JS-RAKE: Judiciously trained Selective RAKE receiver for UWB systems

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Abstract—Hardware complexity reduction is critically important for Rake receivers in ultra-wide band systems, where the number of channel multi-path components (MPCs) can be very large. Though several designs (S-Rake, P-Rake) have been proposed, the impact of channel estimation overhead has not been accounted for in prior literature. In this paper, we propose a new Judiciously trained Selective Rake receiver that performs channel estimation for only a subset of the many channel multi-path components. The intuition behind the design is that only a few channel MPCs have enough strength to make them sufficiently likely to contribute significantly to the received signal power. By restricting the channel estimation to only these MPCs, the estimation overhead can be reduced significantly without degrading capacity. In this paper, we first show that if the MPCs have independent Rayleigh amplitude fading, it is data rate optimal to perform channel estimation for only a subset of the MPCs with the highest second-order moments. We also present a closed form expression for the achievable data-rate of a JS-Rake receiver in Rayleigh fading and find the data-rate maximizing size of this subset. Simulations in several practically relevant channel settings show that JS-Rake boosts data rate by 30 – 40% and 50%, in comparison to S-Rake and P-Rake, respectively.

I. INTRODUCTION

Ultra-Wide Band (UWB) transmission has been extensively researched due to its unique capabilities like resistance to fading, localization abilities, immunity to narrow-band interference etc. In addition, recent results suggest that UWB transmission schemes like impulse radio are good candidates for transmission over mm-wave frequency bands [1].

To exploit the good delay resolvability, UWB systems are equipped with Rake receivers (RXs) which can accumulate power from the many channel diversity paths, i.e., resolvable multi-path components (MPCs) [2]. However since a dedicated rake finger and a corresponding correlator is required for each resolvable MPC, the cost and complexity of such a Rake RX can be prohibitively large especially at large bandwidths. As an example, a typical UWB channel may have around $N = 50$ resolvable MPCs. Therefore, one of the major challenges of UWB schemes for 5G systems is enabling low complexity transceivers, which are ideal for internet of things and sensor networks applications.

Several low complexity Rake RXs have therefore been proposed in the literature. The most popular among them being: 1) the partial Rake (P-Rake) RX which has a fixed number $K < N$ of Rake fingers and accumulates power from the first arriving $K$ MPCs [3] and 2) the selective Rake (S-Rake) RX which also has $K$ Rake fingers but utilizes/combines the $K$ strongest MPCs [2], [3]. In the absence of interference, the capacity optimal combining technique for an S-Rake RX is Generalized Selection Combining (GSC) [5], also known as Hybrid-Selection Maximal Ratio Combining [2]. In GSC, the signal from the $K$ MPCs with largest amplitude are combined using maximum ratio combining.

Unlike a P-Rake RX, an S-Rake RX requires the channel state information (CSI) for all the MPCs. This CSI can be acquired using pilot training wherein the TX transmits a known signal to aid channel estimation at the RX. Since the RX has only $K$ correlators, at any time CSI can only be acquired for $K$ MPCs. Therefore, for full CSI acquisition, the pilot symbol has to be re-transmitted $\lceil N/K \rceil$ times. This pilot repetition consumes useful spectrum resources and the corresponding training overhead can be significant if either the number of MPCs $N$ is large, number of correlators $K$ is small or the pilot symbol duration is long. Therefore, though an S-Rake offers a higher signal-to-noise ratio (SNR) and diversity order, it may not necessarily lead to a larger data-rate than a P-Rake RX. The performance analysis of S-Rake with GSC has been studied in great detail under varied channel assumptions (see [6]–[8] and references therein), but the cost of the channel estimation has not been accounted for in prior literature.

In this paper, we propose a new RX called Judiciously trained Selective Rake (JS-Rake) that optimizes the trade-off between diversity and the training overhead to maximize the achievable data rate. JS-Rake is structurally similar to S-Rake but is functionally very different. In JS-Rake, the CSI is acquired only for a subset of the $N$ MPCs, and GSC is performed only on this subset of MPCs. The choice of this subset of MPCs is decided based on only the channel average statistics like the Power Delay Profile (PDP). The PDP changes very slowly and can therefore be estimated at the RX

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with negligible overhead using standard covariance estimation algorithms. By varying the size of this subset \( L \), we can trade-off between the channel diversity and the training overhead.

