Multi-antenna FSR Receivers: Low Complexity, Non-coherent, Massive Antenna Receivers

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Abstract—Many 5G applications require low complexity transceivers that can exploit the benefits of massive antenna arrays while still maintaining low hardware, energy and computation costs. As a solution, this paper proposes a novel multi-antenna frequency shift reference (MA-FSR) receiver that uses only one down-conversion chain, supports single spatial-stream wide-band transmission with non-coherent demodulation, and can perform receive beamforming without requiring phase-shifters, explicit channel estimation, or complicated signal processing. In MA-FSR a reference tone is transmitted along with the data. At each receive antenna, the received signal for the data is correlated with the received signal for the reference, via a squaring operation, thereby compensating for the inter-antenna phase shift. The resulting signals are then summed and fed to a single down-conversion chain. In this paper, performance of the MA-FSR receiver is studied analytically and a near-optimal power allocation policy for the reference and data signals is proposed. Simulations suggest that the signal-to-noise ratio for MA-FSR is around 9 dB lower than that of coherent analog beamforming, although the bandwidth efficiency is also reduced to 50%. Several extensions to MA-FSR that can achieve better performance, though with a higher hardware cost, are also discussed.

I. INTRODUCTION

Massive Multiple-input-multiple-output (MIMO) systems, where the transceivers are equipped with a large number of antenna elements, offer large spatial multiplexing or beamforming gains while also enabling simplified signal processing. Such massive MIMO systems are especially essential at the millimeter (mm) wave frequencies to compensate for the large path-loss. However full complexity massive MIMO systems are often unfeasible to implement due to the large hardware and energy cost associated with equipping each antenna with a dedicated up/down-conversion chain. To make the technology practically viable, low complexity massive MIMO transceivers have been proposed such as: hybrid beamforming [1] and use of low-resolution ADCs [2]. While these approaches help reduce the transceiver hardware cost and energy consumption to some extent, they may impose other challenges. For example, these architectures require coherent signal demodulation, which is hard to implement at the high phase noise levels encountered at mm-wave frequencies. Furthermore, the acquisition of the channel state information and the design of the beamformer may impose significant channel estimation overheads and computational burdens on the transceivers [3], [4]. Consequently, to alleviate some of these issues, non-coherent transmission for massive MIMO has also received significant interest [5]–[8]. However most prior non-coherent schemes either require an up/down-conversion chain at each antenna element or work well only in rich scattering channels or narrow-band channels, all of which may limit their applicability to low-cost, high data rate applications at mm-wave frequencies. With the advent of new 5G verticals that require low-cost, low-energy solutions, such as internet of things (IoT), the time is ripe for new transceiver architectures that allow high data-rate communication with low system and hardware costs, while also providing a good beamforming gain to improve link budget.

As a solution for such scenarios, in this paper we propose a novel multi-antenna frequency shift reference (MA-FSR) receiver. The MA-FSR receiver (RX) uses only one down-conversion chain, supports wide-band transmission with non-coherent demodulation, and can perform receive beamforming without requiring phase-shifters, explicit channel estimation, or complicated signal processing – thus alleviating the drawbacks of the above mentioned schemes. Inspired by the frequency shift reference (FSR) schemes for single-input-single-output (SISO) ultra-wideband (UWB) systems [9]–[11], in this scheme the transmitter (TX) transmits a reference signal and several data signals on different frequency sub-carriers via orthogonal frequency division multiplexing (OFDM). At each RX antenna, the received waveform corresponding to the data sub-carriers is then correlated with the received waveform corresponding to the reference signal via a simple squaring operation. The outputs are then summed up and fed to a single down-conversion chain for data demodulation. As shall be shown later, this operation emulates maximal ratio combining (MRC) at the RX with imperfect channel estimates. Since the RX beamforming is enabled without channel estimation, MA-FSR is especially suitable for fast time-varying channels, such as in V2V or V2X networks. Furthermore, due to the non-coherent RX architecture, the phase noise of the transmit signal has negligible influence on the performance. The RX also exploits power from all the channel multi-path components (MPCs) and is therefore resilient to blocking of MPCs. Unlike conventional UWB FSR systems, there is no bandwidth spreading of the data signal involved. Therefore, the noise enhancement due to the non-linear RX architecture is significantly smaller, making it practically viable. On the flip side, the proposed scheme only uses 50% of frequency sub-carriers for data transmission, can only support a single spatial data-stream, cannot suppress interference and can only be used for beamforming in the receive mode of a node. Therefore, MA-FSR is more suitable for scenarios with abundant spectrum and where beamforming at the TX is

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unnecessary or where beamforming at TX is achieved using conventional channel estimation methods. Examples include device-to-device networks where beamforming at RX provides sufficient link margin or infrastructure based networks where down-link traffic is dominant. The contributions of this paper are as follows:

1) We propose an MA-FSR RX architecture for massive MIMO systems, that allows non-coherent transmission, lowers implementation cost and energy consumption at the cost of 50% bandwidth efficiency and that does not require phase-shifters or channel estimation at the RX.

