(\tau)\right]^2\right\}\\&\leq Nt\left[1+(N+A_{\max})^2\right]\triangleq B(t),\end{aligned}\quad(42)$$

and $\mathbf{Q}^*(t_0)$ is dropped from the expectation condition because $Policy^*$ makes decisions independent from backlog state.

To prepare for the later proof, we propose the following lemma:

Lemma 4. *For link (n,k) in a network satisfying Assumption 1, under a stationary randomized policy with RMIA, for any* *given $\varepsilon > 0$, there exists an integer $D_{nk}^{(c)} > 0$, such that, for all $t_0 \geq 0$ (t_0 is integer), whenever $t \geq D_{nk}^{(c)}$, the mean time average of $b_{nk}^{(c)}(\tau)$ over the interval from timeslot t_0 to timeslot $t_0 + t - 1$ satisfies:*

$$\left| \frac{1}{t} \sum_{\tau=t_0}^{t_0+t-1} \mathbb{E}\left\{ b_{nk}^{(c)}(\tau) \right\} - b_{nk}^{(c)} \right| \leq \varepsilon. \tag{43}$$

The proof of Lemma 4 is shown in Appendix K and is non-trivial due to the fact that $b_{nk}^{(c)}(\tau)$ is not i.i.d. across timeslots with RMIA and the requirement that the value of $D_{nk}^{(c)}$ does not depend on t_0. Based on Lemma 4, there exists an integer $D_{nk}^{*(c)} > 0$, such that, for all $t_0 \geq 0$, whenever $t \geq D_{nk}^{*(c)}$, we have

$$\left| \frac{1}{t} \sum_{\tau=t_0}^{t_0+t-1} \mathbb{E}\left\{ b_{nk}^{*(c)}(\tau) \right\} - b_{nk}^{*(c)} \right| \leq \frac{\varepsilon}{4N}, \tag{44}$$

Choose $t = D^* \triangleq \max\{D_{nk}^{*(c)} : \ n, c \in \mathcal{N}, k \in \mathcal{K}_n\}$, consider the fact $\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1} \mathbb{E}\{a_{nk}^{(c)}(\tau)\} = \lambda_n^{(c)}$ and plug (39) and (44) into (41), it follows that

$$\Delta_{D^*}^*\left(\mathbf{Q}^*(t_0)\right) \leq B\left(D^*\right) - \varepsilon \sum_{n,c} Q_n^{*(c)}(t_0), \tag{45}$$

Note that (45) satisfies the condition required by Lemma 1, and therefore the strong stability can be achieved:

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} \mathbb{E}\left\{ Q_n^{(c)}(\tau) \right\} \leq \frac{B\left(D^*\right)}{\varepsilon}. \tag{46}$$

Appendix E
Proof of Theorem 3

For a network with an input rate matrix $(\lambda_n^{(c)}) \in \Lambda_{\mathrm{REP}}$, according to Theorem 1, there exists a stationary randomized policy $Policy^{**}$ that stably supports $(\lambda_n^{(c)})$ by forming a flow rate matrix $(b_{nk}^{**(c)})$ with REP satisfying (9)-(11). If the network satisfies Assumption 1, an intuitive proof is to construct another stationary randomized policy, denoted as $Policy^1$, that forms the same flow rate matrix $(b_{nk}^{**(c)})$ but with RMIA.

Note that, when a node n transmits a unit, with RMIA, Ω_n is the first successful receiver set in the first-decoding timeslot, while in the same timeslot, with REP, the successful receiver set would be Ψ_n instead (Ψ_n could be empty indicating no successful decoding). Then, due to MIA, we must have $\Psi_n \subseteq \Omega_n$. Moreover, for node n, the decoding timeslots with REP is a subset of the first-decoding timeslots with RMIA. Base on these facts, define the following variables for all nodes $n \in \mathcal{N}$ and all commodities $c \in \mathcal{N}$:

- $q_{n,\Psi_n}^{\mathrm{rep},(c)}(t)$: the number of times units of commodity c transmitted by node n with REP are decoded by the set of nodes $\Psi_n \subseteq \mathcal{K}_n$ in the first t timeslots.
- $q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia},(c)}(t)$: the number of times units of commodity c transmitted by node n with RMIA are first-decoded by the set of Ω_n ($\Omega_n \neq \emptyset$, $\Omega_n \subseteq \mathcal{K}_n$) in the first t timeslots, while in the same timeslots of transmitting units of commodity c, would be decoded by the set of nodes Ψ_n with REP.

Then we have

$$q_{n,\Psi_n}^{\mathrm{rep},(c)}(t) = \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia},(c)}(t), \quad \text{if } \Psi_{\mathrm{n}} \neq \emptyset. \tag{47}$$

According to the law of large numbers, let $\alpha_n^{(c)}(t) \to \infty$, we have

$$\lim_{\alpha_n^{(c)}(t)\to\infty} \frac{q_{n,\Psi_n}^{\mathrm{rep},(c)}(t)}{\alpha_n^{(c)}(t)} = q_{n,\Psi_n}^{\mathrm{rep}}, \text{ with prob. 1, if } \Psi_{\mathrm{n}} \neq \emptyset. \tag{48}$$

Likewise, since the occurrences of Ψ_n (with REP) and Ω_n (with RMIA) in the first-decoding timeslots for node n are i.i.d. across different epochs (with RMIA), then according to the law of large numbers, we have

$$\begin{aligned} &\lim_{\alpha_n^{(c)}(t)\to\infty} \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} \frac{q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia},(c)}(t)}{\alpha_n^{(c)}(t)} \\ &= \lim_{\alpha_n^{(c)}(t)\to\infty} \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} \frac{\beta_n^{\mathrm{rmia},(c)}(t)}{\alpha_n^{(c)}(t)} \frac{q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia},(c)}(t)}{\beta_n^{\mathrm{rmia},(c)}(t)} \\ &= \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} \beta_n^{\mathrm{rmia}} q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}}, \text{ with prob. 1, if } \Psi \neq \emptyset. \end{aligned} \tag{49}$$

Divide both sides of (47) by $\alpha_n^{(c)}(t)$ and plug (48) and (49) in, we get

$$q_{n,\Psi_n}^{\mathrm{rep}} = \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} \beta_n^{\mathrm{rmia}} q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}}, \text{ if } \Psi_{\mathrm{n}} \neq \emptyset. \tag{50}$$

Consider the flow rate under $Policy^{**}$ shown as (11) in Theorem 1. Plugging (50) into (11) yields:

$$\begin{aligned} b_{nk}^{**(c)} &= \alpha_n^{**(c)} \sum_{\Psi_n : k \in \Psi_n} \sum_{\Omega_n : \Psi_n \subseteq \Omega_n} \beta_n^{\mathrm{rmia}} q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}} \theta_{nk}^{**(c)}(\Psi_n) \\ &= \alpha_n^{**(c)} \beta_n^{\mathrm{rmia}} \sum_{\Omega_n : k \in \Omega_n} q_{n,\Omega_n}^{\mathrm{rmia}} \theta_{nk}^{1(c)}(\Omega_n), \end{aligned} \tag{51}$$

where we define

$$\theta_{nk}^{1(c)}(\Omega_n) \triangleq \sum_{\Psi_n : \Psi_n \subseteq \Omega_n, k \in \Psi_n} \frac{q_{n,\Psi_n,\Omega_n}^{\mathrm{rep,rmia}}}{q_{n,\Omega_n}^{\mathrm{rmia}}} \theta_{nk}^{**(c)}(\Psi_n). \tag{52}$$

In (52), $q_{n,\Omega_n}^{\mathrm{rmia}}$ is positive due to Assumption 1 and Lemma 2:

$$\begin{aligned} q_{n,\Omega_n}^{\mathrm{rmia}} = \sum_{m=1}^{\infty} &\prod_{k : k \in \Omega_n} \left[F_{R_{nk}}^{(m-1)}(H_0) - F_{R_{nk}}^{(m)}(H_0) \right] \\ &\times \prod_{k : k \notin \Omega_n, k \in \mathcal{K}_n} F_{R_{nk}}^{(m)}(H_0) > 0. \end{aligned} \tag{53}$$

Comparing (51) with (14) in Theorem 2, if there is a stationary randomized policy $Policy^1$ with RMIA, under which each node n transmits a unit of commodity c with probability $\alpha_n^{**(c)}$ in each timeslot, and forwards the decoded unit to node $k \in \mathcal{K}_n$ with probability $\theta_{nk}^{1(c)}(\Omega_n)$, given that the first successful receiver set is Ω_n, the same flow rate $(b_{nk}^{**(c)})$ will be formed. Then the remaining part of the proof is to show that the $\theta_{nk}^{1(c)}(\Omega_n)$ in (52) are valid probability values.

To validate the $\theta_{nk}^{1(c)}(\Omega_n)$, first consider the definitions of

$q^{\text{rmia},(c)}_{n,\Omega_n}(t)$ and $q^{\text{rep,rmia},(c)}_{n,\Psi_n,\Omega_n}(t)$, and we have

$$q^{\text{rmia},(c)}_{n,\Omega_n}(t) = \sum_{\Psi_n:\Psi\subseteq\Omega_n} q^{\text{rep,rmia},(c)}_{n,\Psi_n,\Omega_n}(t),\ \Omega_n \neq \emptyset. \quad (54)$$

Divide both sides of (54) by $\beta^{\text{rmia},(c)}_n(t)$ and let $\beta^{\text{rmia},(c)}_n(t) \to \infty$, by applying the law of large number, we have

$$q^{\text{rmia}}_{n,\Omega_n} = \sum_{\Psi_n:\Psi_n\subseteq\Omega_n} q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n},\ \Omega_n \neq \emptyset. \quad (55)$$

By plugging (55) into (52), we check the validity of $\{\theta^{1(c)}_{nk}(\Omega_n) : k \in \mathcal{K}_n t\}$ as follows:

$$\sum_{k:k\in\Omega_n} \theta^{1(c)}_{nk}(\Omega_n) = \sum_{\Psi_n:\Psi_n\subseteq\Omega_n,\Psi_n\neq\emptyset} \frac{q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n}}{q^{\text{rmia}}_{n,\Omega_n}} \sum_{k:k\in\Psi_n} \theta^{**(c)}_{nk}(\Psi_n)$$
$$\leq \frac{\sum_{\Psi_n:\Psi_n\subseteq\Omega_n,\Psi_n\neq\emptyset} q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n}}{\sum_{\Psi_n:\Psi_n\subseteq\Omega_n,\Psi_n\neq\emptyset} q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n} + q^{\text{rep,rmia}}_{n,\emptyset,\Omega_n}} \leq 1, \quad \Omega_n \neq \emptyset. \quad (56)$$

Thus, for $(\lambda^{(c)}_n) \in \Lambda_{\text{REP}}$ that can be supported by $Policy^{**}$ with REP, there also exists a $Policy^1$ with RMIA that forms the same flow rate matrix $(b^{**(c)}_{nk})$ and stably supports $(\lambda^{(c)}_n)$, i.e., $\Lambda_{\text{REP}} \subseteq \Lambda_{\text{RMIA}}$.

Appendix F
Proof of Theorem 4

Suppose $(\lambda^{(c)}_n) \in \Lambda_{\text{REP}}$ has a positive entry $\lambda^{(c_0)}_{n_0}$ and can be stably supported by a stationary randomized policy with REP: $Policy^{**}$, which forms a flow path l_{n_0,c_0} on which each link has a positive time average flow rate. The goal of the proof is to construct a policy with RMIA that can stably support the input rate matrix $(\lambda^{'(c)}_n)(l_{n_0,c_0})$.