The contributions of this paper are as follows:

- A new Rake RX is proposed that can outperform both the S-Rake and P-Rake.
- It is proved that for a JS-Rake in Rayleigh fading channels, for a given subset size \( L \), the corresponding data-rate maximizing CSI acquisition set is the one that considers the \( L \) MPCs with the highest power in the PDP.
- A closed form expression for the achievable rate for a JS-Rake with given subset size \( L \) is presented and the problem of finding the optimal size \( L_{\text{opt}} \) is formulated.
- The performance of JS-Rake is compared to S-Rake and P-Rake in some practically important simulation settings.

**Notation:** Scalars are represented by light-case letters; vectors by bold-case letters; matrices by capitalized bold-case letters; and sets by calligraphic letters. Additionally, \( a_i \) represents the \( i \)-th element of a vector \( a \) and \( |A| \) the cardinality of a set \( A \). Also, \( \mathbb{E}\{\cdot\} \) represents the expectation operator, \( \mathbb{I}_A \) the \( A \times A \) identity matrix, \( \mathbb{O}_{A \times B} \) the \( A \times B \) all-zero matrix, \( \lceil a \rceil \) the smallest integer larger than or equal to \( a \) and \( f_x \) the probability density and cumulative distribution for a random variable \( x \), respectively.

**II. GENERAL ASSUMPTIONS AND CHANNEL MODEL**

We consider a UWB system with a single antenna TX and a single antenna, JS-Rake RX having \( K \) correlators that can be placed at \( N \) possible delay tap positions. Without loss of generality, we assume each delay tap is associated with a, possibly zero magnitude, resolvable MPC. Assuming perfect inter-symbol interference equalization, the baseband equivalent received signal at the RX correlator outputs can then be expressed as:

\[
y = \sqrt{\rho} \mathbf{S} \mathbf{h} x + \mathbf{S} n
\]  

(1)

where \( y \) is the \( K \times 1 \) received signal vector corresponding to the \( K \) correlator outputs, \( \rho \) is the average signal-to-noise ratio (SNR) at the correlator outputs, \( \mathbf{S} \) is a \( K \times N \) sub-matrix of \( \mathbb{I}_N \) that picks the best \( K \) taps for down-conversion, \( \mathbf{h} \) is the \( N \times 1 \) normalized channel vector corresponding to the \( N \) delay taps, \( x \) is the transmit data symbol and \( \mathbf{n} \sim \mathcal{CN}(\mathbb{O}_{N \times N}, \mathbb{I}_N) \) is the \( N \times 1 \) normalized additive white Gaussian noise vector. Without loss of generality, we assume \( N \) is a multiple of \( K \).\(^2\)

The channel resolvable MPCs are assumed to be independent and distributed with Rayleigh amplitude fading\(^3\) with probability density given by:

\[
f_{|h_i|}(x) = \frac{2x}{\Omega_i} \exp\left\{-\frac{x^2}{\Omega_i}\right\}
\]

(2)

where \( \Omega \) is the vector of second order moments for the different resolvable MPCs and it corresponds to samples of the PDP

\[\text{ taken at the different delay taps. Since } \Omega \text{ is an average channel statistic, it changes very slowly and therefore can be tracked very easily at the RX with minimal overhead. Without loss of generality, the channel is normalized such that } \sum_{i=1}^{N} \Omega_i = 1. \]

We also assume the channel to undergo block fading, wherein, the channel \( h \) remains constant for a coherence time interval and then changes to another random realization (drawn from the distribution in (2)). During each coherence time interval, the TX transmits the pilot symbol \([L/K]\) times so that the RX can acquire CSI for \( L \) MPCs. The set of these \( L \) MPCs is represented by \( \mathcal{L} \), where \( \mathcal{L} \subset \{1, \ldots, N\} \). Assuming perfect channel estimation, the instantaneous signal-to-noise ratio (SNR) using the capacity optimal GSC at the RX can be expressed as:

\[
\gamma_{\text{GSC}}(\mathcal{L}) = \max_{S \subseteq \mathcal{L}, |S| = K} \left\{ \rho \sum_{i \in S} |h_i|^2 \right\}
\]