2) We characterize the achievable throughput for the proposed MA-FSR system, both analytically and via simulations, for the single-input-multiple-output (SIMO) scenario in a wide-band channel.

3) We also present a class of improved MA-FSR architectures that can further improve performance, albeit, with a higher hardware complexity.

The system model is discussed in Section II; the signal and noise components of the demodulated outputs are characterized in Section III; the system throughput and power allocation problem are analyzed in Section IV; simulation results are provided in Section V; a class of improved MA-FSR architectures are proposed in Section VI and the conclusions are summarized in Section VII.

**Notation:** scalars are represented by light-case letters; vectors by bold-case letters; and sets by calligraphic letters. Additionally, \( j = \sqrt{-1} \), \( \mathbb{E}\{\} \) represents the expectation operator, \( c^* \) is the complex conjugate of a complex scalar \( c \), \( c^\dagger \) is the Hermitian transpose of a complex vector \( c \), \( \delta(t) \) represents the Dirac delta function, \( \delta_{a,b} \) represents the Kronecker delta function and \( \mathbb{R}\{\cdot\}|\mathbb{I}\{\cdot\} \) refer to the real/imaginary component, respectively. Furthermore \( a^* \) and \( a^\dagger \) denote the complex conjugate and the conjugate transpose of a vector \( a \), respectively.

**II. General Assumptions and System Model**

We consider a SIMO link (which can be part of a larger system) where the TX has a single antenna and the RX has \( M \gg 1 \) antennas and one down-conversion chain. Note that this model also covers a MIMO link where the TX transmits a single spatial data stream, since the combination of TX precoding vector and propagation channel creates an effective SIMO link. The TX transmits OFDM symbols with \( 2K \) sub-carriers, indexed as \( \{0, \ldots, 2K−1\} \). A reference tone and data signals mix to the sub-carriers \( K \). Here \( g \) ensures that the transmit signal lies within the system bandwidth, and is usually small, determined by the TX phase noise. The remaining sub-carriers, i.e., \( \{1, \ldots, K−1\} \cup \{2K−g, \ldots, 2K−1\} \) are unused. While it uses only \( \approx 50\% \) of the sub-carriers for data transmission, this OFDM structure is necessary to prevent inter-stream interference, as shall be shown in section III.\(^1\) Then, the complex equivalent transmit signal for the 0-th symbol for \( −T_p ≤ t ≤ T_s \) can be expressed as:

\[
s(t) = \sqrt{\frac{2}{T_s}} \left[ \sqrt{E_r} + \sum_{k \in K} x_k e^{j2\pi f_k t} \right] e^{j(2\pi f_c t + \Theta(t))},
\]

where \( E_r \) is the energy allocated to the reference signal, \( x_k \) is the data signal for \( k \)-th OFDM sub-carrier, \( f_k \) is the carrier frequency, \( f_k = k/T_s \) represents the frequency offset of \( k \)-th sub-carrier from the reference signal, \( \Theta(t) \) represents the phase noise process at the TX and \( T_s, T_p \) are the symbol duration and the cyclic prefix duration, respectively. Here we define the complex equivalent signal such that the actual (real) transmit signal is given by \( \mathbb{R}\{s(t)\} \). We assume further that the data signals on the sub-carriers \( \{x_k|k \in K\} \) are mutually independently distributed with zero means. The total average transmit symbol energy is then given by \( E_r = E_t + \sum_{k \in K} E_{d}^{(k)} \), where \( E_{d}^{(k)} = \mathbb{E}\{|x_k|^2\} \) is the energy allocated to the \( k \)-th sub-carrier.\(^2\)