Based on the proof of Theorem 3, there exists a stationary randomized policy $Policy^1$ with RMIA that can also stably support $(\lambda^{(c)}_n)$ and forms the flow rate matrix $(b^{**(c)}_{nk})$. For link (n,k) on path l_{n_0,c_0}, regardless of flow conservation constraints, the time average flow rate of commodity c_0 over link (n,k) has a increase potential based on $b^{**(c_0)}_{nk}$ formed by $Policy^1$ with RMIA, by increasing the forwarding probability from $\theta^{1(c_0)}_{nk}(\Omega_n)$ to $\theta^{1'(c_0)}_{nk}(\Omega_n)$ if $k \in \Omega_n$:

$$\theta^{1'(c_0)}_{nk}(\Omega_n) \triangleq \begin{cases} \theta^{1(c_0)}_{nk}(\Omega_n) + \left[q^{\text{rep,rmia}}_{n,\emptyset,\Omega_n} + q^{\text{rep,rmia}}_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}\right] \Big/ q^{\text{rmia}}_{n,\Omega_n}, & \text{if } n \neq n_0; \\ \theta^{1(c_0)}_{n_0k}(\Omega_n) + q^{\text{rep,rmia}}_{n_0,\emptyset,\Omega_{n_0}} \Big/ q^{\text{rmia}}_{n_0,\Omega_{n_0}}, & \text{if } n = n_0. \end{cases} \quad (57)$$

Consequently, if we maintain the forwarding probabilities to the nodes in Ω_n other than node k, i.e. if $\exists j \in \Omega_n$ but $j \neq k$, then $\theta^{1'(c_0)}_{nj}(\Omega_n) = \theta^{1(c_0)}_{nj}(\Omega_n)$, the potential time average flow increase on link (n,k), denoted as $\delta^{(c_0)}_{nk}$, can be obtained, as is shown in (16) in the theorem statement. Here we check the validity of the forwarding probabilities $\{\theta^{1'(c_0)}_{nj}(\Omega_n) : j \in \mathcal{K}_n\}$ as follows:

- If $k \notin \Omega_n$, we have $\sum_{j:j\in\mathcal{K}_n} \theta^{1'(c_0)}_{nj}(\Omega_n) = \sum_{j:j\in\mathcal{K}_n} \theta^{1(c_0)}_{nj}(\Omega_n) \leq 1$ according to (56).
- If $n \neq n_0$ and $\{k, p_{l_{n_0,c_0}}(n)\} \subseteq \Omega_n$, note that the time average flow on link $(p_{l_{n_0,c_0}}(n), n)$ is positive under $Policy^{**}$ (see Footnote 7), and we can assume that the time average flow rate on the reverse link $(n, p_{l_{n_0,c_0}}(n))$ is zero under $Policy^{**}$, i.e., $\theta^{**(c_0)}_{np_{l_{n_0,c_0}}(n)}(\Psi_n) = 0$, if $p_{l_{n_0,c_0}}(n) \in \Psi_n$. Then it follows that

$$\sum_{j:j\in\Omega_n} \theta^{1'(c_0)}_{nj}(\Omega_n)$$
$$= \sum_{j:j\in\Omega_n, j\neq p_{l_{n_0,c_0}}(n)} \ \sum_{\Psi_n:\Psi_n\subseteq\Omega_n, j\in\Psi_n} \frac{q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n}\theta^{**(c_0)}_{nj}(\Psi_n)}{q^{\text{rmia}}_{n,\Omega_n}} + \frac{q^{\text{rep,rmia}}_{n,\emptyset,\Omega_n} + q^{\text{rep,rmia}}_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}}{q^{\text{rmia}}_{n,\Omega_n}}$$
$$= \sum_{\Psi_n:\Psi_n\subseteq\Omega_n,\Psi_n\neq\emptyset,\Psi_n\neq\{p_{l_{n_0,c_0}}(n)\}} \frac{q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n}}{q^{\text{rmia}}_{n,\Omega_n}} \times \sum_{j:j\in\Psi_n, j\neq p_{l_{n_0,c_0}}(n)} \theta^{**(c_0)}_{nj}(\Psi_n) + \frac{q^{\text{rep,rmia}}_{n,\emptyset,\Omega_n} + q^{\text{rep,rmia}}_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}}{q^{\text{rmia}}_{n,\Omega_n}}$$
$$\leq \frac{\sum_{\Psi_n:\Psi_n\subseteq\Omega_n} q^{\text{rep,rmia}}_{n,\Psi_n,\Omega_n}}{q^{\text{rmia}}_{n,\Omega_n}} = 1. \quad (58)$$

- If $n = n_0$ and $k \in \Omega_n$, or if $n \neq n_0$ and $p_{l_{n_0,c_0}}(n) \notin \Omega_n$, we also guarantee that $\sum_{j:j\in\mathcal{K}_{n_0}} \theta^{1'(c)}_{n_0j}(\Omega_{n_0}) \leq 1$ with the similar derivation in (58) but without the term $q^{\text{rep,rmia}}_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}$.

With the time average flow rate increase potential $\delta^{(c_0)}_{nk}$ on each link $(n,k) \in l_{n_0,c_0}$, let $\delta^{(c_0)}_{l_{n_0,c_0}}$ represent the minimum flow increase potential among the links on path l_{n_0,c_0}, i.e., $\delta^{(c_0)}_{l_{n_0,c_0}} \triangleq \min_{(n,k)\in l_{n_0,c_0}}\{\delta^{(c_0)}_{nk}\}$. Therefore, for commodity c_0, each link (n,k) along path l_{n_0,c_0} can support a flow rate increase of $\delta^{(c_0)}_{l_{n_0,c_0}}$ just by assigning a new forwarding probability $\theta^{2(c_0)}_{nk}(\Omega_n)$ such that, i.e., $\exists\ \xi^{(c_0)}_{nk} \in [0,1]$,

$$\theta^{2(c_0)}_{nk}(\Omega_n) \triangleq \begin{cases} \theta^{1(c_0)}_{nk}(\Omega_n) + \left[q^{\text{rep,rmia}}_{n,\emptyset,\Omega_n} + q^{\text{rep,rmia}}_{n,\{p_{l_{n_0,c_0}}(n)\},\Omega_n}\right]\xi^{(c_0)}_{nk} \Big/ q^{\text{rmia}}_{n,\Omega_n}, & \text{if } n \neq n_0; \\ \theta^{1(c_0)}_{n_0k}(\Omega_n) + q^{\text{rep,rmia}}_{n_0,\emptyset,\Omega_{n_0}}\xi^{(c_0)}_{n_0k} \Big/ q^{\text{rmia}}_{n_0,\Omega_{n_0}}, & \text{if } n = n_0; \end{cases} \quad (59)$$

$$\delta^{(c_0)}_{l_{n_0,c_0}} = \delta^{(c_0)}_{nk}\xi^{(c_0)}_{nk}. \quad (60)$$

Then we construct a stationary randomized policy with RMIA, denoted as $Policy^2$, under which each node n chooses to transmit a unit of commodity c with probability $\alpha^{**(c)}_n$ in each timeslot and forwards the decoded unit to node $k \in \Omega_n$ with probability $\theta^{2(c)}_{nk}(\Omega_n)$. The $\theta^{2(c)}_{nk}(\Omega_n)$ satisfy (59) for

$(n,k) \in l_{n_0,c_0}$ and $c = c_0$; $\theta_{nk}^{2(c)}(\Omega_n) = \theta_{nk}^{1(c)}(\Omega_n)$ for $(n,k) \notin l_{n_0,c_0}$ or $c \neq c_0$. Correspondingly, under $Policy^2$, the time average flow rate over each link can be expressed as follows:

$$b_{nk}^{2(c_0)} = b_{nk}^{**(c_0)} + \delta_{l_{n_0,c_0}}^{(c_0)}, \quad \text{for } (n,k) \in l_{n_0,c_0}; \tag{61a}$$

$$b_{nk}^{2(c)} = b_{nk}^{**(c)}, \text{ for } (n,k) \notin l_{n_0,c_0} \text{ or } c \neq c_0. \tag{61b}$$

Following from (61), the flow rate matrix $(b_{nk}^{2(c)})$ under $Policy^2$ and the input rate matrix $(\lambda_n^{'(c)})(l_{n_0,c_0})$ satisfy (12)-(14) in Theorem 2, and therefore $(\lambda_n^{'(c)})(l_{n_0,c_0}) \in \Lambda_{\text{RMIA}}$.

Appendix G
Proof of Corollary 1

Consider an arbitrary input rate matrix $(\lambda_n^{(c)})$ within Λ_{REP} having a positive entry $\lambda_{n_0}^{(c_0)}$. According to Theorem 4, there exists a simple path l_{n_0,c_0} with positive time average flow such that corresponding $(\lambda_n^{'(c)})(l_{n_0,c_0})$ belongs to Λ_{RMIA}. For each link (n,k) on path l_{n_0,c_0}, we have

$$\begin{aligned}\delta_{nk}^{(c_0)} &\geq \eta_{nk}^{\text{rep,rmia}} \alpha_n^{**(c_0)} \beta_n^{\text{rmia}} \sum_{\Omega_n:\Omega_n \subseteq \mathcal{K}_n, \Omega_n \neq \emptyset} q_{n,\Omega_n}^{\text{rmia}} \theta_{nk}^{1(c)}(\Omega_n) \\ &= \eta_{nk}^{\text{rep,rmia}} b_{nk}^{**(c_0)},\end{aligned} \tag{62}$$

where

$$\eta_{nk}^{\text{rep,rmia}} \triangleq \sum_{\Omega_n:k\in\Omega_n} q_{n,\emptyset,\Omega_n}^{\text{rep,rmia}} \Bigg/ \sum_{\Omega_n:\Omega_n \subseteq \mathcal{K}_n, \Omega_n \neq \emptyset} q_{n,\Omega_n}^{\text{rmia}}. \tag{63}$$

The value of $\eta_{nk}^{\text{rep,rmia}}$ only depends on the average channel state and is positive:

$$\begin{aligned}& q_{n,\Psi_n,\Omega_n}^{\text{rep,rmia}} \\ =& \sum_{m=1}^{\infty} \prod_{k:k\in\Psi_n} F_{R_{nk}}^{(m-1)}(H_0)\left[1 - F_{R_{nk}}(H_0)\right] \cdot \prod_{k:k\notin\Omega_n} F_{R_{nk}}^{(m)}(H_0) \\ & \cdot \prod_{k:k\in\Omega_n\backslash\Psi_n} \left[F_{R_{nk}}^{(m-1)}(H_0) F_{R_{nk}}(H_0) - F_{R_{nk}}^{(m)}(H_0)\right] \\ >& 0, \quad \text{for } \Psi_\text{n} \subseteq \Omega_\text{n}.\end{aligned} \tag{64}$$

Define $b_{l_{n_0,c_0}}^{**} = \min\{b_{nk}^{**(c_0)} : (n,k) \in l_{n_0,c_0}\}$ for each path l_{n_0,c_0}; let $\mathcal{L}_{n_0,c_0}^{**}$ represent the set of simple paths with positive flow from node n_0 to node c_0 under $Policy^{**}$; define $l_{n_0,c_0}^{**\max}$ as the simple path with the maximum $b_{l_{n_0,c_0}}^{**}$ among the paths in $\mathcal{L}_{n_0,c_0}^{**}$; let L represent the number of geometric simple paths from node n_0 to node c_0. Then, for each link $(n,k) \in l_{n_0,c_0}^{**\max}$, we have

$$b_{l_{n_0,c_0}^{**\max}} \geq \frac{1}{L} \sum_{l_{n_0,c_0}:l_{n_0,c_0}\in\mathcal{L}_{n_0,c_0}^{**}} b_{l_{n_0,c_0}}^{**} \geq \frac{1}{L}\lambda_{n_0}^{(c_0)}. \tag{65}$$

Define $\eta_{\min}^{\text{rep,rmia}} = \min\left\{\eta_{nk}^{\text{rep,rmia}} : n \in \mathcal{N}, k \in \mathcal{K}_n\right\}$, it follows from (62) and (65) that, for path $l_{n_0,c_0}^{**\max}$, we have

$$\delta_{l_{n_0,c_0}^{**\max}}^{(c_0)} \geq \eta_{\min}^{\text{rep,rmia}} b_{l_{n_0,c_0}^{**\max}} \geq \frac{1}{L}\eta_{\min}^{\text{rep,rmia}} \lambda_{n_0}^{(c_0)}, \tag{66}$$

where $\eta_{\min}^{\text{rep,rmia}} \big/ L$ is a positive constant value that only depends on the average channel state and geometric topology of the network. Thus, according to (66), Λ_{RMIA} extends from Λ_{REP} by at least a factor of $\eta_{\min}^{\text{rep,rmia}} \big/ L$ in the (n_0, c_0)th dimension. Combing with Theorem 3, Λ_{RMIA} is strictly larger than Λ_{REP}.