(3)

Taking the pilot training overhead into account the achievable data-rate can be expressed as:

\[
R(\mathcal{L}) = (1 - \lceil L/K \rceil \theta_p) C(\mathcal{L})
\]

(4)

where, \( C(\mathcal{L}) = \mathbb{E}\{\log(1 + \gamma_{\text{GSC}}(\mathcal{L}))\} \) is the ergodic capacity and \( \theta_p \) is the fraction of time-frequency resources consumed by the pilot sequence. Clearly, from (4) there is a trade-off between the ergodic capacity and the amount of feedback overhead. In this work we aim to find the diversity order \( L_{\text{opt}} \) and also find the corresponding set \( L_{\text{opt}} \) that maximize the achievable data-rate. It is worth mentioning that \( L = K \) for P-Rake and \( L = N \) for S-Rake and therefore these are two extreme, albeit sub-optimal, cases of a JS-Rake.

**III. OPTIMAL CSI ACQUISITION SET**

For any \( L \in \{K, \ldots, N\} \), let the corresponding data-rate maximizing CSI acquisition set be given by:

\[
\mathcal{L}^*(L) = \arg\max_{\mathcal{L} \subseteq \{1, \ldots, N\}} \{ R(\mathcal{L}) \} \quad \text{if } |\mathcal{L}| = L
\]

(5)

Then clearly, \( L_{\text{opt}} = \mathcal{L}^*(L_{\text{opt}}) \), where,

\[
L_{\text{opt}} = \arg\max_{K \leq L \leq N} \{ R(\mathcal{L}^*(L)) \}
\]

(6)

**Theorem III.1.** An optimal solution \( \mathcal{L}^*(L) \) to (5) is given by:

\[
\mathcal{L}^*(L) = \{ \eta_1, \eta_2, \ldots, \eta_L \}
\]

(7)

where \( \eta \) is a permutation of the vector \([1, \ldots, N]\) such that \( \Omega_{\eta_i} \geq \Omega_{\eta_j} \) for all \( i \neq j \).

**Proof.** Suppose \( \{\eta_1, \eta_2, \ldots, \eta_L\} \) is not an optimal solution to (5). Consider any optimal solution \( \mathcal{L}^*(L) \neq \{\eta_1, \eta_2, \ldots, \eta_L\} \). Then there exist distinct numbers \( a_1, \ldots, a_p, b_1, \ldots, b_p \) (\( 1 \leq a_1, \ldots, a_p \leq L < b_1, \ldots, b_p \leq N \)) such that:

\[
(\mathcal{L}^*(L) \cup \{\eta_{a_1}, \ldots, \eta_{a_p}\}) \setminus \{\eta_{b_1}, \ldots, \eta_{b_p}\} = \{\eta_1, \ldots, \eta_L\}
\]

Note that, from the definition of \( \eta \), we have \( \Omega_{\eta_{a_j}} \geq \Omega_{\eta_{b_j}} \) for all \( 1 \leq j \leq p \). From (3), we then have:

\[
\gamma_{\text{GSC}}(\mathcal{L}^*(L)) \leq \max_{S \subseteq \mathcal{L}^*(L), |S| = K} \left\{ \rho \sum_{i \in S} |h_{\eta_i}|^2 \right\}
\]

(8)
where, we define constants:
\[
\alpha_i = \begin{cases} 
\frac{\Omega_{\eta_j}}{\eta_j} & \text{for } i = \eta_j, 1 \leq j \leq p \\
1 & \text{otherwise}
\end{cases} \quad (9)
\]