The channel is assumed to have \( L \ll M \) scatterers with the \( M \times 1 \) channel impulse response vector given as [12]:

\[
h(t) = \sum_{\ell=0}^{L-1} \alpha_{\ell} a_{\ell} \delta(t - \tau_{\ell}),
\]

where \( \alpha_{\ell} \) is the complex gain, \( \tau_{\ell} \) is the delay and \( a_{\ell} \) is the RX array response vector, respectively, of the \( \ell \)-th MPC. As an illustration, the array response vector for a \( \lambda/2 \)-spaced uniform linear array is given by: \( |a_{\ell}| = e^{j\pi(i-1)\sin(\phi_{\ell})} \), where \( \lambda \) is the wavelength of the carrier signal and \( \phi_{\ell} \) is the angle of arrival of the \( \ell \)-th MPC. Note that here we implicitly assume the system bandwidth is small enough to ignore beam squinting effects. For ease of analysis, we assume that the array response vectors for the scatterers are mutually orthogonal i.e. \( a_{\ell}^\dagger a_{\ell} = M\delta_{\ell,i} \). This assumption is reasonable if the scatterers are well separated and \( M \gg L \) [13]. Later, in section V we shall also study the system performance when the above assumption is relaxed. To prevent inter symbol interference, we let the cyclic prefix be longer than maximum channel delay: \( T_p > \tau_{L−1} \). To model a generic time varying channel, we assume that the MPC parameters remain constant for at least a coherence time interval \( T_{coh} \), and may/may not change afterwards.

The RX front-end is assumed to have a low noise amplifier followed by a band-pass filter (BPF) at each antenna element, as depicted in Fig. 1. The BPF has a cut-off frequency of \( f_{2K} \) and leaves the transmitted signal un-distorted but suppresses the out-of-band noise. We also assume the RX to have perfect timing and clock synchronization with the TX. The filtered

\(^1\)This assumption ensures that through the RX non-linearity, the products of reference tone and data signals mix to the sub-carriers \( \{K, \ldots, 2K−g\} \) in base-band, while the data-data inter-modulation products mix to the sub-carriers \( \{1, \ldots, K−1\} \), which can be filtered out. This sub-carrier allocation is different from the frequency off-sets used in a UWB FSR RX [9], and leads to a much lower amount of noise enhancement.

\(^2\)For ease of notation, we do not consider the additional energy per symbol required to transmit the cyclic prefix viz., \( E_r T_p/T_s \).
complex equivalent received waveform for the 0-th symbol for $0 \leq t \leq T_s$ can then be expressed as:

$$r(t) = \sum_{k=0}^{L-1} \alpha_k s_{BB}(t - \tau_k) e^{j2\pi f_c(t - \tau_k)} + n(t) e^{j2\pi f_c t}, \quad (3)$$

where $s_{BB}(t) = s(t) e^{-j2\pi f_c t}$ is the base-band transmit signal including the carrier phase noise, and $n(t)$ is the $M \times 1$ base-band equivalent, stationary, complex additive Gaussian noise process vector, with individual entries being circularly symmetric, independent and identically distributed (i.i.d.), and having a power spectral density $S_n(f) = 2N_0$ for $0 \leq f \leq f_{2K}$. The filtered signals at each antenna are then squared and summed up, as depicted in Fig 1. Note that such a squaring operation can be performed using square law devices or multipliers with identical inputs. For the purpose of this paper we shall assume that the squaring operation, and also the filtering, mixing and adding operations at the RX can be performed exactly. Since it is the actual, real received signal which gets squared, the output for the 0-th symbol, after the squaring and summing, can be expressed as:

$$r_{sq}(t) = \sum_{m=1}^{M} \Re\{|r_m(t)|^2\} \leq \frac{M}{4} \sum_{m=1}^{M} \left| r_m(t) \right|^2 \right|^2 + \frac{r_m(t)^2 + r_m^*(t)^2}{4}, \quad (4)$$

where $r_m(t) = |r(t)|_m$ is the complex equivalent received signal at the $m$-th antenna. Note that both $r_m(t)^2$, $r_m^*(t)^2$ are high pass signals with a carrier frequency of $2f_c$. This summed signal $r_{sq}(t)$ is then low-pass filtered (with a cut-off frequency of $2K/T_s$) to get:

$$r_{LPF}(t) = \frac{\|r(t)\|^2}{2} = \sum_{k=0}^{L-1} \frac{M^2 |\alpha_k|^2}{4} |s_{BB}(t - \tau_k)|^2 + \frac{\|n(t)\|^2}{2} + \sum_{k=0}^{L-1} \Re\{\alpha_k s_{BB}(t - \tau_k) n(t) e^{j2\pi f_c t} \}, \quad (5)$$

where we use the orthogonality assumption for the array response vectors. Finally, $r_{LPF}(t)$ is sampled by an ADC at a sampling rate of $4K/T_s$ samples/sec and conventional OFDM demodulation follows. Note that since $r_{LPF}(t)$ is a real signal with maximum frequency $2K/T_s$, the ADC sampling rate must be at least $4K/T_s$ samples/sec to prevent aliasing. However it can be shown that the signal of interest i.e., the product between the reference and data sub-carriers only lies within the frequency range $K/T_s \leq |f| \leq (2K - g - 1)/T_s$. Thus the same performance can also be obtained by replacing the low-pass filter by a band-pass filter with a pass-band of $K/T_s \leq |f| \leq 2K/T_s$, and using an ADC sampling rate of $2K/T_s$ samples/sec.