Appendix H
Proof of Lemma 3

Firstly, according to (21), the expectation of $Z_n(i, \hat{\mathbf{Q}}(u_{n,i}))$, given the backlog state $\hat{\mathbf{Q}}(u_{n,i})$, can be upper bounded as follows:

$$\begin{aligned}& \mathbb{E}\left\{ Z_n\left(i, \hat{\mathbf{Q}}(u_{n,i})\right) \middle| \hat{\mathbf{Q}}(u_{n,i}) \right\} \\ \overset{(a)}{\leq} & \sum_c \mathbb{E}\left\{ \sum_{\tau=u_{n,i}}^{u_{n,i+1}-1} \sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(\tau) \hat{W}_{nk}^{(c)}(u_{n,i}) \middle| \hat{\mathbf{Q}}(u_{n,i}) \right\} \\ \overset{(b)}{=} & \sum_c \mathbb{E}\left\{ \sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(u_{n,i+1}-1) \hat{W}_{nk}^{(c)}(u_{n,i}) \middle| \hat{\mathbf{Q}}(u_{n,i}) \right\},\end{aligned} \tag{67}$$

In (67), the upper bound condition of (a) is achieved by the following activity: $b_{nk}^{(c)}(\tau) = 0$ when $W_{nk}^{(c)}(u_{n,i}) = 0$, i.e., node n never forwards a packet of commodity c to node $k \in \mathcal{K}_n$ if node k has non-positive differential backlog (zero differential backlog coefficient) of commodity c, which is consistent with the description in step 5) of the algorithm summary of DIVBAR-RMIA; the equality (b) in (67) holds true because of the fact that, for any policy in $\mathcal{P}$, $b_{nk}^{(c)}(\tau) = 0$ when $u_{n,i} \leq \tau < u_{n,i+1} - 1$.

Then we define the following variables for the policies with RMIA in $\mathcal{P}$:

- $\mu_n^{(c)}(i)$: the variable that takes value 1 if node n decides to transmit a unit of commodity c in the ith epoch, and takes value 0 otherwise.
- $\mu_n(i)$: the variable that takes value 1 if node n decides to transmit a unit having a commodity (the unit is not null) in the ith epoch, and takes value 0 if node n decides to transmit a null packet.
- $X_{nk}^{\mathcal{P}}(i)$: the random variable that takes value 1 if node $k \in \mathcal{K}_n$ is in the first successful receiver set of the ith epoch for node n under a policy in $\mathcal{P}$, and takes value 0 otherwise. Given the policy set $\mathcal{P}$, the value of $X_{nk}^{\mathcal{P}}(i)$ only depends on the channel realizations in epoch i for node n.
- $\hat{1}_{nk}^{(c)}(i)$: the indicator variable that takes value 1 if and only if $X_{nk}^{\mathcal{P}}(i) = 1$ and $X_{nj}^{\mathcal{P}}(i) = 0$ for all $j \in \hat{\mathcal{R}}_{nk}^{\text{high},(c)}(u_{n,i})$.

Considering the fact that $X_{nk}^{\mathcal{P}}(i)\mu_n^{(c)}(i) \in \{0,1\}$ and $b_{nk}^{(c)}(u_{n,i+1}-1) = b_{nk}^{(c)}(u_{n,i+1}-1) X_{nk}^{\mathcal{P}}(i)\mu_n^{(c)}(i)$, it follows from (67) that

$$\begin{aligned}& \mathbb{E}\left\{ Z_n\left(i, \hat{\mathbf{Q}}(u_{n,i})\right) \middle| \hat{\mathbf{Q}}(u_{n,i}) \right\} \\ \leq & \sum_c \mathbb{E}\left\{ \mu_n^{(c)}(i) \sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(u_{n,i+1}-1) \right. \\ & \qquad \left. \times X_{nk}^{\mathcal{P}}(i) \hat{W}_{nk}^{(c)}(u_{n,i}) \middle| \hat{\mathbf{Q}}(u_{n,i}) \right\}\end{aligned}$$

$$\overset{(a)}{\le}\sum_c \mathbb{E}\Big\{\max_{k:k\in\mathcal{K}_n}\Big\{X_{nk}^{\mathcal{P}}(i)\,\hat{W}_{nk}^{(c)}(u_{n,i})\Big\}\Big|\,\hat{\mathbf{Q}}(u_{n,i}),\mu_n^{(c)}(i)=1\Big\} \times\mathbb{E}\Big\{\mu_n^{(c)}(i)\Big|\,\hat{\mathbf{Q}}(u_{n,i})\Big\} \quad (68)$$

The inequality (a) in (68) holds true due to the fact that $\sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}(u_{n,i+1}-1)\le 1$; (a) becomes an equality with the following activity: $b_{nk}^{(c)}(u_{n,i+1}-1)=1$ when node k has the largest positive term $X_{nk}^{\mathcal{P}}(i)\,W_{nk}^{(c)}(i)$, i.e., node n forwards a packet to node k only if node k is the successful receiver with the largest positive differential backlog of commodity c, which is consistent with step 5) of the algorithm summary of DIVBAR-RMIA.

Moreover, note that $\hat{1}_{nk}^{(c)}(i)$ takes value 1 with probability $\varphi_{nk}^{(c)}(i)$, given the backlog state $\hat{\mathbf{Q}}(u_{n,i})$ and that node n decides to transmit a unit of commodity c in epoch i, and therefore, we have with the definition of $\hat{1}_{nk}^{(c)}(i)$ that

$$\begin{aligned}&\mathbb{E}\left\{\max_{k:k\in\mathcal{K}_n}\left\{X_{nk}^{\mathcal{P}}(i)\,\hat{W}_{nk}^{(c)}(u_{n,i})\right\}\middle|\,\hat{\mathbf{Q}}(u_{n,i}),\mu_n^{(c)}(i)=1\right\}\\&=\mathbb{E}\left\{\sum_{k:k\in\mathcal{K}_n}\hat{W}_{nk}^{(c)}(u_{n,i})\,\hat{1}_{nk}^{(c)}(i)\middle|\,\hat{\mathbf{Q}}(u_{n,i}),\mu_n^{(c)}(i)=1\right\}\\&=\sum_{k:k\in\mathcal{K}_n}\hat{W}_{nk}^{(c)}(u_{n,i})\,\hat{\varphi}_{nk}^{(c)}(i),\end{aligned}\quad (69)$$

which is the backpressure metric (18) in step 3) of its algorithm summary. Then plugging (69) into (68) yields :

$$\begin{aligned}&\mathbb{E}\left\{Z_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i})\right\}\\&\overset{(a)}{\le}\sum_{k\in\mathcal{K}_n}\hat{W}_{nk}^{(\hat{c}_n(i))}(u_{n,i})\,\hat{\varphi}_{nk}^{(\hat{c}_n(i))}(i)\sum_c\mathbb{E}\left\{\mu_n^{(c)}(i)\middle|\,\hat{\mathbf{Q}}(u_{n,i})\right\}\\&=\hat{\Xi}_n(i)\,\mathbb{E}\left\{\mu_n(i)\middle|\,\hat{\mathbf{Q}}(u_{n,i})\right\}\overset{(b)}{\le}\hat{\Xi}_n(i)\,.\end{aligned}\quad (70)$$

The upper bound condition of (a) in (70) is achieved by the following activity: node n only transmits a unit whose belonging commodity maximizes the metric of (18) if it decides to transmit a packet having a commodity; the upper bound condition of (b) in (70) can be achieved by the following activity: node n transmits a unit having a commodity in the ith epoch if and only if $\hat{\Xi}_n(i)>0$. These two upper bounds achieving activities are consistent with step 4) in the algorithm description of DIVBAR-RMIA.

In summary, the upper bound achieving conditions of (67), (68), and (70) prove that DIVBAR-RMIA maximizes $\mathbb{E}\{Z_n(i,\hat{\mathbf{Q}}(u_{n,i}))\big|\,\hat{\mathbf{Q}}(u_{n,i})\}$ among the policies in $\mathcal{P}$.

Appendix I
Proof of Theorem 5

Reviewing Theorem 2, for a network satisfying Assumption 1 with an input rate matrix $(\lambda_n^{(c)})$ interior to Λ_{RMIA}, there exists a stationary randomized policy with RMIA: $Policy^*$, under which the t-timeslot average Lyapunov drift satisfies the condition (4) given in Lemma 1, and therefore the strong stability can be achieved.

In this proof, the goal is to show that the t-timeslot Lyapunov drift under DIVBAR-RMIA, denoted as $\hat{Policy}$, satisfies the similar condition. Correspondingly, given the ε satisfying $(\lambda_n^{(c)}+\varepsilon)\in\Lambda_{\mathrm{RMIA}}$, the main proof strategy is to compare the upper bounds of t-timeslot Lyapunov drifts respectively under $\hat{Policy}$ and under a policy that is a "modified version" of $Policy^*$ and is denoted as $Policy'^*$. Here $Policy'^*$ is defined as follows: *it is the same as $\hat{Policy}$ from timeslot 0 to timeslot t_0-1; starting from timeslot t_0, it makes the stationary randomized transmitting and forwarding decisions with the same probabilities as $Policy^*$, while the transmissions with RMIA does not use the pre-accumulated partial information before timeslot t_0.*[10]

Start by doing some manipulations on the t-timeslot queueing relation similar as (40)-(41) under an arbitrary policy, the t-timeslot average Lyapunov drift starting at any timeslot t_0 is upper bounded as follows:

$$\begin{aligned}\Delta_t(\mathbf{Q}(t_0))\le B(t)&+\frac{2}{t}\sum_{n,c}Q_n^{(c)}(t_0)\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{a_n^{(c)}(\tau)\right\}\\&-2\sum_n\mathbb{E}\left\{Z_n(\mathbf{Q}(t_0))\big|_{t_0}^{t_0+t-1}\middle|\,\mathbf{Q}(t_0)\right\},\end{aligned}\quad (71)$$

where, the summation metric $Z_n(\mathbf{Q}(t_0))\big|_{t_0}^{t_0+t-1}$ represents the following expression:

$$Z_n(\mathbf{Q}(t_0))\big|_{t_0}^{t_0+t-1}\triangleq\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\sum_c\sum_{k:k\in\mathcal{K}_n}b_{nk}^{(c)}(\tau)\left[Q_n^{(c)}(t_0)-Q_k^{(c)}(t_0)\right].\quad (72)$$

The comparison between $\hat{Policy}$ and $Policy'^*$ focuses on comparing the term $\sum_n\mathbb{E}\{Z_n(\mathbf{Q}(t_0))\big|_{t_0}^{t_0+t-1}\big|\,\mathbf{Q}(t_0)\}$ on the right hand side of (71). In this paper, we call this term as the *key metric*.

In order to facilitate the comparison between $\hat{Policy}$ and $Policy'^*$, an intermediate policy $\tilde{Policy}$ is introduced: *it is the same as $\hat{Policy}$ from timeslot 0 to timeslot t_0-1; in timeslot t_0, each node n makes a transmitting decision based on $\hat{\mathbf{Q}}(t_0)$ using the same strategy as under $\hat{Policy}$, and according to this transmission decision, either keeps transmitting units of the chosen commodity with RMIA or keeps silent (by transmitting the null units) from then on but without using the pre-accumulated partial information before timeslot t_0; in each first-decoding timeslot for node n since timeslot t_0, node n makes the forwarding decision based on $\hat{\mathbf{Q}}(t_0)$* using the same strategy as $\hat{Policy}$.

The proof proceeds into two steps: comparing $\tilde{Policy}$ and $Policy'^*$ as is shown Subsection I-A and comparing $\hat{Policy}$ and $\tilde{Policy}$ as is shown in Subsection I-B.

