A Survey on the Impact of Multipath on Wideband Time-of-Arrival Based Localization

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Abstract—Localization using the time-of-arrival of a wideband signal has the potential to achieve high accuracy, thereby making it a promising choice for a variety of applications. However, a major challenge impacting localization accuracy is multipath propagation, i.e., the existence of indirect paths from a target to an anchor (transceiver) via one or more obstacles present in the environment. In this paper, we provide an overview of the techniques proposed in the literature to analyze and address the effects of multipath on localization accuracy. For this purpose we first cast the localization of one or more targets as a maximum a priori estimation problem, where the distribution of the amplitudes and times of arrival of the indirect paths serves as a prior. Under this framework, we show that multipath can either be a blessing or a curse depending on the extent of prior knowledge available about the multipath statistics. Specifically, in the absence of any indirect path information, only the direct paths that go straight from a target to an anchor contain useful position information; in other words, multipath negatively impacts localization performance. Thus, in this case, it is important to detect the anchors having line-of-sight to the target(s) and we review the techniques reported in the literature to solve this problem. On the other hand, if complete indirect path information is available (i.e., the propagation paths of all multipath components are known), then each indirect path is equivalent to a direct path from a virtual anchor and hence, the spatial diversity offered by multipath can be exploited for improving localization accuracy.

Index Terms—Localization; Time-of-arrival (ToA); Tracking; Multipath; Line-of-sight (LoS) identification; Multipath exploitation; Tracking algorithms

I. INTRODUCTION

The ability to accurately determine the location of (i.e., localize) one or more targets remotely is a necessary and vital component of numerous applications such as navigation, search and rescue operations, surveillance, medical imaging etc., including an ever-growing list of emerging applications such as location-based advertising [1] and social networks [2], crowd-sensing [3] and crowd-sourcing [4], inventory tracking [5], assisted living [6] and so on. Generally speaking, a typical use-case scenario consists of a network of nodes with known coordinates, called anchors, deployed over a region of interest that contains one or more (possibly moving) targets that need to be localized, as shown in Fig. 1. Under this setting, target localization reduces to a relatively simple application of geometric and trigonometric principles, as shown in Fig. 2. In Fig. 2a, the unknown coordinates of the target on the plane can be determined unambiguously if its distance (also known as range) to at least three anchors are known. Each range value constrains the target to lie on a circle of radius equal to the range, with the corresponding anchor at the center, and the target location can be unambiguously determined by solving for the intersection of three or more such circles. In a similar manner, the difference between pairs of ranges can also be used for localization, as illustrated in Fig. 2b. In this case, each range difference constrains a target to lie on a hyperbola and the intersection of two or more such curves unambiguously determines the target location. These range-based localization techniques are referred to as multilateration. Alternately, in Fig. 2c, the target coordinates can be determined if the angles formed with respect to the horizontal at two or more anchor locations are known. This technique is called triangulation.

An accurate method to determine the distance between two points involves transmitting a short pulse [7], known as a ranging signal, from one of the locations and measuring its time-of-arrival (ToA) at the other point using a common reference clock. Thus, if the anchors in Fig. 1 are a network of wireless transceivers, then localization using ranges obtained from the ToA of the ranging signal (Fig. 2a) is commonly

1For 3D localization, the ranges to at least four anchors are required for unambiguous localization.

2In some texts, the technique in Fig. 2a using absolute range values is referred to as trilateration, regardless of the number of anchors, whereas multilateration is used to exclusively refer to the technique in Fig. 2b involving range differences.
(a) Each distance (range) value constrains the target to lie on a circle of radius equal to the range, with the corresponding anchor at the center, and the intersection of three or more such circles provides an unambiguous solution for the target location.

(b) The difference of a pair of range values from two different anchors constrains the target to lie on a hyperbola. In this figure, the range to Anchor 1 is subtracted from all the other range values. A minimum of four anchors is required for unambiguous 2D localization in this case.

(c) The angles formed by the target (with respect to the horizontal) at two or more anchor locations can be used to obtain an unambiguous solution for the target location.