### III. Analysis of the Demodulation Outputs

Inspired by our similar analysis for the UWB FSR RX in [11], the current section analyzes the OFDM demodulation outputs. The OFDM demodulated output for the $k$-th sub-carrier of the 0-th symbol can be expressed as $(-2K \leq k \leq 2K - 1)$:

$$Y_k = \frac{T_s}{4K} \sum_{m=0}^{L-1} r_{LPF}(m T_s) e^{-j2\pi k_m f_c m T_s} \quad \frac{e^{-j2\pi k m f_c}}{4K_4} \quad (6)$$

We shall express each demodulation output as $Y_k = S_k + Z_k$ where $S_k$, referred to as the signal component, involves terms in (6) not containing the channel noise and $Z_k$, referred to as the noise component, containing the remaining terms. It can be verified from (5) and the expression for $s_{BB}(t)$ that only the demodulation outputs $\{Y_k|k \leq |k| \leq (2K - g)\}$ involve signal components. We shall therefore consider a sub-optimal, albeit simple, demodulation approach where only the outputs $\{Y_k|k \in K\}$ are utilized for demodulating the data, and the noise components are treated as noise.

#### A. Signal component analysis

From (5)–(6), the signal components of the OFDM demodulated outputs $Y_k$, for $k \in K$, can be expressed as:

$$S_k = \sum_{n \in K} \frac{M |\alpha_k|^2}{4K} \sqrt{E_t} + \sum_{k \in K} x_k e^{j2\pi f_k (\frac{2T_s}{4K} - \tau_k)} \left| e^{-j2\pi n_k f_c} \right|^2 \quad (1)$$

$$\left\{ M \sqrt{E_t} \sum_{\ell=0}^{L-1} |\alpha_k|^2 e^{-j2\pi f_k \tau_k} \right\} x_k, \quad (7)$$

where (1) follows from the sub-carrier allocation, which ensures that, despite the non-linear RX architecture, only the cross-product between the reference signal and the data on the $k$-th sub-carrier contribute to $S_k$, for $k \geq K$. Essentially, the MA-FSR RX utilizes the received vector corresponding to the reference tone as weights to combine the received signal corresponding to the data, thus emulating maximal ratio combining of the contributions from the different antennas with imperfect channel estimates. Since this combining takes place via squaring in the analog domain, the proposed RX enables beamforming without channel estimation or use of phase-shifters. However, as is evident from (7), the signals from the $L$ multi-path components do not add up in-phase, at the demodulation outputs. This is due to the fact that the reference signal and the $k$-th data stream pass through slightly different channels owing to the difference in modulating frequency. This leads to some amount of frequency selective fading, as shall be explained later in Section IV.

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3Since $r_{LPF}(t)$ is real, $Y_k = Y^*_{-k}$ i.e. $\{Y_k|k < 0\}$ do not contain any additional information.
B. Noise component analysis

From (6), the noise component of the OFDM demodulation output $Y_k$, for $k \in K$, can be expressed as:

$$Z_k = \sum_{u=0}^{4K-1} T_u e^{-j2\pi ku/4K} \left[ \frac{1}{2} \left| \mathbf{h}(uT_k/4K) \right|^2 \right] + \sum_{\ell=0}^{L-1} \text{Re} \left[ \alpha_{\ell}\mathcal{BB} \left( \frac{uT_k}{4K} - \tau_{\ell} \right) \mathbf{n}(uT_k/4K) \right] \left( \alpha_{\ell} e^{-j2\pi \ell \tau_{\ell}} \right).$$

Note that the noise consists of both: noise-noise cross product and data-noise cross product terms. Given the transmit data vector $\mathbf{x}$ and channel impulse response $h(t)$, the conditional mean of the noise components can be computed as:

$$\mathbb{E}\{Z_k|\mathbf{x}, h\} = 0,$$

where we use the fact that the noise process $\mathbf{n}(t)$ is stationary, has a zero mean and $\sum_{k=0}^{4K-1} e^{-j2\pi ku/4K} = 0$ for $k > 0$. Similarly, the conditional second order statistics of the noise components can be computed as (detailed steps are given in Appendix A):

$$K_{a,b}(\mathbf{x}, h) \triangleq \mathbb{E}\{Z_aZ_b\} \approx \text{MSE}_{a,b} \left[ (2K-b)N_0^2 + \text{MSE}_{0,0}\gamma_t \right] + \sum_{(k_1,k_2)\in \mathcal{A}(a,b)} \text{MSE}_{a,b}\gamma_t x_{k_1}^* x_{k_2}$$