It can be easily verified from (2), that \( |h_{\eta_j}|^2 \overset{d}{=} \alpha_{\eta_j} |h_{\eta_j}|^2 \forall j \), where \( \overset{d}{=} \) denotes equality in distribution. Now since \( h_i \) are independently distributed for \( 1 \leq i \leq N \), from (8) we have:
\[
\gamma_{\text{GSC}}(\{\eta_1, ..., \eta_L\}) \overset{d}{=} \max_{S \subseteq \mathcal{L}^*(L), |S| = K} \left\{ \rho \sum_{i \in S} \alpha_i |h_i|^2 \right\}
\]
\[
\Rightarrow \gamma_{\text{GSC}}(\{\eta_1, ..., \eta_L\}) \overset{d}{\geq} \gamma_{\text{GSC}}(\mathcal{L}^*(L)) \quad (10)
\]

where \( \overset{d}{=} \) represents first order stochastic dominance of the left hand side over the right hand side. Using (4) and (10) we further have:
\[
R(\mathcal{L}^*(L)) \leq R(\{\eta_1, ..., \eta_L\}) \quad (11)
\]
which is in contradiction to our initial assumption. This concludes the proof.

This intuitively pleasing result states that it is data-rate optimal to acquire CSI for only the resolvable MPCs having the highest power in the PDP. From theorem III.1, since \( \mathcal{L}^*(L_1) \subset \mathcal{L}^*(L_2) \) for \( L_2 > L_1 \), it can be easily verified from (3)-(4) that: \( C(\mathcal{L}^*(L)) \) is a non-decreasing function of \( L \). Therefore, \( R(\mathcal{L}^*(L)) \leq R(\mathcal{L}^*(K \left[ \frac{L}{K} \right])) \) for all \( K \leq L \leq N \) and the optimization problem (6) can be reduced to:
\[
L_{\text{opt}} = \arg \max_{L \in \{K, 2K, ..., N\}} \{R(\mathcal{L}^*(L))\} \quad (12)
\]
where \( R(\mathcal{L}^*(L)) \) is as given in (4). Since the resolvable MPCs are independent and non-identically distributed with Rayleigh amplitude fading, the ergodic capacity in (4) can be expressed as (see [11]):
\[
C(\mathcal{L}^*(L)) = \sum_{i_1, i_2, ..., i_L = 1}^L \left( \prod_{\ell = 1}^L \frac{\Omega_{\eta_i}}{\min\{\ell, K\}} \right) \sum_{j=1}^L A_j e^{a_j} E_1(a_j) \quad (13)
\]
where, \( E_1(x) = \int_x^\infty t^{-1} e^{-t} dt \) is the exponential integral function and
\[
a_j = \sum_{k=1}^j \frac{\Omega_{\eta_k}}{\min\{j, K\}} \quad (14a)
\]
\[
A_j = \prod_{n=1, n \neq j}^L (a_n - a_j)^{-1} \quad (14b)
\]

A comparison of the theoretical ergodic capacity derived in (13) to Monte-Carlo simulations for a sample JS-Rake RX is presented in Fig 1 for both a light tail PDP (exponential) and a heavy tail PDP (Zipf). Since we now know a closed form expression for the objective function of (12), \( L_{\text{opt}} \) can be found using a simple search over the set \( \{K, 2K, ..., N\} \).

It is worth mentioning that though (13) is in closed form, its computational complexity is exponential in \( L \). This may lead to a significant computational overhead in finding \( L_{\text{opt}} \) when \( N \) is large, as exemplified in the next section.4