Fig. 2: Basics of localization.

referred to as ToA-based localization. Similarly, the technique in Fig. 2b using range differences is also known as time-difference-of-arrival (TDoA)-based localization. Alternately, the range between a target and an anchor can also be obtained from the ratio between the transmitted and received powers of the ranging signal, as the signal strength decreases monotonically with distance. Thus, localization using ranges obtained in this manner is known as signal strength (SS) or received signal strength indicator (RSSI) based localization. The ranging signal can also be used to measure the angles required for triangulation if the anchors contain an antenna array, as shown in Fig. 2c. In particular, the angle-of-arrival (AoA) of the ranging signal at an anchor can be estimated from the variation of the signal phase across the array elements [8]. As a result, the localization technique illustrated in Fig. 2c is also known as AoA-based localization.

In the above discussion, there is an implicit assumption that the targets in Fig. 1 have radio-frequency (RF) circuitry that enables them to either transmit or receive ranging signals. This holds true for a number of applications (e.g., location-based advertising where the targets are smartphones), which are collectively categorized as active localization scenarios. On the other hand, there are many applications where the targets do not have any RF circuitry and only reflect or scatter the incoming signals from the anchors (e.g., tumors in medical imaging); these are collectively categorized as passive localization scenarios. Furthermore, for passive localization, the transceivers can also be replaced by disjoint transmitter (TX) and receiver (RX) nodes. In this case, an anchor is functionally equivalent to a TX-RX pair. Such an anchor architecture is especially popular in the radar community, where it is known as distributed MIMO radar [9]. Finally, the classification of localization techniques, based on the properties of the ranging signal (e.g., ToA, AoA etc.), can be extended to passive localization as well, with the understanding that the propagation path of the ranging signal involves two hops (i.e., anchor→target→anchor for the transceiver anchor model and TX→target→RX for the distributed MIMO radar anchor model). For instance, for passive ToA-based localization under the distributed MIMO radar anchor model, each range value associated with an anchor-target pair constrains the target to lie on an ellipse instead of a circle, with the corresponding TX and the RX lying at its foci, and the target location can be obtained from the intersection of three or more ellipses, similar to Fig. 2a.

The choice of a suitable localization technique for a given application depends on a number of factors, such as cost, spatial constraints, accuracy of the range measurements, desired localization accuracy, algorithmic complexity etc. For instance, angle-based localization may not be suitable for low-cost sensor network-based applications as antenna arrays are typically larger and more expensive to realize than single-antenna solutions. On the other hand, while RSSI-based localization is simple and relatively inexpensive, the accuracy of the range measurements are especially sensitive to channel propagation phenomena such as log-normal shadowing, where the received signal power varies spatially in a random
manner\(^3\) as a result of propagation through obstacles in the environment [10]. For high accuracy (e.g., centimeter-level) applications, ToA-based localization using a high bandwidth (of the order of GHz) ranging signal is especially attractive, since a large bandwidth provides fine time resolution (because time and frequency domains form a Fourier-transform pair), which improves the accuracy of the range measurements [11]. As a result, wideband ToA-based localization has attracted a lot of research interest [12], and is therefore, the main focus of this paper\(^4\). However, in order to consistently achieve high localization accuracy under a variety of environmental conditions, a number of system-level challenges need to be addressed, which are briefly described below:

a) **Accurate ranging:** It is easy to see that the localization accuracy is highly dependent on the ability to accurately measure the ranges between anchors and targets, a process known as *ranging*. A number of challenges impact the ranging accuracy, such as the presence of thermal noise in receiver electronics, clock synchronization between targets and anchors, multipath propagation, interfering transmissions etc. (Fig. 3). A brief preview of the principles of ToA-based ranging, including some of the techniques to address the challenges involved, is provided in Section II-A.

b) **Resource Allocation:** In addition to accurate ranging, system-level issues, such as the efficient use of power and bandwidth resources [14]–[17], interference management [18] and scheduling [19], also plays an important role in the performance of localization systems.

c) **Multipath:** For accurate ranging, the ToA of the ranging signal along the direct path (DP), corresponding to the line-of-sight (LoS) link between a target and an anchor, needs to be estimated accurately. However, the presence of obstacles in the environment (Fig. 1) causes the ranging signal to also be reflected off them, which results in multiple copies of the ranging signal arriving at different times with varying strengths (Fig. 1). This phenomenon is referred to as *multipath* and each such copy is referred to as a multipath component (MPC). In particular, the DP component is also an MPC and the other MPCs which correspond to reflections off obstacles are known as *indirect paths* (IPs, Fig. 1). It is easy to see that the ToA of the DP component is smaller than that of the IPs. Multipath poses the following challenges to accurate ranging:

(i) The strongest MPC (i.e., having the highest signal-to-noise ratio (SNR)) need not correspond to the DP signal from a particular target. This is because the LoS link between a target and an anchor may be obstructed by obstacles in the environment, which can either result in the DP component being completely blocked or severely attenuated with respect to the other IPs. As a result, ranging using the ToA of the strongest MPC can cause large errors.