(9a)

$$\tilde{K}_{a,b}(\mathbf{x}, h) \triangleq \mathbb{E}\{Z_aZ_b\} \approx \sum_{(k_2,k_1)\in \mathcal{B}(a,b)} \text{MSE}_{a,b}\gamma_t x_{k_1}^* x_{k_2}N_0,$$

(9b)

where $a, b \geq K$, $\beta_{a,b}(h) \triangleq \sum_{\ell} |\alpha_{\ell}|^2 e^{-j2\pi(f_s-f_k)\tau_{\ell}}$, $\mathcal{A}(a,b) \triangleq \{(k_1,k_2)|k_1, k_2 \in K, k_1-k_2 = a-b, k_2 \geq b\}$ and $\mathcal{B}(a,b) \triangleq \{(k_1,k_2)|k_1, k_2 \in K, k_1-k_2 = a+b-4K, k_2 \geq b\}$. Clearly, from (9a)-(9b) we see that the noise components at the OFDM outputs are mutually correlated and are further dependent on the data vector $\mathbf{x}$. For reducing computational complexity, we consider the sub-optimal approach where each sub-carrier is decoded independently. Under this assumption, the noise variances, averaged over the transmit data vector $\mathbf{x}$ reduce to:

$$K_{k,k}(h) = \text{MSE}_{k,k} \left[ (2K-k)N_0^2 + \text{MSE}_{0,0}\gamma_t + \sum_{k_2=k}^{2K-g-1} F_{d(k_2)} \right]$$

(10a)

$$\tilde{K}_{k,k}(h) = 0,$$

(10b)

where $k \in K$ and we use the fact that the data streams have a zero mean and are mutually independent (see Section II). We shall henceforth approximate the noise component at each OFDM output $\{Z_k|k \in K\}$ to be jointly Gaussian distributed. While this allows finding a lower bound to the system capacity, the accuracy of this assumption is also justified via simulations in Section V.

IV. PERFORMANCE ANALYSIS

Using (7) and (10), the effective SISO channel between the transmit data and the $k$-th demodulation output ($k \in K$) can be expressed as:

$$Y_k = M \sqrt{E_r E_d(k)} \beta_{k,0}(h) \bar{x}_k + Z_k,$$

(11)

where $\bar{x}_k \triangleq x_k/\sqrt{E_d(k)}$ and $Z_k \sim \mathcal{CN}(0, K_{k,k}(h))$. We assume that at regular intervals the TX transmits pilot symbols using which the RX can estimate the ‘fading coefficients’ $\{\beta_{k,0}|k \in K\}$. Similarly, using blanked pilots i.e., symbols where no signal is transmitted, the RX can also estimate the average channel parameters, they change slowly and can be tracked accurately and with low overhead. Henceforth, we shall assume the RX has perfect knowledge of these channel parameters for each channel realization $h(t)$. These values are further assumed to be fed back to the TX, via a feedback channel, for bit and power allocation. Note that since these pilots are used only to estimate the average channel parameters and not the actual SIMO channel, the advantages of simplified channel estimation are still applicable for MA-FSR RX. From the Gaussian assumption for $\{Z_k|k \in K\}$ and from (10), note that (11) represents a parallel Gaussian channel across the sub-carriers. The effective SNR for the $k$-th demodulation output ($k \in K$) is then given by:

$$\gamma_k(h(t)) = \frac{ME_r E_d(k) |\beta_{k,0}(h)|^2}{(2K-k)N_0^2 + \text{MSE}_{0,0}\gamma_t + \sum_{k_2=k}^{2K-g-1} F_{d(k_2)} \gamma_t},$$

(12)

Note that even though the RX does not explicitly perform channel estimation, we still observe a beamforming gain of $M$ in $\gamma_k(h(t))$. However since the different MPCs do not add up in phase at the RX and the noise power varies with $k$, the system suffers from frequency selective fading, which causes some loss in performance. Similarly, we define the instantaneous sum rate (ISR) as:

$$\text{ISR}(h(t)) \triangleq \sum_{k \in K} \frac{1}{T_s} \log \left( 1 + \gamma_k(h(t)) \right),$$

(13)

where we neglect the cyclic prefix overhead for convenience.

