Bit and Power Allocation in QAM capable Multi-Differential Frequency-Shifted Reference UWB Radio

Vishnu V. Ratnam*, Student Member, IEEE, Andreas F. Molisch*, Fellow, IEEE, Amr Alasaad†, Member, IEEE, Faisal Alawwad† and Hatim Behairy†

*Ming Hsieh Dept. of Electrical Engineering, University of Southern California
Los Angeles, California, USA. Email: {ratnam,molisch}@usc.edu
†National Center for Electronics and Photonics Technology, King Abdulaziz City for Science and Technology
Riyadh, Saudi Arabia. Email: {alasaad, falouwad, hbehairy}@kacst.edu.sa

Abstract—Auto-correlation receivers, such as frequency shift reference systems, offer a low-complexity alternative to Rake receivers, for low data-rate ultra-wideband applications. In such systems, a data signal and a reference signal are simultaneously transmitted. At the receiver, the received signal corresponding to the data is correlated with the received signal corresponding to the reference signal, thereby accumulating the multi-path channel energies in-phase. For improving bandwidth and energy efficiency, multi-differential schemes have also been proposed where multiple data signals share the same reference signal. With the aim of further boosting the achievable data rates, in this work we propose a multi-differential frequency shift reference receiver that supports higher order modulation formats. We design the corresponding receiver architecture that can exploit both the in-phase and quadrature-phase signal components and analytically characterize the signal and noise components. We also study the problem of optimal bit and power allocation to the multiple data signals, both for uncoded and coded systems, as a function of channel metrics that can be easily tracked at the receiver.

I. INTRODUCTION

Due to its inherent advantages of offering resilience to narrow-band interference, channel fading and its precision in localization, ultra-wide band (UWB) communication has received significant attention. Recent results have also shown that impulse-radio, a form of UWB communication, is a good candidate for transmission over millimeter wave frequency bands [1]. For coherent UWB systems, while the transmitter can be designed with fairly low complexity, the receiver complexity may be prohibitively high. Issues such as timing synchronization and requirement of a large number of Rake correlators to tap sufficient channel power, especially in dense multi-path environments, make the receivers both power-hungry and expensive [2]. While several low-complexity alternatives have been proposed in the literature [3], [4], the required channel estimation overhead for such systems may be large [5]. Therefore, as a compromise between performance and receiver complexity, non-coherent transmission schemes such as energy detection and auto-correlation reception have been proposed. While such schemes have inferior performance to coherent reception, they admit a simpler receiver architecture. Furthermore, such schemes do not require explicit channel estimation and can therefore be used in rapidly time varying channels [2], such as in vehicular networks.

Among the non-coherent approaches, auto-correlation schemes have the added advantage over energy detection that they can utilize the signal phase and can therefore achieve higher data rates. In such schemes, a data signal and a reference signal are simultaneously transmitted, separated either in time, frequency or code domain. The receiver performs a non-linear operation that effectively correlates the received waveform corresponding to the data signal with that of the reference signal, thereby, leading to an in-phase accumulation of the multi-path component (MPC) energies. The first such scheme called transmit-reference was explored in [6], where the data and reference signals are separated in the time domain. The corresponding receiver however requires a wide-band analog delay element, which may be hard to implement in an integrated circuit. Consequently, frequency-shift reference systems (FSR) [7] and code-shift reference systems [8] were proposed, where the data and reference signals are separated in frequency and code domains, respectively. In such systems, the delay element is replaced by mixers or analog correlators, which are easier to implement. Several modifications to reduce the noise accumulation have also been considered [9]. To improve the energy efficiency and throughput of auto-correlation receivers, multi-differential schemes, where multiple data signals share the same reference signal, have also been proposed [10], [11]. However, most prior works on multi-differential FSR (MD-FSR) systems only exploit the in-phase component of each data signal i.e., they can support only binary phase shift keying (BPSK) or amplitude shift keying (ASK). This limits the data-rate such systems can achieve. In this work, we explore a more generic MD-FSR receiver that can exploit both the in-phase and quadrature phase components of the signal and therefore can support higher order modulation formats, such as, quadrature amplitude modulation (QAM). The contributions of this work are as follows:

1) We propose a MD-FSR system where the data streams support QAM with a channel adaptive modulation order.
2) We design the corresponding receiver that can exploit both the in-phase and quadrature-phase signal compo-
ments and analyze its performance in terms of bit error rate and input-output mutual information.

3) We show that the system performance depends only on a few channel metrics which can be estimated very easily at the receiver.

4) Using these channel metrics, we also find the optimal bit and power allocation to each data stream, both for un-coded and coded systems.

Notation used in this work is as follows: scalars are represented by light-case letters; vectors by bold-case letters; matrices are represented by capitalized bold-case letters; and sets are represented by calligraphic letters. Additionally, \( \mathbb{E}\{\} \) represents the expectation operator, \( \mathbb{C}\) represents the set of complex numbers. Also, \( j = \sqrt{-1} \) and \( \text{Re}\{\cdot\} \), \( \text{Im}\{\cdot\} \), \( c^* \) refer to the real component, the imaginary component and the complex conjugate of a complex number \( c \), respectively.