[10]Note that $Policy'^*$ in principle does not belong the RMIA policy space we discussed in this paper, because using no pre-accumulated partial information before timeslot t_0 is equivalent to making forwarding decisions and renewal operations for all commodities in timeslot t_0-1 and starting a new epoch in timeslot t_0, while timeslot t_0-1 may not be a first-decoding timeslot for all commodities. But it is convenient to introduce $Policy'^*$ as an intermediate policy in the proof, because starting from timeslot t_0, it is statistically the same as $Policy^*$ starting from timeslot 0 but with initial backlog state $\hat{\mathbf{Q}}_n(t_0)$. Likewise, the intermediate policies $\tilde{Policy}$ and $\tilde{Policy}^*$, which will be introduced later, do not belong to the RMIA policy space for the same reason but are introduced to facilitate the proof.

A. Comparison on the key metric between $\tilde{Policy}$ and $Policy'^{*}$

In this part of proof, the key metrics under $\tilde{Policy}$ and $Policy'^{*}$ are analyzed on the interval from timeslot t_0 to timeslot t_0+t-1. In order to facilitate the comparison, we further introduce an intermediate policy: $\tilde{Policy}^{*}$, which is defined as follows: *it is the same as $\hat{Policy}$ from timeslot 0 to timeslot t_0-1; starting from timeslot t_0, each node n makes the same transmitting decision as under $Policy'^{*}$ in each timeslot, according to which node n chooses units to transmit with RMIA but without using the pre-accumulated partial information before timeslot t_0; in each first-decoding timeslot for node n since timeslot t_0, node n makes the forwarding decision based on* $\hat{\mathbf{Q}}(t_0)$ using the same strategy as $\tilde{Policy}$.

To prepare the later proof, we count the first epoch of commodity c that ends in or after timeslot t_0 as epoch 1 of commodity c. Then we define the following variables:

- $\alpha_n^{(c)}(t_0,t)$: the number of times node n decides to transmit the units of commodity c from timeslot t_0 to timeslot t_0+t-1.
- $\beta_n^{(c)}(t_0,t)$: the number of epochs for transmitting units of commodity c from node n with RMIA that end within the interval from timeslot t_0 to timeslot t_0+t-1.
- $\tau_j^{(c)}$: the index of the subsequence of timeslots which are used to transmit units of commodity c.
- $u_{n,i}^{(c)}$: the index of the starting timeslot of epoch i of commodity c in $\{\tau_j^{(c)}\}$.
- $X_{nk}^{(c)}(i)$: the random variable that takes value 1 if node $k\in\mathcal{K}_n$ is in the first successful receiver set of epoch i of commodity c with RMIA and takes value 0 otherwise.
- $Z_n^{(c)}(i,\mathbf{Q}(\tau))$: the metric over the epoch i of commodity c for node n under a policy based on a CPQ backlog state in timeslot τ shown as

$$Z_n^{(c)}(i,\mathbf{Q}(\tau)) \triangleq \sum_{j=u_{n,i}^{(c)}}^{u_{n,i+1}^{(c)}-1}\sum_{k:k\in\mathcal{K}_n} b_{nk}^{(c)}\left(\tau_j^{(c)}\right)\left[Q_n^{(c)}(\tau)-Q_k^{(c)}(\tau)\right]. \quad (73)$$

*1) **Comparing the key metrics under** $\tilde{Policy}^{*}$ **and** $Policy'^{*}$:* Because $\tilde{Policy}^{*}$ uses the same forwarding strategy as $\hat{Policy}$ since timeslot t_0, and $Policy'^{*}$ has synchronous epochs as $\tilde{Policy}^{*}$, resembling the derivations in the proof of Lemma 3 (see Appendix H), it follows that

$$\begin{aligned}&\sum_n \mathbb{E}\left\{\tilde{Z}_n^{*}\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\hat{\mathbf{Q}}(t_0)\right\}\\ =&\sum_n\sum_c\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\beta'^{*(c)}_n(t_0,t)}\max_{k:k\in\mathcal{K}_n}\left\{X'^{*(c)}_{nk}(i)\,\hat{W}_{nk}^{(c)}(t_0)\right\}\middle|\hat{\mathbf{Q}}(t_0)\right\}\\ \ge&\sum_n\sum_c\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\beta'^{*(c)}_n(t_0,t)}Z'^{*(c)}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\middle|\hat{\mathbf{Q}}(t_0)\right\}\\ =&\sum_n \mathbb{E}\left\{Z'^{*}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\hat{\mathbf{Q}}(t_0)\right\},\end{aligned} \quad (74)$$

where we use the facts that $\tilde{\beta}_n^{*(c)}(t_0,t)=\beta'^{*(c)}_n(t_0,t)$, $\tilde{u}_{n,i}^{*(c)}=u'^{*(c)}_{n,i}$ and $\tilde{X}_{nk}^{*(c)}(i)=X'^{*(c)}_{nk}(i)$.

*2) **Comparing the key metrics under** $\tilde{Policy}$ **and** $\tilde{Policy}^{*}$:* To facilitate the later proof, we first consider the policy set, denoted as $\mathcal{Y}$, that consists of the policies each of which is defined as follows: *it is the same as $\hat{Policy}$ from timeslot 0 to timeslot t_0-1; from timeslot t_0, each node n uses fixed probabilities to choose commodities to transmit with RMIA without using the pre-accumulated partial information before timeslot t_0; in each first-decoding timeslot for node n since timeslot t_0, node n makes the forwarding decisions based on* $\hat{\mathbf{Q}}_n(t_0)$ using the same strategy as $\tilde{Policy}$. Note that both $\tilde{Policy}$ or $\tilde{Policy}^{*}$ belong to $\mathcal{Y}$.

Under a policy in $\mathcal{Y}$, the key metric can be expressed as follows:

$$\begin{aligned}&\sum_n \mathbb{E}\left\{Z_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\hat{\mathbf{Q}}(t_0)\right\}\\ =&\sum_n\sum_c\mathbb{E}\left\{\frac{\alpha_n^{(c)}(t_0,t)}{t}\frac{\frac{1}{\beta_n^{(c)}(t_0,t)}\sum_{i=1}^{\beta_n^{(c)}(t_0,t)}Z_n^{(c)}\left(i,\hat{\mathbf{Q}}(t_0)\right)}{\frac{\alpha_n^{(c)}(t_0,t)}{\beta_n^{(c)}(t_0,t)}}\middle|\hat{\mathbf{Q}}(t_0)\right\},\end{aligned} \quad (75)$$

where we define $0/0\triangleq 0$ for terms in the above equation. Since the transmission decisions under a policy in $\mathcal{Y}$ are i.i.d. over timeslots, according to the law of large number, we have

$$\lim_{t\to\infty}\frac{\alpha_n^{(c)}(t_0,t)}{t}=\alpha_n^{(c)} \text{ with prob. 1.} \quad (76)$$

Additionally, since $\sum_{i=1}^{\beta_n^{(c)}(t_0,t)}T_n^{(c)}(i)\le\alpha_n^{(c)}(t_0,t)<\sum_{i=1}^{\beta_n^{(c)}(t_0,t)+1}T_n^{(c)}(i)$,[11] we have according to the law of large number, if $\alpha_n^{(c)}>0$,

$$\lim_{t\to\infty}\frac{\alpha_n^{(c)}(t_0,t)}{\beta_n^{(c)}(t_0,t)}=\mathbb{E}\left\{T_n^{(c)}(i)\right\}=\frac{1}{\beta_n^{\mathrm{rmia}}}, \text{ with prob. 1.} \quad (77)$$

Moreover, the value of $Z_n^{(c)}(i,\hat{\mathbf{Q}}(t_0))$ under a policy in $\mathcal{Y}$ only depends on $\hat{\mathbf{Q}}(t_0)$ and the channel realizations in epoch i, therefore $Z_n^{(c)}(i,\hat{\mathbf{Q}}(t_0))$ is i.i.d. across epochs given $\hat{\mathbf{Q}}(t_0)$, and we have, if $\alpha_n^{(c)}>0$,

$$\lim_{t\to\infty}\frac{1}{\beta_n^{(c)}(t_0,t)}\sum_{i=1}^{\beta_n^{(c)}(t_0,t)}Z_n^{(c)}\left(i,\hat{\mathbf{Q}}(t_0)\right)=z_n^{(c)}\left(\hat{\mathbf{Q}}(t_0)\right), \text{ with prob. 1,} \quad (78)$$

where $z_n^{(c)}(\hat{\mathbf{Q}}(t_0))\triangleq\mathbb{E}\{Z_n^{(c)}(i,\hat{\mathbf{Q}}(t_0))\,|\,\hat{\mathbf{Q}}(t_0)\}$. According to (75)-(78) and incorporating the trivial case $\alpha_n^{(c)}=0$, it follows from (75) that

$$\lim_{t\to\infty}\sum_n\mathbb{E}\left\{Z_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\hat{\mathbf{Q}}(t_0)\right\}$$

[11] For any policy in $\mathcal{Y}$, we assume that a forwarding decision is made at each node n in timeslot t_0-1, and the epoch 1 ($i=1$) of each commodity for node n starts in or after timeslot t_0.

$$= \sum_n \sum_c \alpha_n^{(c)} \beta_n^{\mathrm{rmia}} z_n^{(c)} \left(\hat{\mathbf{Q}}(t_0) \right). \tag{79}$$

With (79), given the ε satisfying $(\lambda_n^{(c)} + \varepsilon) \in \Lambda_{\mathrm{RMIA}}$, under a policy in $\mathcal{Y}$, there exists an integer D_y, such that, for all $t_0 \geq 0$, whenever $t \geq D_y$, we have

$$\left| \sum_n \mathbb{E} \left\{ Z_n \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} - \sum_n \sum_c \alpha_n^{(c)} \beta_n^{\mathrm{rmia}} z_n^{(c)} \left(\hat{\mathbf{Q}}(t_0) \right) \right| \leq \frac{\varepsilon}{16} \sum_{n,c} \hat{Q}_n^{(c)}(t_0). \tag{80}$$

Since $\sum_c \alpha_n^{(c)} \leq 1$, we have

$$\sum_n \sum_c \alpha_n^{(c)} \beta_n^{\mathrm{rmia}} z_n^{(c)} \left(\hat{\mathbf{Q}}(t_0) \right) \leq \sum_n \beta_n^{\mathrm{rmia}} \max_c \left\{ z_n^{(c)} \left(\hat{\mathbf{Q}}(t_0) \right) \right\}. \tag{81}$$

If defining $\tilde{c}_n = \arg\max_c \{ z_n^{(c)}(\hat{\mathbf{Q}}(t_0)) \}$, (81) becomes an equality when node n chooses commodity $\tilde{c}_n$ to transmit from timeslot t_0 on, which is consistent with the strategy of making transmitting decisions under $Pol\tilde{i}cy$. Under $Pol\tilde{i}cy$ and $Pol\tilde{i}cy^*$, define $\tilde{D}_1 \triangleq D_y$ and $\tilde{D}^* \triangleq D_y$ as the respective threshold integers. Choose $\tilde{D} \triangleq \max\{\tilde{D}^*, \tilde{D}_1\}$, based on (81) and (80), we can get, for all $t_0 \geq 0$, whenever $t \geq \tilde{D}$,

$$\sum_n \mathbb{E} \left\{ \tilde{Z}_n \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} \geq \sum_n \mathbb{E} \left\{ \tilde{Z}_n^* \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} - \frac{\varepsilon}{8} \sum_{n,c} \hat{Q}_n^{(c)}(t_0). \tag{82}$$

3) ***Comparing the key metrics under*** $Pol\tilde{i}cy$ ***and*** $Policy'^*$*:* In summary, plugging (74) into (82), and it follows that, for all $t_0 \geq 0$, whenever $t \geq \tilde{D}$,

$$\sum_n \mathbb{E} \left\{ \tilde{Z}_n \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} \geq \sum_n \mathbb{E} \left\{ {Z'}_n^* \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} - \frac{\varepsilon}{8} \sum_{n,c} \hat{Q}_n^{(c)}(t_0). \tag{83}$$

B. Comparison on the key metric between $Pol\hat{i}cy$ and $Pol\tilde{i}cy$

Note that each epoch either $Pol\hat{i}cy$ or $Pol\tilde{i}cy$ consists of contiguous timeslots. In this subsection, for a policy, under which each epoch consists of contiguous timeslots, we count the first epoch for node n that ends in or after timeslot t_0 as epoch 1 (without specifying the commodity) and denote the starting timeslot of epoch i as timeslot $u_{n,i}$.