A. Power Allocation

Since both the signal and noise variances in (11) are affected by the transmit powers in a non-linear way, finding the ISR maximizing power allocation to the data and reference tone is difficult. We shall therefore rely on the following sub-optimal solution.

**Lemma 1.** For any feasible power allocation $\{E_r, E_d(k)|k \in K\}$, we have: $\gamma_k(h(t)) \leq \gamma_k(h(t))$ for all $k \in K$, where $\gamma_k(h(t))$ is the effective SNR with:

$$\tilde{E}_t = \frac{E_s}{2}, \quad \tilde{E}_d = \frac{E_d(k)}{E_s} \left( \frac{E_d - \tilde{E}_r}{E_d - E_t} \right).$$

(14)

While not considered here, differential modulation can also be used on the sub-carriers when such channel estimation is infeasible.
shown that this allocation is optimal, as maximizing power allocation for Eq. (13) with $E_d$ yields $\gamma_k(h(t)) \geq \gamma_k(h(t))$.

Case 2: If on the other hand $E_r < E_a/2$, then from (4), we can write for any $k \in \mathcal{K}$:

$$2\tilde{E}_r \gamma_k(h(t)) = \frac{M E_r E_d(k) |\beta_{k0}(h)|^2}{(2-k) N_0^2 + N_0 \beta_{k0} E_r + 2 (2-k) g k E_d(k)} \geq \frac{M E_r E_d(k) |\beta_{k0}(h)|^2}{(2-k) N_0^2 + N_0 \beta_{k0} E_r + 2 (2-k) g k E_d(k)} = \gamma_d(h(t)),$$

where $\gamma(k) \leq \gamma_k(h(t))$ follows from the fact that $\tilde{E}_r > E_r$ and $2\tilde{E}_r > E_d(k) > E_d(k)$ (see (14)). Therefore the theorem follows.

As a consequence of Lemma 1, using $E_r = E_a/2$ can at worst cause a 3 dB loss in the SNR of the data stream. Note that the SNR expression in (12) can be approximated as:

$$\tilde{\gamma}_k(h(t)) = \frac{M E_r E_d(k) |\beta_{k0}(h)|^2}{(2-k) N_0^2 + N_0 \beta_{k0} E_r + 2 (2-k) g k E_d(k)},$$

which is obtained by using $E_d(k)$ by $(E_r - E_r)/(K - g)$. Now using $\gamma_k(h(t))$ instead of $\tilde{\gamma}(h(t))$ in (13) with $E_r = E_a/2$ and $\sum_{k} E_d(k) = E_a/2$, a sub-optimal iSR containing power allocation for $\{E_d(k) | k \in \mathcal{K}\}$ can be obtained by the water-filling algorithm. In fact, it can be shown that this allocation is optimal, as $\beta_{k0}(h) E_r \rightarrow 0$.

V. SIMULATION RESULTS

For simulations we consider a SIMO system, where the RX has a half-wavelength spaced uniform linear array ($M = 64$) with one down-conversion chain and is equipped with a MA-FSR RX. The TX transmits OFDM symbols with $T_s = 2$ μs, $T_p = 0.2$ μs, $g = 5$ and $f_c = 30$ GHz. The phase noise at the TX is modeled as a Wiener process with $\mathbb{E} \{ |\theta(t) + T_s| - |\theta(t)| \}^2 = 2$. We consider a sample channel impulse response $h(t)$ with: $L = 3, r = 50/(L - 1)$ ns, $\phi = (L - 1) \pi/10, \alpha = (-1)^{L-1} \exp \{-|\phi|/\sigma\}, \sigma = \pi/10$ and $|a_k| = e^{i\pi(-1)\sin(\phi)}$. We also assume perfect timing synchronization, and perfect knowledge of $\{\beta_{k0} | k \in \mathcal{K}\}$.

In practice, the value of $E_r$ may further be limited by spectral mask regulations, a discussion of which is beyond the scope of this paper.

Such large arrays are expected in mm-wave systems to compensate for the path-loss.