II. GENERAL ASSUMPTIONS AND SYSTEM MODEL

We consider an impulse radio system implementing MD-FSR transmission [10] with \( K \) parallel data streams, sharing a common reference signal. Each symbol is composed of \( F \) frames, each of duration \( T_f \) i.e., symbol duration \( T_s = F T_f \). The transmit UWB pulse waveform \( p(t) \) is assumed to have a narrow support on \( t \in [0, T_p] \), where \( T_p \ll T_f \). Further, we assume the pulse is normalized such that \( \int_{-\infty}^{\infty} p^2(t)dt = 1/F \). Without loss of generality, throughout this paper, a baseband notation is assumed. Let \( x = [x_1, \ldots, x_K] \) be the scaled complex transmit symbol vector\(^1\) for the 0-th symbol, where \( E_d^{(k)} = \mathbb{E}\{|x_k|^2\} \) is the average energy per symbol allocated to the \( k \)-th data stream. Then, the baseband signal for the 0-th symbol is given by:

\[
s(t) = \text{Re}\left\{ \sqrt{E_s} + \sum_{k=1}^{K} \sqrt{2E_k e^{j2\pi f_k t}} \right\} u(t)
\]

where \( u(t) = \sum_{n=1}^{F} p(t - nT_f) \), \( E_s \) is the energy per symbol allocated to the reference signal and \( f_k \) represents the frequency offset of \( k \)-th data stream from the reference signal. The total average transmit symbol energy is then given by \( E_s = E_r + \sum_{k=1}^{K} E_d^{(k)} \). To minimize inter-stream interference [10], we assume these frequency offsets are chosen as: \( f_k = (2k - 1)/T_s \). Note that in (1), \( u(t) \) effectively samples \( x_k e^{j2\pi f_k t} \) at every \( T_f \) seconds. Therefore, to prevent aliasing, we further have \( K < (F + 2)/4 \). In practice, the UWB pulses in \( u(t) \) may actually be jittered for improved interference rejection and shaping of the emission spectrum. Such jittering may slightly degrade the performance of the FSR system, especially at the higher frequency data streams. An analysis of the impact of jittering is beyond the scope of this paper. For brevity, we assume the radio-frequency transmission happens via double side-band modulation [12]. However, it can be verified the presented results are also applicable to single-side band transmission. Using partial channel information fed back by the receiver, the transmitter chooses the appropriate modulation and/or coding scheme for each data stream, which shall be discussed later in section V.

The channel is assumed to have \( L \) MPCs with the base-band channel impulse response given as:

\[
h(t) = \sum_{\ell=1}^{L} \alpha_\ell \delta(t - \tau_\ell)
\]

where \( \alpha_\ell \) and \( \tau_\ell \) denote the complex amplitude and delay of the \( \ell \)-th channel MPC, with \( \tau_{\ell+1} > \tau_\ell \). For simplicity of analysis, we let \( \tau_a - \tau_b \geq T_p \) for all \( a \neq b \). Similarly, to prevent inter-frame interference, we assume that the maximum channel delay \( \tau_\ell \) is smaller than \( T_f \). Furthermore, the channel coherence time is assumed to be at least an order of magnitude larger than the symbol duration.

At the receiver, we assume to have an ideal filter that leaves the desired signal un-distorted but suppresses the out-of-band noise. Assuming perfect symbol timing synchronization, the filtered base-band received signal for the 0-th symbol can be expressed as:

\[
r(t) = \sum_{\ell=1}^{L} \text{Re}\left\{ \sum_{k=1}^{K} \sqrt{2E_k e^{j2\pi f_k(t - \tau_\ell)}} \right\} \alpha_\ell u(t - \tau_\ell) + n(t)
\]

where \( \text{Re}\{\cdot\} \), \( \text{Im}\{\cdot\} \) represent the base-band received signals generated by cosine and sine components of the carrier, respectively, \( n(t) \) is the zero-mean, two-sided, circularly symmetric, complex, additive Gaussian noise process with power spectral density \( S_n(f) = N_0 \) for \( |f| \leq W \), \( W \) being the system bandwidth. Without loss of generality, the carrier phase offset and the phase-shift due to the delay of the carrier frequency component is included into the complex MPC amplitude \( \alpha_\ell \), for each MPC \( \ell \). The base-band signal is then squared and frequency shifted using a bank of mixers as depicted in Fig. 1. Note that since the transmitter may use higher order modulation formats, both the in-phase (I) and quadrature-phase (Q) signal components: \( Y_{k,1}, Y_{k,Q} \) are required. For a given \( K \), this doubles the required number of mixers and integrators, in comparison to the receiver in [10], which exploits only the I-component. However, as shall be shown in section VI, the proposed receiver outperforms the design in [10] even if restricted to have the same number of mixers and integrators.