The later proof can be facilitated by introducing another intermediate but *non-causal* policy: $Pol\tilde{\tilde{i}}cy$, which is defined as follows: *it is the same as $Pol\hat{i}cy$ from timeslot 0 to timeslot $\hat{u}_{n,1} - 1$ at each node n; starting from timeslot $\hat{u}_{n,1}$, node n keeps transmitting the units of commodity $\tilde{c}_n$ with RMIA; in each first-decoding timeslot for node n since timeslot t_0, node n makes the forwarding decision based on $\hat{\mathbf{Q}}_n(t_0)$ using the same strategy as $Pol\hat{i}cy$ (or $Pol\tilde{i}cy$).* Here note that $\tilde{c}_n$ is decided based on the value $\hat{\mathbf{Q}}(t_0)$, which is formed by $Pol\hat{i}cy$, and we have $u_{n,1} \leq t_0$. With this non-causality feature, $Pol\tilde{\tilde{i}}cy$ is non-realizable but is used to facilitate the theoretical analysis.

1) ***Comparison between*** $Pol\hat{i}cy$ ***and*** $Pol\tilde{\tilde{i}}cy$*:* For a policy, under which each epoch consists of contiguous timeslots, define $M_n(t_0, t)$ as the minimum number of epochs that covers the time interval $[t_0, t_0 + t - 1]$, i.e.,

$$M_n(t_0, t) \triangleq \min \left\{ m : u_{n,1} + \sum_{i=1}^{m} T_n(i) - 1 \geq t_0 + t - 1 \right\}, \tag{84}$$

where $T_n(i)$ is defined as the number of timeslots in the ith epoch for node n. Additionally, as is defined in (21), given $\hat{\mathbf{Q}}_n(t_0)$, $\tilde{\tilde{Z}}_n^{(c)}(i, \hat{\mathbf{Q}}(t_0))$ is i.i.d. across epochs under $Pol\tilde{\tilde{i}}cy$, and we can notate its conditional expectation as follows:

$$\mathbb{E} \left\{ \tilde{\tilde{Z}}_n \left(i, \hat{\mathbf{Q}}(t_0) \right) \Big| \hat{\mathbf{Q}}(t_0) \right\} \triangleq \tilde{\tilde{z}}_n \left(\hat{\mathbf{Q}}(t_0) \right). \tag{85}$$

Incorporating $M_n(t_0, t)$ and $\tilde{\tilde{z}}_n^{(c)}(\hat{\mathbf{Q}}(t_0))$, we propose the following lemma that compares $Pol\hat{i}cy$ and $Pol\tilde{\tilde{i}}cy$:

Lemma 5. *For a network satisfying Assumption 1, there exists a positive integer $\hat{D}$ such that, for all $t_0 \geq 0$, whenever $t \geq \hat{D}$, $Pol\hat{i}cy$ and $Pol\tilde{\tilde{i}}cy$ satisfy the following relationship given the backlog state $\hat{\mathbf{Q}}(t_0)$:*

$$\sum_n \mathbb{E} \left\{ \hat{Z}_n \left(\hat{\mathbf{Q}}(t_0) \right) \Big|_{t_0}^{t_0+t-1} \Big| \hat{\mathbf{Q}}(t_0) \right\} \geq \frac{1}{t} \sum_n \tilde{\tilde{z}}_n \left(\hat{\mathbf{Q}}(t_0) \right) \mathbb{E} \left\{ \tilde{\tilde{M}}_n(t_0, t) \right\} - \left[N C_2(t) + C_1(t) + \frac{\varepsilon}{8} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \right], \tag{86}$$

where we have $C_1(t) \triangleq Nt(N + A_{\max} + 1)$ and $C_2(t) \triangleq t(N + A_{\max} + 1)$.

The detailed proof of Lemma 5 is shown in Appendix L. Note that the metric under $Pol\tilde{\tilde{i}}cy$ on the right hand side of (86) is not exactly the key metric we defined before but is more convenient to use for the later derivations.

2) ***Comparison between*** $Pol\tilde{\tilde{i}}cy$ ***and*** $Pol\tilde{i}cy$*:* For epoch i under $Pol\tilde{\tilde{i}}cy$ and epoch j under $Pol\tilde{i}cy$, where $1 \leq i \leq \tilde{\tilde{M}}_n(t_0, t)$ and $1 \leq j \leq \tilde{M}_n(t_0, t)$, note that $\tilde{\tilde{Z}}_n(i, \hat{\mathbf{Q}}(t_0))$ and $\tilde{Z}_n(j, \hat{\mathbf{Q}}(t_0))$ are identically distributed because of the i.i.d channel state across timeslots and the fact that $Pol\tilde{\tilde{i}}cy$ and $Pol\tilde{i}cy$ choose the same commodity to transmit in these two epochs, respectively. Therefore, we get

$$\tilde{\tilde{z}}_n \left(\hat{\mathbf{Q}}(t_0) \right) = \tilde{z}_n \left(\hat{\mathbf{Q}}(t_0) \right). \tag{87}$$

On the other hand, under the two policies, since $\tilde{\tilde{u}}_{n,1} \leq t_0 = \tilde{u}_{n,1}$, with any channel realization (common for both policies), we have

$$\tilde{\tilde{M}}_n(t_0, t) \geq \tilde{M}_n(t_0, t). \tag{88}$$

Resembling part of the proof in Lemma 5 (see Appendix L) and considering that $\tilde{M}_n(t_0, t) \leq t$ because $\tilde{T}_n(i) \geq 1$, define the following indicator function of integer $i = 1, 2, \cdots, t$ under $\tilde{Policy}$:

$$\tilde{1}_n(i) = \begin{cases} 1, & 1 \le i \le \tilde{M}_n(t_0, t) \le t \\ 0, & \tilde{M}_n(t_0, t) < i \le t,. \end{cases} \quad (89)$$

Consider that $\tilde{Z}_n(i, \hat{\mathbf{Q}}(t_0))$ and $\tilde{1}_n(i)$ are independent because $\{\tilde{Z}_n(i, \hat{\mathbf{Q}}(t_0)) : i \ge 1\}$ are i.i.d. and the value of $\tilde{1}_n(i)$ only depends on $\tilde{T}_n(1), \cdots, \tilde{T}_n(i-1)$. With (87) and (88), we have the following relationship:

$$\begin{aligned} &\mathbb{E}\left\{ \sum_{i=1}^{\tilde{M}_n(t_0,t)} \tilde{Z}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ =&\sum_{i=1}^{t} \mathbb{E}\left\{ \tilde{Z}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \middle| \hat{\mathbf{Q}}(t_0) \right\} \mathbb{E}\left\{\tilde{1}_n(i)\right\} \\ =&\tilde{z}_n\left(\hat{\mathbf{Q}}(t_0)\right) \mathbb{E}\left\{\tilde{M}_n(t_0, t)\right\} \\ \le&\tilde{\tilde{z}}_n\left(\hat{\mathbf{Q}}(t_0)\right) \mathbb{E}\left\{\tilde{\tilde{M}}_n(t_0, t)\right\}, \end{aligned} \quad (90)$$

which completes the comparison between $\tilde{\tilde{Policy}}$ and $\tilde{Policy}$.

3) ***Comparison between*** $\hat{Policy}$ ***and*** $\tilde{Policy}$*:* Plug (90) back into (86), it follows that, for all $t_0 \ge 0$, whenever $t \ge \hat{D}_2$, we have

$$\begin{aligned} &\sum_n \mathbb{E}\left\{ \hat{Z}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ \ge&\sum_n \mathbb{E}\left\{ \frac{1}{t} \sum_{i=1}^{\tilde{M}_n(t_0,t)} \tilde{Z}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ &- \left[NC_2(t) + C_1(t) + \frac{\varepsilon}{8} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \right] \\ \ge&\sum_n \mathbb{E}\left\{ \tilde{Z}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ &- \left[NC_2(t) + C_1(t) + \frac{\varepsilon}{8} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \right], \end{aligned} \quad (91)$$

which completes the comparison between $\hat{Policy}$ and $\tilde{Policy}$.

C. Strong stability achieved under $\hat{Policy}$

Combining (83) in Subsection I-A and (91) in Subsection I-B, the comparison on the key backpressure metric between $\hat{Policy}$ and $Policy'^*$ is shown as follows: for all $t_0 \ge 0$, whenever $\forall t \ge \max\{\hat{D}, \tilde{D}\}$, we have

$$\begin{aligned} &\sum_n \mathbb{E}\left\{ \hat{Z}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ \ge&\sum_n \mathbb{E}\left\{ Z'^*_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ &- \left[NC_2(t) + C_1(t) + \frac{\varepsilon}{4} \sum_{n,c} \hat{Q}_n^{(c)}(t_0,) \right]. \end{aligned} \quad (92)$$

After plugging (92) back into (71), $\hat{\Delta}_t(\hat{\mathbf{Q}}(t_0))$ can be further upper bounded as follows:

$$\begin{aligned} \hat{\Delta}_t\left(\hat{\mathbf{Q}}(t_0)\right) \le& B(t) + 2\left[C_1(t) + NC_2(t)\right] \\ &+ \frac{\varepsilon}{2} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) - 2\Upsilon\left(\hat{\mathbf{Q}}(t_0)\right), \end{aligned} \quad (93)$$

where $\Upsilon(\hat{\mathbf{Q}}(t_0))$ is as follows:

$$\begin{aligned} \Upsilon\left(\hat{\mathbf{Q}}(t_0)\right) \triangleq& \sum_n \mathbb{E}\left\{ Z'^*_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ &- \frac{1}{t} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \sum_{\tau=t_0}^{t_0+t-1} \mathbb{E}\left\{a_n^{(c)}(\tau)\right\} \\ =& \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \frac{1}{t} \sum_{\tau=t_0}^{t_0+t-1} \mathbb{E}\left\{ \sum_{k:k\in\mathcal{K}_n} b'^{*(c)}_{nk}(\tau) \right. \\ &\left. - \sum_{k:k\in\mathcal{K}_n} b'^{*(c)}_{kn}(\tau) - a_n^{(c)}(\tau) \right\} \end{aligned} \quad (94)$$

Since from timeslot t_0, $Policy'^*$ is the same as the stationary randomized policy $Policy^*$ starting from timeslot 0, according to the derivations from (41) to (45) in Appendix D, there exists a positive integer D^*, such that, for all $t_0 \ge 0$, whenever $t \ge D^*$, we have

$$\begin{aligned} \Upsilon\left(\hat{\mathbf{Q}}(t_0)\right) =& \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{ \sum_{k:k\in\mathcal{K}_n} b^{*(c)}_{nk}(\tau) \right. \\ &\left. - \sum_{k:k\in\mathcal{K}_n} b^{*(c)}_{kn}(\tau) - a_n^{(c)}(\tau) \right\} \ge \frac{\varepsilon}{2} \sum_{n,c} \hat{Q}_n^{(c)}(t_0) \end{aligned} \quad (95)$$

Plugging (95) back into (93) and letting $D \triangleq t \ge \max\{\hat{D}, \tilde{D}, D^*\}$, we have, for all $t_0 \ge 0$,

$$\hat{\Delta}_D\left(\hat{\mathbf{Q}}(t_0)\right) \le B(D) + C(D) - \frac{\varepsilon}{2} \sum_{n,c} \hat{Q}_n^{(c)}(t_0), \quad (96)$$

where we have $C(D) \triangleq 2\left[C_1(D) + NC_2(D)\right] = 4ND(N + A_{\max} + 1)$.