We next compare the analytical iSR of MA-FSR (13) to the iSR of a coherent RX with analog beamforming that only occupies half the bandwidth, i.e., $|f| \leq K/T_s$. In Fig. 3. For analog beamforming, we assume the use of statistical beamforming with perfect channel estimates, where the beamformer is $a_k/|a_k|$, assume perfect phase noise cancellation, and do not include the channel estimation overhead. The results show that for $\beta_{k0} E_r/N_0 \geq 10$ dB, MA-FSR suffers from an SNR loss of $\leq 9$ dB in comparison to analog beamforming. However at lower values, the SNR loss increases significantly, as is also evident from (13). Note that $\beta_{k0} E_r/N_0 = 10$ dB corresponds to a per sub-carrier SNR of around $-10$ dB without the RX beamforming gain, and thus, indeed represents a scenario at the RX. For this $h(t)$, the symbol error rate (SER) for (6), obtained by Monte-Carlo simulations, is compared to the analytical SER for the effective channel (11) in Fig. 2. The perfect match between the analytical and simulation results validates our analysis and the effective channel model in (11). Due to the frequency selective signal and noise powers, we also observe that the SER changes with $k$.

![Fig. 2. SER for data streams $k = 50, 74, 94$ for an MA-FSR RX with QAM modulation $\{K = 50, E_r = E_a/2, E_d(k) = E_a/(2|K|)\}$, $K = 50$, $g = 5\}$](image-url)

![Fig. 3. Comparison of the iSR (without channel estimation overhead) of MA-FSR and analog beamforming (a) For MA-FSR, $E_r = E_a/2, K = 128$ and $g = 5$ (b) For analog beamforming, we only use the subcarriers $\{0, \ldots, K - g - 1\}$](image-url)
where the RX beamforming gain is essential. Furthermore, we observe that the performance of equal data power allocation is comparable to water-filling. However these results depend on L. Larger values of L intensify the frequency selective fading of MA-FSR, thereby possibly increasing the noise loss in Fig. 3. Therefore, MA-FSR is more suited to sparse channels with very few MPCs.

VI. IMPROVED MA-FSR DESIGNS

Note that the MA-FSR RX performance degrades significantly below a certain threshold SNR. This is mainly due to the noise enhancement resulting from the squaring operation, which leads to the large noise-noise cross term in (8). Since the transmit signal is mainly restricted to frequencies |f| ≤ \frac{f_c}{2} and \frac{f_c^2}{4L} ≤ |f - f_c| ≤ \frac{2K}{L} (ignoring phase noise), the impact of this noise enhancement can be significantly reduced by suppressing the noise at lower frequencies by a factor of \sqrt{\varepsilon} using a filter, as illustrated in Fig. 4a. Using a similar analysis as presented here, it can be shown that the effective SNR on the k-th sub-carrier for this design (with no phase noise) is:

$$\gamma_k^N(h(t)) = \frac{M E_k E_d^{(k)}|\beta_k,0(h)|^2}{(2K - k + 1)\varepsilon + 1N_0^2 + N_0|\beta_0,0 E_{sum}^{(k)}} (16)$$

where $E_{sum}^{(k)} = E_t + E_d^{(k)} + \sum_{k' = k+1}^{K} E_d^{(k')}$. This design reduces the noise enhancement of the RX, note that the RX still only has a 50% bandwidth efficiency. This efficiency can be boosted to ≈ 100% by using an alternate design where the squaring circuit is replaced by a multiplier, with one input being the received signal processed via a narrow bandpass filter that isolates only the reference sub-carrier, as illustrated in Fig. 4b. This design, called reference tone aided transmission has been analyzed in detail in [14]. Note that while these designs show significant improvements in performance, implementing such sharp filters at the carrier frequency is difficult and may have to rely on carrier recovery techniques, thus increasing RX complexity.

![Illustration of MA-FSR designs with improved performance](image)

(a) With noise suppression (b) With narrow band-pass filter

Fig. 4. MA-FSR designs with improved performance

VII. CONCLUSION

In this work a novel non-coherent massive antenna RX is proposed, that only requires a single down-conversion chain, can support high data-rates and can perform RX beamforming without phase shifters or channel estimation. The MA-FSR RX essentially uses the received signal for a reference tone to combine the received signal corresponding to the data, via a squaring operation at each antenna. The carefully designed sub-carrier allocation prevents inter-carrier interference. The analysis suggests that the effective channel between the sub-carrier inputs and the demodulated outputs behaves like a parallel Gaussian channel with frequency selective fading, where the frequency selectivity arises due to modulating frequency mismatch between the reference and data sub-carriers and due to the varying noise levels. These varying noise levels arise due to the noise enhancement experienced by the squaring operation at the RX. The simulation results show that MA-FSR suffers only ≈ 6 dB SNR loss in comparison to analog beamforming in sparse channels, as long as the mean received power is above a certain threshold. This threshold behavior is due to the noise enhancement due to the squaring operation, and several improved FSR designs that can reduce its impact are also proposed.