\(^1\)Here, the in-phase component of the signal is represented as real and the quadrature-phase component as imaginary.
III. ANALYSIS OF THE INTEGRATOR OUTPUTS

Throughout this paper we will encounter several cases where a low frequency phasor, say $e^{j2\pi f t}$, is “sampled” by the UWB pulse waveform. In such cases we shall use the following results:

$$e^{j2\pi f t}p(t-a) \approx e^{j2\pi fa}p(t-a)$$

$$\sum_{n=1}^{F} \frac{e^{j2\pi fnT_s}}{T_s} = \int_{0}^{T_s} \frac{e^{j2\pi fnT_s}dt}{T_s}$$

where $f$ is a multiple of $1/T_s$, $f \ll W$ and $f_{min} = f - \frac{1}{2T_s}$ for an integer $b$ such that $f_{min} \in \left(\frac{1}{2T_s}, \frac{1}{T_s}\right]$. Note that the squared output in (1) follows from (4) and the facts that $\alpha_{min} \in \left(\frac{1}{2T_s}, \frac{1}{T_s}\right]$. The expression for the squared signal-plus-noise energy as:

$$|r(t)|^2 = |r_s(t)|^2 + |n(t)|^2 + r_s^*(t)n(t)$$

where $r_s(t)$ denotes the noise-free component of the received signal $r(t)$.

A. Signal plus noise power estimation

Taking an average of the 0-th integrator output in Fig. 1, over several symbols, the receiver can estimate the received signal-plus-noise power energy as:

$$\mathbb{E}\{Y_0|h(t)\} = \mathbb{E}\left\{\int_{0}^{T_s} |r_s(t)|^2 + |n(t)|^2 \right\} dt$$

$$\approx \frac{2}{L} \sum_{\ell=1}^{L} |\alpha_\ell|^2 \left[E_r + \sum_{k=1}^{K} E_d^k\right] + \int_{0}^{T_s} R_n(0) dt$$

$$= \sigma_{rss}^2 + 2T_s W N_0$$

where $\mathbb{E}\{Y_0|h(t)\}$ follows from (4) and the facts that $n(t)$ has zero mean and is independent of $r_s(t)$; (2) follows by using (2)-(3) and the facts that $f_k$ is a multiple of $1/T_s$; $R_n(\cdot)$ is the auto-correlation function for the noise process and we define $\alpha_{rss}(h) \triangleq \sqrt{\sum_{\ell=1}^{L} |\alpha_\ell|^2}$ as a measure of the instantaneous channel gain. Since the channel coherence time is typically several orders of magnitude larger than the symbol duration, we assume that the receiver has an accurate knowledge of $\mathbb{E}\{Y_0|h(t)\}$. From (5), it is evident that our receiver accumulates twice the noise power in comparison to [10]. However, it also accumulates twice the channel power ($\alpha_{rss}^2$) as opposed to the $\sum_{\ell=1}^{L} \text{Re}\{\alpha_\ell\}^2$ in [10]. The remaining $2K$ integrator outputs in Fig. 1, which are used for demodulating the data streams, are analyzed in the following subsections.

B. Signal Component Analysis

For ease of representation, we define $Y_k \triangleq Y_{k,1} + jY_{k,2}$ as the k-th I/Q integrator output in Fig.1. From (4), the signal component of $Y_k$ can then be expressed as:

$$S_k = \int_{0}^{T_s} |r_s(t)|^2 e^{-j2\pi f_k \tau_k} dt$$

$$\approx \int_{0}^{T_s} \sum_{\ell=1}^{L} |\alpha_\ell|^2 \sqrt{2E_r x_k} e^{-j2\pi f_k \tau_k} du^2(t - \tau_k) dt$$

which $\approx$ follows by using (2)-(3) and observing that the other terms vanish since $f_k$ is an odd multiple of $1/T_s$ $\forall k$. As is evident from (6), even if $\text{Im}\{x_k\} = 0$ i.e., the transmitted signal has only I-phase component, the corresponding received signal has both I and Q components. This apparent rotation is introduced because the reference signal and the k-th data stream pass through slightly different channels, owing to the difference in the modulating frequency. Such a rotation of the constellation points can easily be tracked at the receiver with negligible overhead, as explained in section III-D.