Thus, given the positive ε satisfying $(\lambda_n^{(c)} + \varepsilon) \in \Lambda_{\text{RMIA}}$, (96) is achieved under $\hat{Policy}$. According to Lemma 1, we achieve (22), which completes the proof.

Appendix J
Proof of Theorem 6

Given ε satisfying $(\lambda_n^{(c)} + \varepsilon) \in \Lambda_{\text{RMIA}}$, the goal of the proof is to show that, the t-timeslot Lyapunov drift under DIVBAR-FMIA, denoted as $\hat{\hat{Policy}}$, has an upper bound satisfying the condition (4) required by Lemma 1. The proof procedure is also similar to the proof of Theorem 5 (see Appendix I) except for minor modifications.

To facilitate the proof, we also introduce the intermediate policies: $\tilde{Policy}$, $Policy'^*$, $\tilde{\tilde{Policy}}$ and $\tilde{Policy}^*$, which are similar to the ones proposed in Appendix I except for the following modifications:

- the $Policy'^*$ in this proof is the same as $\hat{\hat{Policy}}$ from timeslot 0 to timeslot $t_0 - 1$;

- the $\tilde{Policy}$ in this proof is the same as $\hat{\hat{Policy}}$ from timeslot 0 to timeslot $t_0 - 1$, and from timeslot t_0 on, the transmitting and forwarding decisions of each epoch are made based on $\hat{\hat{\mathbf{Q}}}(t_0)$;
- the $\tilde{Policy}^*$ in this proof is the same as $\hat{\hat{Policy}}$ from timeslot 0 to timeslot $t_0 - 1$, and from timeslot t_0 on, the forwarding decisions of each epoch are made based on $\hat{\hat{\mathbf{Q}}}(t_0)$;
- the $\tilde{\tilde{Policy}}$ in this proof is the same as $\hat{\hat{Policy}}$ from timeslot 0 to timeslot $\hat{\hat{u}}_{n,1} - 1$ at each node n, and from timeslot $\hat{\hat{u}}_{n,1}$ on, the transmitting and forwarding decisions of epoch are made based on $\hat{\hat{\mathbf{Q}}}(t_0)$.

To achieve the proof goal, the strategy is to compare the key metric $\sum_n \mathbb{E}\{ Z_n(\hat{\hat{\mathbf{Q}}}(t_0))\Big|_{t_0}^{t_0+t-1} \Big| \hat{\hat{\mathbf{Q}}}(t_0)\}$ under the introduced policies.

A. Comparison on the key metric between $\tilde{Policy}$ and $Policy'^*$

The comparison between $\tilde{Policy}$ and $Policy'^*$ on the key metric is the same as that in the proof shown in Appendix I-A, except that the backlog coefficient here is $\hat{\hat{\mathbf{Q}}}(t_0)$. The final comparison results is that, for all $t_0 \geq 0$, there exists an integer $\tilde{D} > 0$, such that, whenever $t \geq \tilde{D}$,

$$\sum_n \mathbb{E}\left\{ \tilde{Z}_n\left(\hat{\hat{\mathbf{Q}}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\hat{\mathbf{Q}}}(t_0)\right\} \geq \sum_n \mathbb{E}\left\{ Z'^*_n\left(\hat{\hat{\mathbf{Q}}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\hat{\mathbf{Q}}}(t_0)\right\} - \frac{\varepsilon}{8}\sum_{n,c}\hat{\hat{Q}}_n^{(c)}(t_0). \tag{97}$$

B. Comparison on the key metric between $\hat{\hat{Policy}}$ and $\tilde{Policy}$

With the similar strategy as Appendix I-B, the comparison on the key backpressure metric between $\hat{\hat{Policy}}$ and $\tilde{Policy}$ consists of two steps: compare $\hat{\hat{Policy}}$ and $\tilde{\tilde{Policy}}$ and then compare $\tilde{\tilde{Policy}}$ and $\tilde{Policy}$. In this part of proof, the comparison between $\hat{\hat{Policy}}$ and $\tilde{\tilde{Policy}}$ over a single epoch is summarized as Lemma 6 shown as follows:

Lemma 6. *For each node n in a network satisfying Assumption 1 and for all $t_0 \geq 0$, whenever $t \geq T_{\max}$, we have*

$$\mathbb{E}\left\{ \hat{\hat{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1\right\} \geq \mathbb{E}\left\{ \tilde{\tilde{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(t_0)\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1\right\} - C_2(t), \tag{98}$$

where $T_{\max} \triangleq \max_{n:n\in\mathcal{N}}\{\mathbb{E}\{T_n(i)\}\}$; $C_2(t) = t(N + A_{\max} + 1)$; $\hat{\hat{1}}_n(i)$ is equal to $\hat{1}_n(i)$ because $\hat{\hat{Policy}}$ and $\hat{Policy}$ have synchronous epochs.

The proof of Lemma 6 is shown in Appendix M.

With (98), the remaining proof is the same as in Appendix I-B except changing backlog coefficients to $\hat{\hat{\mathbf{Q}}}(u_{n,i})$ and $\hat{\hat{\mathbf{Q}}}(t_0)$, we have: there exists an integer $\hat{\hat{D}} \geq T_{\max}$, such that, for all $t_0 \geq 0$, whenever $t \geq \hat{\hat{D}}$,

$$\sum_n \mathbb{E}\left\{ \hat{\hat{Z}}_n\left(\hat{\hat{\mathbf{Q}}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\hat{\mathbf{Q}}}(t_0)\right\} \geq \sum_n \mathbb{E}\left\{ \tilde{Z}_n\left(\hat{\hat{\mathbf{Q}}}(t_0)\right)\Big|_{t_0}^{t_0+t-1} \middle| \hat{\hat{\mathbf{Q}}}(t_0)\right\} - \left[NC_2(t) + C_1(t) + \frac{\varepsilon}{8}\sum_{n,c}\hat{\hat{Q}}_n^{(c)}(t_0)\right], \tag{99}$$

where $C_1(t) = Nt(N + A_{\max} + 1)$.

C. Strong stability achieved under $\hat{\hat{Policy}}$

With the similar manipulations as in Appendix I-C, it follows from (97) and (99) that, for all $t_0 \geq 0$ and with the integer $D \triangleq \max\{\hat{D}_2, \tilde{D}_2, D^*\}$, we have

$$\hat{\hat{\Delta}}_D\left(\hat{\hat{\mathbf{Q}}}(t_0)\right) \leq B(D) + C(D) - \frac{\varepsilon}{2}\sum_{n,c}\hat{\hat{Q}}_n^{(c)}(t_0), \tag{100}$$

where $C(D) = 4ND(N + A_{\max} + 1)$. According to Lemma 1, we achieve (23), which completes the proof.

Appendix K
Proof of Lemma 4

To facilitate the proof, we first define the *extended-epoch of commodity c for link (n,k)* as the interval consisting of contiguous timeslots between two timeslots, in each of which a unit of commodity c is forwarded from node n to node k. Specifically, this interval starts from the timeslot right after the timeslot when a unit of commodity c is forwarded from node n to node k, and ends at the timeslot when the next forwarding of a unit of commodity c from node n to node k happens.

Suppose from timeslot 0 up to an arbitrary timeslot t_0, $M_0 - 1$ units of commodity c have been forwarded from node n to node k, where $M_0 \geq 1$. Therefore, if we define $\bar{t}_{nk,i}^{(c)}$ as the starting timeslot of the ith extended epoch of commodity c for link (n,k), with which we further define $\bar{t}_{nk,1}^{(c)} = 0$, we have

$$\max\left\{\bar{t}_{nk,M_0}^{(c)} - 1, 0\right\} \leq t_0 < \bar{t}_{nk,M_0+1}^{(c)} - 1, \tag{101}$$

where, if $M_0 > 1$, $\bar{t}_{nk,M_0}^{(c)} - 1$ is the ending timeslot of the $(M_0 - 1)$th extended-epoch of commodity c; if $M_0 = 1$, timeslot t_0 must be located within the 1st extended-epoch of commodity c, and therefore t_0 is lower bounded by 0. Additionally, for each extended epoch i of commodity c for link (n,k), we have $b_{nk}^{(c)}(\bar{t}_{nk,i+1}^{(c)} - 1) = 1$.

Under a stationary randomized policy with the RMIA transmission scheme, because of the the renewal operations, and the stationarity of the decision makings and channel states, $\frac{1}{t}\sum_{\tau=0}^{t-1} b_{nk}^{(c)}(\tau)$ and $\frac{1}{t}\sum_{\tau=\bar{t}_{nk,i}^{(c)}}^{\bar{t}_{nk,i}^{(c)}+t-1} b_{nk}^{(c)}(\tau)$ are identically distributed. This property will be used in the following derivations.

On the one hand, if $t_0 \geq \bar{t}_{nk,M_0}^{(c)}$, then $\sum_{\tau=\bar{t}_{nk,M_0}^{(c)}}^{t_0-1} b_{nk}^{(c)}(\tau) =$

0 and $\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}+t}^{t_0+t-1} b^{(c)}_{nk}(\tau) \geq 0$, and we have

$$\begin{aligned}&\sum\nolimits_{\tau=t_0}^{t_0+t-1} b^{(c)}_{nk}(\tau)\\&=\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau)-\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{t_0-1} b^{(c)}_{nk}(\tau)+\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}+t}^{t_0+t-1} b^{(c)}_{nk}(\tau)\\&\geq\sum\nolimits_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau),\end{aligned}\tag{102}$$

where, for any summation term $\sum_{\tau=x}^{y} f(\tau)$, the summation value is defined as zero when $y < x$. Additionally, if $t_0 = \bar{t}^{(c)}_{nk,M_0} - 1$, since $b^{(c)}_{nk}(\bar{t}^{(c)}_{nk,M_0} - 1) = 1$ and $b^{(c)}_{nk}(\bar{t}^{(c)}_{nk,M_0} + t - 1) \leq 1$, we have

$$\begin{aligned}&\sum\nolimits_{\tau=t_0}^{t_0+t-1} b^{(c)}_{nk}(\tau)\\&=\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau)+b^{(c)}_{nk}\left(\bar{t}^{(c)}_{nk,M_0}-1\right)-b^{(c)}_{nk}\left(\bar{t}^{(c)}_{nk,M_0}+t-1\right)\\&\geq\sum\nolimits_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau).\end{aligned}\tag{103}$$

Note that, given any $\varepsilon > 0$, there exists an integer $D^{(c)}_{nk,0} > 0$, such that, whenever $t \geq D^{(c)}_{nk,0}$, we have

$$\left|\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{b^{(c)}_{nk}(\tau)\right\}-b^{(c)}_{nk}\right|\leq\frac{\varepsilon}{2}.\tag{104}$$

Then according to (102)-(103) together with the fact that $\frac{1}{t}\sum_{\tau=0}^{t-1} b^{(c)}_{nk}(\tau)$ and $\frac{1}{t}\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau)$ are identically distributed, it follows that, for all $t_0 \geq 0$, whenever $t \geq D^{(c)}_{nk,0}$,

$$\begin{aligned}\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{b^{(c)}_{nk}(\tau)\right\}&\geq\frac{1}{t}\mathbb{E}\left\{\sum_{\tau=\bar{t}^{(c)}_{nk,M_0}}^{\bar{t}^{(c)}_{nk,M_0}+t-1} b^{(c)}_{nk}(\tau)\right\}\\&=\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{b^{(c)}_{nk}(\tau)\right\}\\&\geq b^{(c)}_{nk}-\frac{\varepsilon}{2}.\end{aligned}\tag{105}$$