IV. SIMULATION RESULTS

For simulations, we consider a UWB wireless personal area network (W-PAN) system having a JS-Rake RX and a single antenna TX. The RX has an SNR of \( \rho = 10 \) and has \( K = 10 \) correlators that can be placed at any delay tap positions. The system center frequency is \( f_c = 6 \) GHz, with an operating bandwidth of \( W = 500 \) MHz and with a symbol duration \( T_{\text{symb}} = 8 \mu s \). For modelling the channel, we consider the industrial non line of sight scenarios (pp NLoS-A and pp NLoS-B) from [9] (which is also a part of the IEEE 802.15.4a channel model standard [12]). In both cases, the channel maximum delay is \( \tau_{\text{max}} = 300 \) ns and the coherence time is \( T_{\text{coh}} = 10 \) ms. Therefore, the number of resolvable MPCs is \( N = W \tau_{\text{max}} = 150 \). For such a large value of \( N \), using (13) is not computationally feasible and hence we rely only on Monte-Carlo simulations to compute \( R(\mathcal{L}^*(L)) \) in this section. Also, by sampling the PDP we have:
\[
\Omega_i^A = \sum_{j=1}^5 \kappa \exp \left\{ -\frac{j}{\Delta T} \frac{i/W - j/\Lambda}{\gamma_0 + aj/\Lambda} \right\} u \left( \frac{i}{W} - \frac{j}{\Lambda} \right), \quad (15)
\]
\[
\Omega_i^B = \kappa \frac{\gamma_1 + \gamma_{\text{rise}}(1 - \chi)}{\gamma_1 + \gamma_{\text{rise}}(1 - \chi)} \left( 1 - \chi e^{-\frac{i}{\tau_{\text{max}}}} \right) e^{-\frac{i}{\tau_{\text{coh}}}}, \quad (16)
\]
where the constant \( \kappa \) is chosen such that \( \sum_{i=1}^N \Omega_i = 1 \), \( u(t) = 1 \) for \( t \geq 0 \) & \( u(t) = 0 \) otherwise, and the different parameters are as listed in Table I. While accounting for the training overhead, we assume there are 25 TXs sharing the same spectrum via orthogonal time access. Under this assumption, the fractional pilot overhead can be computed as: \( \theta_p \approx \frac{25 T_{\text{coh}}}{T_{\text{symb}}} = 0.02 \). Using these parameters, the system achievable rate as a function of \( L \) for the two channel models

4A low computational-complexity approximation to the objective function is proposed in our follow-up work [10].
TABLE I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Model</th>
<th>$\gamma_0$ (ns)</th>
<th>$\gamma_1$ (ns)</th>
<th>$\gamma_{\text{rise}}$ (ns)</th>
<th>$K$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLoS-A</td>
<td>13.10</td>
<td>29.78</td>
<td>4.13</td>
<td>1.19</td>
<td>-</td>
</tr>
<tr>
<td>NLoS-B</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>66.80</td>
</tr>
</tbody>
</table>

is plotted in Fig. 2. Notice that $R(L^*(N))$ corresponds to the performance of an S-Rake RX and $R(L^*(K))$ gives an upper-bound to the performance of a P-Rake RX.\(^5\) In Fig 3, increases achievable rate in comparison to S-Rake and P-Rake by 30\% and 40 – 50\%, respectively.

Fig. 2. Achievable data rate for a JS-Rake RX as a function of $L$ for NLoS-A and NLoS-B scenarios (refer [9])

Fig. 3. Comparison of performance of JS-Rake (at $L_{\text{opt}}$), S-Rake and P-Rake RX as a function of the number of correlators ($K$) at the RX for the NLoS-A scenario

we study the influence of number of correlators at the RX on the performance of JS-Rake, S-Rake and upper bound to P-Rake. The results show that JS-Rake outperforms both S-Rake and P-Rake and $L_{\text{opt}} \approx K$ when $K \ll N$ and $L_{\text{opt}} \approx N$ when $K \approx N$. The results also show that for practical values of $K$, JS-Rake (with the optimal choice $L_{\text{opt}}$)

\(^5\)Since P-Rake always uses the first arriving $K$ MPCs (as opposed to $L^*(K)$), $R(L^*(K))$ yields an upper bound to its performance.

V. Conclusion

In this paper, we introduce a novel Rake RX that optimally trades off diversity to reduce the training overhead, thereby outperforming P-Rake and S-Rake receivers. For this RX, we show that it is data-rate optimal to acquire CSI only for a subset of the MPCs with highest second moments. We also find a closed form expression for the achievable data-rate and formulate the problem for finding the optimal subset size. The results suggest that the proposed JS-Rake (with the optimal choice $L_{\text{opt}}$) increases achievable rate in comparison to S-Rake and P-Rake by 30\% and 40 – 50\%, respectively. We also observe that for practical values of $N$, using (13) to find $L_{\text{opt}}$ is not feasible and therefore alternate approximations may be required.

REFERENCES