C. Noise component analysis

From (4), the noise components at the k-th I/Q integrator output ($Y_k = Y_{k,1} + jY_{k,2}$) in Fig. 1 can be expressed as:

$$Z_k = \int_{0}^{T_s} e^{-j2\pi f_k \tau_k} \left[r_s(t)n^*(t) + r^*_s(t)n(t) + |n(t)|^2\right] dt$$

It is well known from literature [7], [10] that such noise components are approximately Gaussian distributed. Given the transmit data vector $x$ and channel impulse response $h(t)$, the conditional mean of the noise components can be computed as:

$$\mathbb{E}\{Z_k|x,h\} = \int_{0}^{T_s} R_n(0) e^{-j2\pi f_k \tau_k} dt = 0$$

Similarly the conditional variance and pseudo-covariance, respectively, of the I/Q noise can be expressed as (detailed steps are given in Appendix A):

$$\sigma_{Z_k|x,h}^2 = \mathbb{E}\{Z_k Z_k^*\}$$

$$= 2N_0^2 W T_s + 2N_0 \alpha_{rss}^2 \sum_{k=1}^{K} |x_k|^2 + E_r$$

$$\tilde{\sigma}_{Z_k|x,h}^2 \triangleq \mathbb{E}\{\hat{Z}_k Z_k^*\}$$

$$= N_0 \sum_{\ell=1}^{L} |\alpha_\ell|^2 e^{-j4\pi f_k \tau_k} \sum_{(k_1,k_2)\in A_k} \hat{x}_{k_1} \hat{x}_{k_2}$$

where $A_k \subseteq \{-K, \ldots, 0, \ldots, K\}$ such that $(k_1,k_2) \in A_k$ if $(k_1,k_2)$ satisfies one of:

$$\hat{f}_{k_1} + \hat{f}_{k_2} = 2f_k$$

$$\hat{f}_{k_1} + \hat{f}_{k_2} + 2f_k = \frac{1}{T_k}$$

with $\hat{f}_k \triangleq f_k$, $\hat{f}_{-k} \triangleq -f_k$, $\hat{x}_k \triangleq x_k$, $\hat{x}_{-k} \triangleq x^*_k$, for $k \in \{1, \ldots, K\}$, $f_0 \triangleq 0$ and $\hat{x}_0 \triangleq \sqrt{2E_r}$. Clearly, from (8)–(9), we see that the noise terms are not circularly symmetric and are further dependent on the data vector $x$. Therefore, for optimal decoding, a joint estimation of the data vector $x$ should be performed. To reduce the decoding complexity, we neglect the dependence of noise on the data vector $x$. Under this assumption, the noise variances, averaged over the data-vector $x$, reduce to:

$$\sigma_{Z_k|x}^2 = 2N_0^2 W T_s + 2N_0 \alpha_{rss}^2 E_s$$
where we use the fact that the data streams have a zero mean and are mutually independent (see Section II), and we approximate $\mathbb{E}\{x^2_k\} \approx 0$. Note that this approximation does not hold for some modulation formats e.g., BPSK. However, as shall be shown in section VI, the mismatch in the results due to approximating (8)-(9) by (11) is small, even in such cases. In fact, in the signal-to-noise ratio (SNR) regime of interest, typically the noise-noise term $N_0^2|Z_k|^2$ predominates $\sigma^2_{Z_k|x,h}$, and therefore any non-circularly and dependence on $x$, if present, is negligible. In conclusion, the noise for each data stream is approximately circularly symmetric and Gaussian distributed.

### D. Pilot Training

Since coherence time is several orders of magnitude larger than the symbol duration, we assume that for each channel realization several pilot symbols can be transmitted without negligible overhead. Using the integrator outputs $\{Y_1, \ldots, Y_K\}$ for these pilot symbols (refer (6)), the receiver can accurately track the phase rotation introduced by the channel as:

$$
\beta_k = \sum_{i=1}^L |\alpha_k|^2 e^{-j2\pi f_i \tau} \quad 1 \leq k \leq K
$$

Furthermore, using blanked pilots i.e., symbols where no signal is transmitted, the receiver can use the $0$-th integrator output to accurately estimate the noise power $N_0$. Henceforth, we shall assume the receiver has perfect knowledge of $\{\beta_1, \ldots, \beta_K\}$, $N_0$ and $\sigma_{\alpha_k}$ (refer (5)), for a given channel realization $h(t)$. We assume that using this knowledge, the receiver picks the appropriate modulation order and/or coding rate for transmission and feeds it back to the transmitter via an error free feedback channel. Note that since these pilots are used only to detect the constellation-rotation and not the actual channel impulse response, the advantage of an FSR system, of not requiring explicit channel estimation, is still applicable.

### IV. PERFORMANCE ANALYSIS

Using (6)-(11), the effective base-band channel between the transmit data and the integrator outputs, can be expressed as:

$$
Y_k = \sqrt{2E_s}E_d^{(k)} \beta_k \hat{x}_k + Z_k \quad 1 \leq k \leq K
$$

where, $\hat{x}_k \triangleq x_k/\sqrt{E_d^{(k)}}$ and $Z_k \sim \mathcal{CN}(0, \sigma^2_{Z_k|h})$. Note that since we utilize both the I-phase and Q-phase components, the magnitude of the channel fading term $|\beta_k|$ is larger than the fading term in [10] viz., $\Re\{\beta_k\}$. This leads to a reasonable boost in performance when either the maximum channel delay $\tau_{\text{max}} \approx T_i$ and $K \approx F/2$ or when the MPC amplitudes ($\alpha_k$’s) have a soft onset behaviour [13]. Assuming the $k$-th data stream uses grey coded $M_k$-QAM modulation, the corresponding uncoded bit error rate (BER), can be approximated as [12]:

$$
P_e^{(k)}(M_k) \approx \begin{cases} 
Q\left(\sqrt{\frac{\sqrt{4E_sE_d^{(k)}|\beta_k|^2}}{\sigma^2_{Z_k|h}}}\right) & \text{for } M_k = 2 \\
\frac{\sqrt{4E_sE_d^{(k)}|\beta_k|^2}}{M_k \log_2 M_k} Q\left(\sqrt{\frac{4E_sE_d^{(k)}|\beta_k|^2}{(M_k-1)^2 \sigma^2_{Z_k|h}}}\right) & \text{for } M_k > 2
\end{cases}
$$

For un-coded systems and systems with a fixed code rate, BER is a good metric for analyzing system performance.

Similarly for systems with adaptive code-rate, we use as a performance metric the mutual information between $Y_k$ and $\hat{x}_k$ for a given modulation order format. This is an upper-bound to the system throughput achievable using practical channel codes. Assuming the $k$-th data stream uses $M_k$-QAM modulation, the mutual information for the $k$-th data stream (in bits/symbol) can be expressed as [14]:

$$
I_k(h(t), M_k) = \sum_{\hat{x}_k \in \mathcal{X}_k} \mathbb{E}_{Y_k \in \mathcal{C}} \left[ \exp \left\{ -\frac{|Y_k - \sqrt{2E_sE_d^{(k)}|\beta_k|^2}}{\sigma^2_{Z_k|h}} \right\} \right] \pi M_k \sigma^2_{Z_k|h} \
\times \log_2 \left( \frac{M_k \exp \left\{ -\frac{|Y_k - \sqrt{2E_sE_d^{(k)}|\beta_k|^2}}{\sigma^2_{Z_k|h}} \right\}}{\sum_{\hat{x}_k \in \mathcal{X}_k} \exp \left\{ -\frac{|Y_k - \sqrt{2E_sE_d^{(k)}|\beta_k|^2}}{\sigma^2_{Z_k|h}} \right\} } \right) dY_k
$$

where $\mathcal{X}_k$ is the set of constellation points on the I-Q plane for $M_k$-QAM.

### V. OPTIMAL BIT AND POWER ALLOCATION

Let the allowed set of QAM modulations for each data stream be: $M = 1, 2, 4, \ldots, M_{\text{max}}$. In this section we shall discuss the optimal bit and power allocation\footnote{These allocations are optimal given the approximations in (11).} to each data stream, for both fixed and adaptive code-rate FSR systems. For these results, we shall utilize the following lemma:

**Lemma 1.** For any power allocation $\{E_r, E_d^{(1)}, \ldots, E_d^{(K)}\}$ such that $E_r + \sum_k E_d^{(k)} \leq E_s$, we have: $E_r E_d^{(k)} \leq \hat{E}_r \hat{E}_d^{(k)}$ for all $1 \leq k \leq K$, where:

$$
\hat{E}_d = \frac{E_d}{2} = \frac{E_d^{(k)}}{E_r - E_r}, \quad \hat{E}_r = \frac{E_r}{2}(E_s - E_r).
$$

**Proof.** Using AM-GM inequality, we have:

$$
\frac{E_r^2}{4} \geq E_r (E_s - E_r) \\
\Rightarrow \frac{E_d^2}{4} \geq E_d (E_s - E_r), \quad \text{for any } 1 \leq k \leq K
$$

This proves the lemma. □

Note that $\{\hat{E}_d, \hat{E}_d^{(1)}, \ldots, \hat{E}_d^{(1)}\}$ also satisfies the sum power constraint i.e., $\hat{E}_d + \sum_k \hat{E}_d^{(k)} \leq E_s$.

### A. Uncoded/ Fixed Code-rate Systems

For an un-coded system, the optimal\footnote{These allocations are optimal given the approximations in (11).} power allocation $\{E_{r,\text{opt}}, \ldots, E_{d,\text{opt}}^{(K)}\}$ and modulation order assignment $M_{1,\text{opt}}, \ldots, M_{K,\text{opt}}$ is the solution to:

$$
\arg\max_{E_r, E_d^{(1)}, \ldots, E_d^{(K)}, M_1, \ldots, M_K} \left\{ \sum_k \log_2 M_k \right\}
$$

subject to: $E_r + \sum_k E_d^{(k)} \leq E_s,$
\[ P_{e}(k) \leq P_{e,th}, \quad M_k \in \{1, 2, 4, \ldots, M_{\text{max}}\} \quad \forall 1 \leq k \leq K \]

where \( P_{e,th} \) is a threshold on the bit error rate. It can be noted from (14) that the BER for stream \( k \) is a decreasing function of \( E_r E_d^{(k)} \). Therefore, using lemma 1, there exists an optimal power allocation with \( E_{r,\text{opt}} = E_r/2 \). Using this value of \( E_{r,\text{opt}} \), (17) reduces to the classical problem of bit and power allocation to multi-carrier systems [15]. This can be solved using the Hughes-Hartogs algorithm or any of its numerous variants [15], [16]. Note that, by replacing \( P_{e}(k) \) with BER for a given code-rate, this analysis also extends to systems with fixed code-rates.