On the other hand, we have $\sum_{\tau=t_0}^{\bar{t}^{(c)}_{nk,M_0+1}-1} b^{(c)}_{nk}(\tau) \leq 2$ and $\sum_{\tau=t_0+t}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau) \geq 0$, and it follows that

$$\begin{aligned}&\sum\nolimits_{\tau=t_0}^{t_0+t-1} b^{(c)}_{nk}(\tau)\\&=\sum_{\tau=\bar{t}^{(c)}_{nk,M_0+1}}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau)-\sum_{\tau=t_0+t}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau)+\sum_{\tau=t_0}^{\bar{t}^{(c)}_{nk,M_0+1}-1} b^{(c)}_{nk}(\tau)\\&\leq\sum\nolimits_{\tau=\bar{t}^{(c)}_{nk,M_0+1}}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau)+2.\end{aligned}\tag{106}$$

Since $\frac{1}{t}\sum_{\tau=0}^{t-1} b^{(c)}_{nk}(\tau)$ and $\frac{1}{t}\sum_{\tau=\bar{t}^{(c)}_{nk,M_0+1}}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau)$ are identically distributed, then for all $t_0 \geq 0$, whenever $t \geq \max\{D^{(c)}_{nk,0}, \lceil 4/\varepsilon\rceil\}$, we have

$$\begin{aligned}\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{b^{(c)}_{nk}(\tau)\right\}&\leq\frac{1}{t}\mathbb{E}\left\{\sum_{\tau=\bar{t}^{(c)}_{nk,M_0+1}}^{\bar{t}^{(c)}_{nk,M_0+1}+t-1} b^{(c)}_{nk}(\tau)\right\}+\frac{2}{t}\\&\leq\left(b^{(c)}_{nk}+\frac{\varepsilon}{2}\right)+\frac{\varepsilon}{2}\\&=b^{(c)}_{nk}+\varepsilon.\end{aligned}\tag{107}$$

Combining (106) and (107), it follows that, given any $\varepsilon > 0$, for all $t_0 \geq 0$, whenever $t \geq \max\{D^{(c)}_{nk,0}, \lceil 4/\varepsilon\rceil\} \triangleq D^{(c)}_{nk}$, we have

$$\left|\frac{1}{t}\sum_{\tau=t_0}^{t_0+t-1}\mathbb{E}\left\{b^{(c)}_{nk}(\tau)\right\}-b^{(c)}_{nk}\right|\leq\varepsilon.\tag{108}$$

Appendix L
Proof of Lemma 5

The comparison procedure between $\hat{Policy}$ and $\tilde{\hat{Policy}}$ on their respective metrics involves two aspects: comparing the metrics over a single epoch; then extending the comparison to multiple epochs.

To facilitate the later comparisons, it is necessary to transform the key metric under $\hat{Policy}$ into certain form that is easier to manipulate. First, given the positive ε satisfying $(\lambda^{(c)}_n + \varepsilon) \in \Lambda_{\mathrm{RMIA}}$, for all $t_0 \geq 0$, whenever $\forall t \geq \lceil 8/\varepsilon\rceil \triangleq \hat{D}_1$, we have

$$\begin{aligned}&\sum_n\mathbb{E}\left\{\hat{Z}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\hat{\mathbf{Q}}(t_0)\right\}\\&=\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\hat{Z}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)-\right.\\&\left.\frac{1}{t}\sum_{\tau=t_0+t}^{u_{\hat{M}_n(t_0,t)+1}-1}\sum_c\sum_{k:k\in\mathcal{K}_n}\hat{b}^{(c)}_{nk}(\tau)\left[\hat{Q}^{(c)}_n(t_0)-\hat{Q}^{(c)}_k(t_0)\right]\middle|\hat{\mathbf{Q}}(t_0)\right\}\\&\geq\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\hat{Z}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\middle|\hat{\mathbf{Q}}(t_0)\right\}-\frac{\varepsilon}{8}\sum_{n,c}\hat{Q}^{(c)}_n(t_0)\\&=\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_i)\right)\middle|\hat{\mathbf{Q}}(t_0)\right\}-\frac{\varepsilon}{8}\sum_{n,c}\hat{Q}^{(c)}_n(t_0)\\&\quad-\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\left[\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_i)\right)-\hat{Z}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\right]\middle|\hat{\mathbf{Q}}(t_0)\right\},\end{aligned}\tag{109}$$

where

$$\begin{aligned}&\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\left[\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_i)\right)-\hat{Z}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\right]\middle|\hat{\mathbf{Q}}(t_0)\right\}\\&=\sum_n\mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\sum_{\tau=u_{n,i}}^{u_{n,i+1}-1}\sum_c\sum_{k:k\in\mathcal{K}_n}\hat{b}^{(c)}_{nk}(\tau)\times\right.\end{aligned}$$

$$\left[\hat{Q}_n^{(c)}(u_{n,i})-\hat{Q}_n^{(c)}(t_0)+\hat{Q}_k^{(c)}(t_0)-\hat{Q}_k^{(c)}(u_{n,i})\right]\Big|\,\hat{\mathbf{Q}}(t_0)\Big\}. \tag{110}$$

In (110), $t_0 < u_{n,2} <, \cdots, < u_{n,M_n(t_0,t)} \le t_0 + t - 1$, while $u_{n,1} \le t_0$ and $t_0 - u_{n,1} + 1 \le T_n(1)$. Then we have the following relationship:

$$\hat{Q}_n^{(c)}(u_{n,i}) - \hat{Q}_n^{(c)}(t_0) \le \sum_{\tau=t_0}^{u_{n,i}-1}\left[\sum_{k:k\in\mathcal{K}_n}\hat{b}_{kn}^{(c)}(\tau) + a_n^{(c)}(\tau)\right] \le t\,(N + A_{\max}), \quad \text{for } 2 \le i \le M_n(t_0,t); \tag{111}$$

$$\hat{Q}_k^{(c)}(t_0) - \hat{Q}_k^{(c)}(u_{n,i}) \le \sum_{\tau=t_0}^{u_{n,i}-1}\sum_{j:j\in\mathcal{K}_k}\hat{b}_{kj}^{(c)}(\tau) \le t, \ \text{for } 2\le i\le M_n(t_0,t),\ k\in\mathcal{K}_n; \tag{112}$$

$$\hat{Q}_n^{(c)}(u_{n,1}) - \hat{Q}_n^{(c)}(t_0) \le \sum_{\tau=u_{n,1}}^{t_0-1}\sum_{k:k\in\mathcal{K}_n}\hat{b}_{nk}^{(c)}(\tau) = 0; \tag{113}$$

$$\hat{Q}_k^{(c)}(t_0) - \hat{Q}_k^{(c)}(u_{n,1}) \le \sum_{\tau=u_{n,1}}^{t_0-1}\left[\sum_{j:j\in\mathcal{K}_k}\hat{b}_{jk}^{(c)}(\tau) + a_k^{(c)}(\tau)\right] \le (N + A_{\max})\,T_n(1),\ \text{for } k\in\mathcal{K}_n. \tag{114}$$

Setting $t \ge T_{\max} \triangleq \max_{n:n\in\mathcal{N}}\{\mathbb{E}\{T_n(i)\}\}$ and incorporating the fact that $\hat{M}_n(t_0,t) \le t$, we plug (111)-(114) into (110) and get

$$\sum_n \mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\left[\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)-\hat{Z}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\right]\middle|\,\hat{\mathbf{Q}}(t_0)\right\} \le Nt\,(N + A_{\max}+1) \triangleq C_1(t). \tag{115}$$

If denoting $\hat{D} = \max\{\hat{D}_1, T_{\max}\}$, and plugging (115) into (109) with $t \ge \hat{D}$, we finally get, for all $t_0 \ge 0$,

$$\sum_n \mathbb{E}\left\{\hat{Z}_n\left(\hat{\mathbf{Q}}(t_0)\right)\Big|_{t_0}^{t_0+t-1}\middle|\,\hat{\mathbf{Q}}(t_0)\right\} \ge \sum_n \mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(t_0)\right\} - C_1(t) - \frac{\varepsilon}{8}\sum_{n,c}Q_n^{(c)}(t_0). \tag{116}$$

A. Comparison between $Po\hat{l}icy$ and $Po\tilde{\hat{l}}icy$ over a single epoch

Define the following indicator function of integer $i = 1, 2, 3, \cdots, t$:

$$\hat{1}_n(i) = \begin{cases} 1, & 1 \le i \le \hat{M}_n(t_0,t) \le t \\ 0, & \hat{M}_n(t_0,t) < i \le t. \end{cases} \tag{117}$$

Since each node n under $Po\hat{l}icy$ makes decisions based on the backlog state observation $\hat{\mathbf{Q}}(u_{n,i})$ for each epoch i, the value of $\hat{Z}_n(i,\hat{\mathbf{Q}}(u_{n,i}))$ is independent of $\hat{\mathbf{Q}}(t_0)$ given $\hat{\mathbf{Q}}(u_{n,i})$. Consequently, we have

$$\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{1}_n(i)=1\right\} = \mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\}. \tag{118}$$

Additionally, according to Lemma 3, the metric $\mathbb{E}\{Z_n(i,\hat{\mathbf{Q}}(u_{n,i}))\big|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{1}_n(i)=1\}$ is maximized under $Po\hat{l}icy$ among all policies within policy set $\mathcal{P}$, to which $Po\tilde{\hat{l}}icy$ also belongs. Thus, given any $\hat{\mathbf{Q}}(t_0)$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\}\\
&\ge \mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\}\\
&= \mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\} -\\
&\quad\mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)-\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}),\hat{\mathbf{Q}}(t_0),\hat{1}_n(i)=1\right\},
\end{aligned} \tag{119}$$

where, similar to (110)-(114) but with the roles of n and k switched, by setting $t \ge \hat{D}$, we have

$$\mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)-\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(u_{n,i}), \hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\} \le t\,(N + A_{\max}+1) \triangleq C_2(t). \tag{120}$$

Plugging (120) into (119) and then taking expectations on both sides over $\hat{\mathbf{Q}}(u_{n,i})$, it follows that, whenever $t \ge \hat{D}$,

$$\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\} \ge \mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\middle|\,\hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\} - C_2(t), \tag{121}$$

which completes the comparison on the key metric over a single epoch between $Po\hat{l}icy$ and $Po\tilde{\hat{l}}icy$.

B. Comparison between $Po\hat{l}icy$ and $Po\tilde{\hat{l}}icy$ over $\hat{M}_n(t_0,t)$ (or $\tilde{\hat{M}}_n(t_0,t)$) epochs

Starting from (116), we rewrite the first term on the right hand side as follows:

$$\sum_n \mathbb{E}\left\{\frac{1}{t}\sum_{i=1}^{\hat{M}_n(t_0,t)}\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\middle|\,\hat{\mathbf{Q}}(t_0)\right\} = \frac{1}{t}\sum_n\sum_{i=1}^{t}\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\hat{1}_n(i)\middle|\,\hat{\mathbf{Q}}(t_0)\right\}. \tag{122}$$

Considering that, if $\hat{1}_n(i) = 0$, we have

$$\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\hat{1}_n(i)\middle|\,\hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=0\right\} = \mathbb{E}\left\{\tilde{\hat{Z}}_n\left(i,\hat{\mathbf{Q}}(t_0)\right)\hat{1}_n(i)\middle|\,\hat{\mathbf{Q}}(t_0), 1_n(i)=0\right\} = 0; \tag{123}$$

if $1_n(i) = 1$, according to (121), we have, for all $t_0 \ge 0$, whenever $t \ge \hat{D}$,

$$\mathbb{E}\left\{\hat{Z}_n\left(i,\hat{\mathbf{Q}}(u_{n,i})\right)\hat{1}_n(i)\middle|\,\hat{\mathbf{Q}}(t_0), \hat{1}_n(i)=1\right\}$$

$$\geq \mathbb{E}\left\{ \tilde{\tilde{Z}}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \hat{1}_n(i) \middle| \hat{\mathbf{Q}}(t_0), 1_n(i) = 1 \right\} - C_2(t). \tag{124}$$

In sum of (123) and (124), it follows that, for all $t_0 \geq 0$, whenever $t \geq \hat{D}$,

$$\mathbb{E}\left\{ \hat{Z}_n\left(i, \hat{\mathbf{Q}}(u_{n,i})\right) \hat{1}_n(i) \middle| \hat{\mathbf{Q}}(t_0) \right\} \geq \mathbb{E}\left\{ \tilde{\tilde{Z}}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \hat{1}_n(i) \middle| \hat{\mathbf{Q}}(t_0) \right\} - C_2(t). \tag{125}$$