APPENDIX A

From (8), the conditional cross-covariance between the noise components at the a-th and b-th sub-carriers can further be computed as:

$$K_{a,b}(x, h) = 4K^{-1} \sum_{u,v=0}^{K-1} \frac{T_k^2 e^{-j\pi (a u + b v)/4K}}{16K^2} \left\{ \frac{1}{4} \left\| n \left( \frac{u T_a}{4K} \right) \right\|^2 \left\| n \left( \frac{v T_b}{4K} \right) \right\|^2 \right. + \left. \sum_{\ell_1, \ell_2=0}^{L-1} \text{Re} \left[ \alpha_{\ell_1 a} s_{BB} \left( \frac{u T_a}{4K} - \tau_{\ell_1} \right) n \left( \frac{v T_b}{4K} \right) \right] \right. \\
\times \left. \text{Re} \left[ \alpha_{\ell_2 b} s_{BB} \left( \frac{v T_a}{4K} - \tau_{\ell_2} \right) n \left( \frac{u T_b}{4K} \right) \right] \right\} \left(2\right)$$

$$\text{where } K_{a,b}^{(1)} \text{ follows from the fact that } n(t) \text{ is zero-mean Gaussian and therefore the odd moments of } n(t) \text{ are zero; (2) follows by using the identity } \text{Re} \{A\} \text{Re} \{B\} = \frac{1}{2} \text{Re} \{AB^* + AB\} \text{ for any scalars } A, B, \text{ and by ignoring the terms involving pseudo-covariance of the circularly symmetric Gaussian noise and (3) follows by defining } R_{u,v} \equiv \mathbb{E} \left\{ n(t) \right\}_u \left\{ n(t + \frac{u T_a}{4K}) \right\}_v \text{ for any } 1 \leq i \leq M, \text{ using the results on expectation of a product of four Gaussian random variables [15], and from the orthogonality of the array response vectors. Defining a new variable } w = v - u \text{ and using change of variables, we can approximate } K_{a,b}(x, h) \text{ as:}$$

$$K_{a,b}(x, h) \approx 4K^{-1} \sum_{u,v=0}^{K-1} \frac{MT_k^2 e^{-j\pi (a u + b v)/4K}}{16K^2} \left\{ R_{u,v} \right\}_u^2$$

$$\text{where } a \text{ and } b \text{ are any scalars, } 1 \leq a,b \leq M, \text{ and } 0 \leq u,v \leq K-1.$$
where \( \approx \) follows by assuming that the phase noise \( e^{i\theta(t)} \) is constant within the support of the noise auto-correlation function \( R_n[w] \) and \( \approx \) follows by changing the summation limits since \( R_n[w] \) has a very narrow support of around \( O(1) \) and by defining \( \beta_{a,b} \equiv \sum_{\ell=0}^{L-1} [\alpha]\ e^{-j2\pi(f_{\ell}-f_0)T_s} \). Note that \( \approx \) is accurate as long as the system bandwidth is much larger than the phase noise bandwidth [16]. Now taking a summation over \( u \), we obtain:

\[
\begin{align*}
K_{a,b}(x,h) & = \int_{-\infty}^{\infty} S_n(f) e^{j2\pi f T_s} df, \\
& \text{(16)}
\end{align*}
\]

where \((6)\) follows from the identity: \( \sum_{a=-\infty}^{\infty} e^{-j2\pi f/T_s} = \sum_{f_{\ell}=0}^{\infty} \sum_{a=-\infty}^{\infty} \delta(f - f_{\ell} T_s) \) and using the fact that \( S_n(f) \) is non-zero only in the range \( 0 \leq f \leq 2K/T_s \) and we define \( A_{a}(a,b) \triangleq \{ (k_1, k_2) \in A(a,b) \mid k_2 \geq b \} \). Using a similar sequence of steps the noise pseudo-covariance can be computed as:

\[
\begin{align*}
\widetilde{K}_{a,b}(x,h) & = \sum_{(k_1, k_2) \in B(a,b)} M \beta_{a,b}^2 \delta_{k_1,k_2} x_k^e x_k^e N_0, \\
& \text{(18)}
\end{align*}
\]

where \( (18) \) follows from the identity: \( \sum_{k=-\infty}^{\infty} x_k^e x_k^e \delta(f - f_{\ell} T_s) \) and using the fact that \( S_n(f) \) is non-zero only in the range \( 0 \leq f \leq 2K/T_s \) and we define \( B_{a}(a,b) \triangleq \{ (k_1, k_2) \in B(a,b) \mid k_2 \geq b \} \).

\section{REFERENCES}