B. Adaptive Code-rate Systems

For systems with adaptive code-rate and with equi-probable symbols, a higher order QAM constellation always provides more mutual information than lower order QAMs [14]. Therefore, without loss of generality, we set \( M_1 = \ldots = M_K = M_{\text{max}} \) and find the optimal power allocation \( \{E_{r,\text{opt}}, \ldots, E_{d,\text{opt}}\} \) as a solution to:

\[
\text{argmax}_{E_r, E_d^{(1)}, \ldots, E_d^{(K)}} \left\{ \sum_k \mathcal{I}_k(h(t), M_{\text{max}}) \right\}
\]

subject to: \( E_r + \sum_k E_d^{(k)} \leq E_s \).

It can be noted from (15) that \( \mathcal{I}_k(h(t), M) \) is a strictly increasing function of \( E_r E_d^{(k)} \). Therefore, again using lemma 1, there exists an optimal power allocation with \( E_{r,\text{opt}} = E_r/2 \). Using this value of \( E_{r,\text{opt}} \), (18) reduces to the problem of power allocation in multi-carrier systems with specific input distributions. Therefore, \( \{E_{d,\text{opt}}^{(1)}, \ldots, E_{d,\text{opt}}^{(K)}\} \) can be obtained using the Mercury water-filling algorithm [17].

VI. SIMULATION RESULTS

For simulation results, we consider a MD-FSR impulse radio system operating at a center frequency of 6 GHz, having a pulse width of \( T_p = 2 \) ns and a frame duration of \( T_f = 160 \) ns. For modeling the channel power delay profile (PDP), we use the industrial non line-of-sight scenario model (DSM bs NLoS-b) from [13]. By using this PDP, we consider a base-band sample channel impulse response (that satisfies the assumptions in Section II) as [18]:

\[
h(t) = \sum_{l=0}^{(T_f/T_p)-1} \alpha_{\text{rss}} \sqrt{\text{PDP}(lT_p)} \eta_l e^{j\phi_l} \delta(t - lT_p)
\]

where, the PDP is normalized as: \( \sum_{l=0}^{(T_f/T_p)-1} \text{PDP}(lT_p) = 1 \) and for reproducible results, we set the small scale fading coefficients \( \eta_l = 1 \) and the phase angles \( \phi_l = \pi l / 5 \). We also define the average receiver signal-to-noise ratio as: \( \text{SNR} = \alpha_{\text{rss}}^2 E_s / N_0 \). For this channel model, the simulated bit error probabilities for a sample MD-FSR receiver are compared to the analytical results from (14), in Fig. 2. The results show an excellent match between the analytic BER expressions and the Monte-Carlo simulations. This suggests that the circularly symmetric Gaussian approximation for the effective noise (in section III-C) is reasonable, at-least for the SNR regime depicted here.

Next, the sum-rates of an un-coded and an adaptive code-
rate system, with optimal bit and power allocation, are studied as a function of \( M_{\text{max}} \) in Fig. 3. Additionally, we also plot the performance of the MD-FSR receiver design in [10], with BPSK transmission. This design only requires one integrator and correlator per data stream, as opposed to the two in our design (see Fig. 1). Hence, for a fair comparison, we allow double the data-streams for the receiver in [10]. The results show that the use of higher order modulation formats improves performance, even at low SNR values. Furthermore, even with the same receiver hardware complexity, our design with \( M_{\text{max}} \geq 4 \) outperforms the conventional BPSK MD-FSR receiver [10] over the whole SNR range studied.\(^3\) Intuitively, this is because the use of both I/Q components allows a slightly higher SNR (\( \|\beta_k\| \) versus \( \text{Re}\{\beta_k\} \) in (13)) and also allows the same data-rate to be achieved using fewer sub-carriers (with better channel gains).

VII. C O NCLUSION

In this paper a novel architecture for a MD-FSR system is proposed that can exploit both the in-phase and quadrature-phase components of the signal. The corresponding received signal and noise components are analyzed, and closed form expressions for the system performance measures are provided. The optimal power allocation to the reference signal is derived, and the bit and data stream power allocation problem is considered, both for fixed and adaptive code-rate systems. Results suggest that the proposed receiver outperforms conventional designs, even when restricted to have the same hardware complexity.