Plug (125) into right hand side of (122), and it follows that, for all $t_0 \geq 0$, whenever $t \geq \hat{D}$,

$$\sum_n \mathbb{E}\left\{ \frac{1}{t} \sum_{i=1}^{\hat{M}_n(t_0,t)} \hat{Z}_n\left(i, \hat{\mathbf{Q}}(u_{n,i})\right) \middle| \hat{\mathbf{Q}}(t_0) \right\} \geq \frac{1}{t} \sum_n \sum_{i=1}^{t} \mathbb{E}\left\{ \tilde{\tilde{Z}}_n\left(i, \hat{\mathbf{Q}}(t_0)\right) \hat{1}_n(i) \middle| \hat{\mathbf{Q}}(t_0) \right\} - NC_2(t). \tag{126}$$

Note that the value of $\hat{1}_n(i)$ only depends on $T'_{n,t_0}(1), T_n(2), \cdots, T_n(i-1)$, where $T'_{n,t_0}(1) = u_{n,2} - t_0$, therefore $\tilde{\tilde{Z}}_n(i, \hat{\mathbf{Q}}(t_0))$ and $\hat{1}_n(i)$ are independent. Moreover, we have $\hat{M}_n(t_0, t) = \tilde{\tilde{M}}_n(t_0, t)$ because $\hat{Policy}$ and $\tilde{\tilde{Policy}}$ have synchronized epochs. Then it follows from (126) that, for all $t_0 \geq 0$, whenever $t \geq \hat{D}$,

$$\begin{aligned} &\sum_n \mathbb{E}\left\{ \frac{1}{t} \sum_{i=1}^{\hat{M}_n(u_{n,i},t)} \hat{Z}_n\left(i, \hat{\mathbf{Q}}(u_{n,i})\right) \middle| \hat{\mathbf{Q}}(t_0) \right\} \\ \geq &\frac{1}{t} \sum_n \tilde{\tilde{z}}_n\left(\hat{\mathbf{Q}}(t_0)\right) \sum_{i=1}^{t} \mathbb{E}\left\{ \hat{1}_n(i) \right\} - NC_2(t) \\ = &\frac{1}{t} \sum_n \tilde{\tilde{z}}_n\left(\hat{\mathbf{Q}}(t_0)\right) \mathbb{E}\left\{ \tilde{\tilde{M}}_n(t_0, t) \right\} - NC_2(t). \end{aligned} \tag{127}$$

Finally, going back to (116) and plugging (127) in, we can get (86) and complete the proof.

Appendix M
Proof of Lemma 6

Define $\hat{\hat{X}}_{nk}^{(c)}(i)$ under $\hat{\hat{Policy}}$ as the random variable that takes value 1 if node $k \in \mathcal{K}_n$ is in the successful receiver set of epoch i for node n when a unit of commodity c is transmitted by node n in epoch i with FMIA, and takes value 0 otherwise. Since each node n under $\hat{\hat{Policy}}$ makes decisions based on the backlog state observation $\hat{\hat{\mathbf{Q}}}(u_{n,i})$ for each epoch i, the value of $\hat{\hat{Z}}_n(i, \hat{\hat{\mathbf{Q}}}(u_{n,i}))$ is independent of $\hat{\hat{\mathbf{Q}}}(t_0)$ and $\hat{\hat{1}}(i)$ given $\hat{\hat{\mathbf{Q}}}(u_{n,i})$. Then, with the similar manipulations as in the proof of Lemma 3 (see Appendix H), we get a similar result for $\hat{\hat{Policy}}$ shown as follows:

$$\begin{aligned} &\mathbb{E}\left\{ \hat{\hat{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}), \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} \\ = &\mathbb{E}\left\{ \max_{k:k\in\mathcal{K}_n} \left\{ \hat{\hat{X}}_{nk}^{(\hat{\hat{c}}_n(i))}(i)\, \hat{\hat{W}}_{nk}^{(\hat{\hat{c}}_n(i))}(u_{n,i}) \right\} \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}), \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1, \hat{\hat{\mu}}_n^{(\hat{\hat{c}}_n(i))}(i) = 1 \right\} \\ &\times \mathbb{E}\left\{ \hat{\hat{\mu}}_n(i) \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}) \right\} \end{aligned} \tag{128}$$

In (128), adding $\hat{\hat{\mathbf{Q}}}(t_0)$ and $\hat{\hat{1}}(i) = 1$ as part of the given condition is for the convenience of later derivations.

To facilitate the later proof, we introduce another intermediate policy, which is denoted as $\hat{Policy}'(i)$, $i \geq 1$, and is defined as follows: *it is the same as $\hat{\hat{Policy}}$ from timeslot 0 to timeslot $\hat{\hat{u}}_{n,i} - 1$ at each node n; starting from timeslot $\hat{\hat{u}}_{n,i}$, node n makes the transmitting and forwarding decisions based on $\hat{\hat{\mathbf{Q}}}_n(u_{n,i})$ using the same strategies as under $\hat{Policy}$, and the transmissions use the RMIA transmission scheme*. Starting from timeslot $\hat{\hat{u}}_{n,i}$, $\hat{Policy}'(i)$ can be treated the same as $\hat{Policy}$ but with initial CPQ backlog state $\hat{\hat{\mathbf{Q}}}_n(u_{n,i})$, and we have

$$\hat{c}'_n(i) = \hat{\hat{c}}_n(i);\ \hat{\mu}_n^{(\hat{c}'_n(i))}(i) = \hat{\mu}'_n(i) = \hat{\hat{\mu}}_n(i) = \hat{\hat{\mu}}_n^{(\hat{\hat{c}}_n(i))}(i). \tag{129}$$

Additionally, in epoch i for each node n, since FMIA is used in the transmissions under $\hat{\hat{Policy}}$, where the retained partial information is used in the decoding process, while RMIA is used in the transmissions under $\hat{Policy}'(i)$, we have, for any commodity c,

$$\hat{\hat{X}}_{nk}^{(c)}(i) \geq \hat{X}'^{(c)}_{nk}(i). \tag{130}$$

With (129) and (130), it follows from (128) that

$$\begin{aligned} &\mathbb{E}\left\{ \hat{\hat{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}), \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} \\ \geq &\mathbb{E}\left\{ \max_{k:k\in\mathcal{K}_n} \left\{ \hat{X}'^{(\hat{c}'_n(i))}_{nk}(i)\, \hat{W}_{nk}^{(\hat{c}'(i))}(u_{n,i}) \right\} \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}), \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1, \hat{\mu}'_n(i) = 1 \right\} \\ &\times \mathbb{E}\left\{ \hat{\mu}'_n(i) \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}) \right\} \\ = &\mathbb{E}\left\{ \hat{Z}'_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(u_{n,i}), \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\}. \end{aligned} \tag{131}$$

Taking expectations over $\hat{\hat{\mathbf{Q}}}(u_{n,i})$ on both sides of (131) yields:

$$\mathbb{E}\left\{ \hat{\hat{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} \geq \mathbb{E}\left\{ \hat{Z}'_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} \tag{132}$$

According to the definition of $\hat{Policy}'(i)$, we follow the similar manipulations as in the proof of Lemma 5 (see Appendix L-A) and get, for all $t_0 \geq 0$, whenever $t \geq T_{\max}$,

$$\mathbb{E}\left\{ \hat{Z}'_n\left(i, \hat{\hat{\mathbf{Q}}}(u_{n,i})\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} \geq \mathbb{E}\left\{ \tilde{\tilde{Z}}_n\left(i, \hat{\hat{\mathbf{Q}}}(t_0)\right) \middle| \hat{\hat{\mathbf{Q}}}(t_0), \hat{\hat{1}}_n(i) = 1 \right\} - C_2(t). \tag{133}$$

Plugging (133) into (132) yields (98), which completes the proof.

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REFERENCES

[1] J. Kleinberg and E. Tardos, *Algorithm Design*. Addison Wesley, 2005.

[2] A. V. Goldberg and S. Rao, "Beyond the flow decomposition barrier," *Journal of the ACM (JACM)*, vol. 45, no. 5, pp. 783–797, 1998.

[3] T. Leighton, F. Makedon, S. Plotkin, C. Stein, É. Tardos, and S. Tragoudas, "Fast approximation algorithms for multicommodity flow problems," *Journal of Computer and System Sciences*, vol. 50, no. 2, pp. 228–243, 1995.

[4] T. Leighton and S. Rao, "Multicommodity max-flow min-cut theorems and their use in designing approximation algorithms," *Journal of the ACM (JACM)*, vol. 46, no. 6, pp. 787–832, 1999.

[5] S. Biswas and R. Morris, "ExOR: Opportunistic multi-hop routing for wireless networks," in *ACM SIGCOMM Computer Communication Review*, vol. 35, no. 4, 2005, pp. 133–144.

[6] D. S. J. deCouto, D. Aguayo, J. Bicket, and R. Morris, "A high-throughput path metric for multi-hop wireless routing," *Wireless Networks*, vol. 11, no. 4, pp. 419–434, 2005.

[7] E. Rozner, J. Seshadri, Y. Mehta, and L. Qiu, "SOAR: Simple opportunistic adaptive routing protocol for wireless mesh networks," *Mobile Computing, IEEE Transactions on*, vol. 8, no. 12, pp. 1622–1635, 2009.

[8] L. Georgiadis, M. J. Neely, and L. Tassiulas, *Resource allocation and cross-layer control in wireless networks*. Now Publishers Inc., 2006.

[9] M. J. Neely, "Stochastic network optimization with application to communication and queueing systems," *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, pp. 1–211, 2010.

[10] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *Automatic Control, IEEE Transactions on*, vol. 37, no. 12, pp. 1936–1948, 1992.

[11] ——, "Dynamic server allocation to parallel queues with randomly varying connectivity," *Information Theory, IEEE Transactions on*, vol. 39, no. 2, pp. 466–478, 1993.

[12] M. J. Neely, E. Modiano, and C. E. Rohrs, "Power and server allocation in a multi-beam satellite with time varying channels," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 2002, pp. 1451–1460 vol.3.

[13] ——, "Dynamic power allocation and routing for time-varying wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 1, pp. 89–103, 2005.

[14] E. M. Yeh and R. A. Berry, "Throughput optimal control of cooperative relay networks," *Information Theory, IEEE Transactions on*, vol. 53, no. 10, pp. 3827–3833, 2007.

[15] M. J. Neely and R. Urgaonkar, "Optimal backpressure routing for wireless networks with multi-receiver diversity," *Ad Hoc Networks*, vol. 7, no. 5, pp. 862–881, 2009.

[16] M. Luby, "LT codes," in *Foundations of Computer Science, 2002. Proceedings. The 43rd Annual IEEE Symposium on*, 2002, pp. 271 – 280.

[17] J. W. Byers, M. Luby, and M. Mitzenmacher, "A digital fountain approach to asynchronous reliable multicast," *Selected Areas in Communications, IEEE Journal on*, vol. 20, no. 8, pp. 1528–1540, 2002.

[18] A. Shokrollahi, "Raptor codes," *Information Theory, IEEE Transactions on*, vol. 52, no. 6, pp. 2551–2567, 2006.

[19] S. C. Draper, L. Liu, A. F. Molisch, and J. S. Yedidia, "Cooperative routing for wireless networks using mutual-information accumulation," *Information Theory, IEEE Transactions on*, vol. 57, no. 8, pp. 5757–5762, 2011.

[20] R. Urgaonkar and M. J. Neely, "Optimal routing with mutual information accumulation in wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 9, pp. 1730 –1737, october 2012.

[21] J. Yang, Y. Liu, and S. C. Draper, "Optimal scheduling policies with mutual information accumulation in wireless networks," in *INFOCOM, 2012 Proceedings IEEE*, March 2012, pp. 1062–1070.

[22] A. F. Molisch, *Wireless communications, 2nd edition*. IEEE-Press - Wiley, 2011.

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