APPENDIX A

From (7), the conditional variance of the noise for the \( k \)-th data stream can be expressed as:

\[
\sigma^2_{Z_k|X_k} = \mathbb{E}\{Z_k\hat{Z}_k^*\} = \mathbb{E}_n\left[\int_0^{T_n} \left[ r_s(t) n_s^*(t) + r_s^*(t)n(t) \right] e^{-j2\pi f_k t} dt \right]^2 + \mathbb{E}_n\left[\int_0^{T_n} \left| n(t) \right|^2 e^{-j2\pi f_k t} dt \right]^2
\]

(19)

Note that the other terms in (19) vanish, since \( n(t) \) is independent of \( r_s(t) \) and the odd moments of \( n(t) \) are zero. Now the first term of (19) can be computed as:

\[
(i) = \int_0^{T_n} e^{-j2\pi f_k (t_1-t_2)} \left[ R_n^*(t_1-t_2) r_s(t_1) \right] \hat{r}_s(t_2) + R_n(t_1-t_2) r_s^*(t_1) \hat{r}_s^*(t_2) dt_1 dt_2
\]

(20)

The first component in (20) can be computed as:

\[
(i)_a = \int_0^{T_n} R_n^*(t_1-t_2) e^{-j2\pi f_k (t_1-t_2)} r_s(t_1) \hat{r}_s^*(t_2) dt_1 dt_2
\]

\[^{\text{Note that the receiver in [10] can potentially also transmit Q-ary ASK modulated signals. By picking \( M_{\text{max}} \geq Q^2 \), our design can outperform it.}}\]

\[\approx \int_0^{T_n} \sum_{t=1}^L \sum_{n=1}^F R_n^*(t_1-t_2) e^{-j2\pi f_k (t_1-t_2)} p(t_1-nT_1-\tau_t) \times p(t_2-nT_1-\tau_t) \text{Re}\left\{ \sum_{k_3=1}^K \sqrt{2} x_{k_3} e^{j2\pi f_{k_3} nT_1} + \sqrt{E_r} \right\} \times \text{Re}\left\{ \sum_{k_2=1}^K \sqrt{2} x_{k_2} e^{j2\pi f_{k_2} nT_1} + \sqrt{E_r} \right\} |\alpha|^2 dt_1 dt_2
\]

(1)

\[= \int_0^{T_n} \sum_{t=1}^L \sum_{n=1}^F \left[ \sum_{k_2=1}^K \frac{x_{k_2}}{\sqrt{N_0}} e^{j2\pi f_{k_2} nT_1} + \sqrt{E_r} \right] \left[ \sum_{k_1=1}^{K-1} \frac{x_{k_1}}{\sqrt{N_0}} e^{j2\pi f_{k_1} nT_1} + \sqrt{E_r} \right] |\alpha|^2 dt_1 dt_2
\]

(2)

\[= \int_0^{T_n} \sum_{t=1}^L \sum_{n=1}^F \left[ \sum_{k_2=1}^K \frac{x_{k_2}}{\sqrt{N_0}} e^{j2\pi f_{k_2} nT_1} + \sqrt{E_r} \right] \left[ \sum_{k_1=1}^{K-1} \frac{x_{k_1}}{\sqrt{N_0}} e^{j2\pi f_{k_1} nT_1} + \sqrt{E_r} \right] |\alpha|^2 dt_1 dt_2
\]

(3)

\[= \int_0^{T_n} \sum_{t=1}^L \sum_{n=1}^F \left[ \sum_{k_2=1}^K \frac{x_{k_2}}{\sqrt{N_0}} e^{j2\pi f_{k_2} nT_1} + \sqrt{E_r} \right] \left[ \sum_{k_1=1}^{K-1} \frac{x_{k_1}}{\sqrt{N_0}} e^{j2\pi f_{k_1} nT_1} + \sqrt{E_r} \right] |\alpha|^2 dt_1 dt_2
\]

(4)

(21)

where, \( \approx \) follows from (2), the fact that \( R_n(t) \approx 0 \) for \( t \gg 1/W \) and by change of the integration limits using that fact that \( p(t) = 0 \) for \( t \notin [0, T_p] \); \( \approx \) follows by change of variables \( t_1 = t_1 - nT_1 - \tau_t \), \( t_2 = t_2 - nT_1 - \tau_t \) and defining \( f_k \triangleq f_k, \hat{f}_k \triangleq -f_k, \hat{x}_k \triangleq x_k, \hat{x}_k \triangleq x_k^* \) for \( k \in \{1, ..., K\} \), \( \hat{f}_0 \triangleq 0 \) and \( \hat{x}_0 \triangleq 2\sqrt{E_r} \); \( \approx \) follows by using \( S_n(f) = \int_0^{T_n} R_n(t) e^{j2\pi ft} dt, P(f) = \int_0^{T_n} p(t) e^{j2\pi ft} dt, S_n(f), P(f) \) being the noise power spectral density and Fourier transform of the pulse, respectively and from (3); \( \approx \) follows from Parseval’s theorem. Using a similar sequence of steps, the other term in (20) can be computed as:

\[
(ii)_b = \int_0^{T_n} \sum_{k=1}^{K} \left| x_k \right|^2 + E_r
\]

(22)

Now the second component of (19) can be computed as:

\[
(ii)_a = \int_0^{T_n} \sum_{k=1}^{K} \left| x_k \right|^2 e^{-j2\pi f_k t} dt
\]

(1)

\[\approx \int_0^{T_n} \sum_{k=1}^{K} \left| x_k \right|^2 e^{-j2\pi f_k t} \mathbb{E}_n\left\{n(t_1) n(t_2) n^*(t_2) \right\} dt_1 dt_2
\]

(2)

\[\approx \int_0^{T_n} e^{-j2\pi f_k (t_1-t_2)} \left| R_n^*(0) + R_n(t_1-t_2)^2 \right| dt_1 dt_2
\]
where $^{(1)}$ follows from the fact that $P_a(t)\approx 0$ for $t \gg 1/W$ and $\approx$ follows by change of the integration limits using that fact that $P_a(t) \approx 0$ for $t \gg 1/W$ and $^{(3)}$ follows from the fact that $f_k \ll W$ and $S_n(f) = N_0$ for $|f| \leq W$. Now using (21)–(23), we arrive at (8). Similarly, the noise pseudo-covariance for the $k$-th data stream can be computed as:

$$\tilde{\sigma}_z^2 \mid \mathcal{H} = \mathbb{E}\{Z_kZ_k\}$$

$$= \mathbb{E}_n\left[\int_0^{T_s} \left[r_n(t)(n(t) + r_k^*(t)(t)) e^{-j2\pi ft} dt\right]^2\right]$$

$$+ \mathbb{E}_n\left[\int_0^{T_s} |n(t)|^2 e^{-j2\pi ft} dt\right]^2$$

Now the first term of (24) can be computed as:

$$\approx \int_0^{T_s} e^{-j2\pi ft_1(t_1 + t_2)} \left[R_n(t_1 - t_2) + R_n(t_1 - t_2) \right] dt_1 dt_2$$

$$\approx \int_0^{T_s} 2R_n(t_1 - t_2) e^{-j2\pi ft_1(t_1 + t_2)} r_k(t_1)(t_1) r_k(t_2) dt_1 dt_2$$

$$\approx \int_0^{T_s} 2R_n(t_1 - t_2) e^{-j2\pi ft_1(t_1 + t_2)} r_k(t_1)(t_1) r_k(t_2) dt_1 dt_2$$

$$\times \sum_{k=1}^{K} \sum_{n=1}^{N_0} \left|\alpha_k\right|^2 e^{-j2\pi ft_2}$$

$$\times \sum_{k=1}^{K} \sum_{n=1}^{N_0} \left|\alpha_k\right|^2 e^{-j2\pi ft_2}$$

$$\approx \int_0^{T_s} S_n(-f) P^*(f + f_k) P(f - f_k) df$$

$$\approx \sum_{l=1}^{L} \left[\sum_{(k_1,k_2) \in A_k} \tilde{x}_{k_1} \tilde{x}_{k_2}\right]$$

$$\approx N_0 \left[\sum_{l=1}^{L} \left[\sum_{(k_1,k_2) \in A_k} \tilde{x}_{k_1} \tilde{x}_{k_2}\right]$$

$$\approx N_0 \left[\sum_{l=1}^{L} \left|\alpha_k\right|^2 e^{-j2\pi ft_2}$$

$$\times \sum_{l=1}^{L} \left[\sum_{(k_1,k_2) \in A_k} \tilde{x}_{k_1} \tilde{x}_{k_2}\right]$$

where $^{(1)}$ follows from the fact that $R_n(t_1 - t_2) = R_n(t_1 - t_2)$; $^{(2)}$ follows from steps similar to (21); $^{(3)}$ also follows from steps similar to (21) with $A_k$ as defined in (10) and $^{(4)}$ follows from the fact that $f_k \ll W$ and hence, $P(f + f_k) \approx P(f)$.

Now the second component of (24) can be computed as:

$$\approx \int_0^{T_s} e^{-j2\pi ft_1(t_1 + t_2)} \left[R_n(t_1 + t_2) + R_n(t_1 - t_2)\right] dt_1 dt_2$$

$$\approx \int_0^{T_s} 0$$

$$\approx \int_0^{T_s} e^{-j2\pi ft_1(t_1 + t_2)\mid R_n(t_1 - t_2)\mid^2 dt_1 dt_2$$

where $^{(1)}$ follows from the fact of results on expectation of a product of Gaussian random variables [19]; $^{(2)}$ follows by change of the integration limits using that fact that $R_n(t) \approx 0$ for $t \gg 1/W$ and $^{(3)}$ follows by change of variable $t_1 = t_1 - t_2$. Now using (26)–(27), we arrive at (9).

REFERENCES