(ii) Due to DP blockage (Fig. 1), the first arriving MPC need not correspond to the DP either. As a result, ranging using the ToA of the earliest arriving MPC can also cause large errors.

(iii) In spite of the large bandwidth, the time resolution is not infinite and therefore, the ranging accuracy is hampered when the DP signal overlaps with the other MPCs and is not resolvable (Fig. 4). This, in turn, affects the localization accuracy.

In essence, multipath is the foremost challenge to accurate wideband ToA-based ranging and localization and over the past two decades, there has been a tremendous amount of research focusing on its impact, along with techniques to overcome it. This survey paper is dedicated to providing a detailed overview of the important results in this area. Since multipath is a generic propagation phenomenon encountered in wireless signal transmission, and is not unique to localization alone, we provide a brief summary in Section II-B of the characterization of multipath in the wireless literature, for the sake of completeness.

A. **Organization**

This paper is divided into eight sections. In Section II, we provide a brief overview of the preliminaries; namely, (i) the principles of ToA-based ranging, (ii) multipath propagation, and (iii) estimation theory principles, that will be used in the later parts of this work. In Section III, we introduce the multipath signal model for both active and passive localization and formulate target localization and tracking as a maximum a posteriori (MAP) estimation problem. In Section IV, we provide bounds for the localization mean square error in the presence of multipath and describe the conditions under which the MAP estimates of the target location(s), formulated in Section III, meet these bounds. In Section V, we analyze the impact of multipath on ToA estimation. Based on the

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\(^3\)This variation is typically modeled as a log-normal random variable, which explains the nomenclature.

\(^4\)Narrowband ToA-based localization using cellular base-stations as anchors has also been extensively studied, although the accuracy is typically of the order of tens of meters [13].
analysis in Section IV, multipath can either be a blessing or a hindrance to location estimation, depending on the extent of prior knowledge available about the multipath statistics. In particular, in the absence of any IP information, multipath negatively impacts localization performance and therefore, it is important to use only the DP signals from the anchors having LoS to the targets for ranging and localization. In this context, we present LoS/non-LoS (NLoS) identification techniques for active and passive localization in Section VI. On the other hand, if complete IP information is available (i.e., the propagation paths of all MPCs are known), then we show that each IP is equivalent to a DP from a virtual anchor (VA) and hence, the spatial diversity (in the form of extra anchors) offered by multipath can be exploited for improving the localization accuracy. This case is investigated in Section VII, where we also describe a template for multipath exploiting localization algorithms. Finally, Section VIII concludes the paper.

B. Notation

Throughout this paper, column vectors and matrices are expressed in boldface by lowercase and uppercase letters, respectively. The transpose, hermitian, determinant, trace and inverse operators acting on matrices are denoted by \((\cdot)^T\), \((\cdot)^H\), \(\text{det}(\cdot)\), \(\text{tr}(\cdot)\) and \((\cdot)^{-1}\), respectively. \([A]_{n \times n}\) denotes the upper \(n \times n\) submatrix of a matrix \(A\), while \(\text{diag}(a_1, \cdots, a_n)\) denotes the \(n \times n\) diagonal matrix with the \(i\)-th diagonal element equal to \(a_i\) \((i \in \{1, \cdots, n\})\). For square matrices \(A\) and \(B\), \(A \succeq B\) implies \(A - B\) is positive semi-definite. \(I_n\) and \(0_n\) respectively denote the \(n \times n\) identity and zero matrices. \(\|\cdot\|_2\) and \((\cdot)^*\) respectively denote the Euclidean norm and complex conjugation, while \(\mathbb{E}[\cdot]\) denotes the expectation operator acting on a random variable. \(\mathcal{N}(\mu, \sigma^2)\) denotes the univariate normal distribution with mean \(\mu\) and variance \(\sigma^2\), while \(\mathcal{N}(\mu, K)\) denotes the multivariate normal distribution with mean vector \(\mu\) and covariance matrix \(K\). Finally, the set of real numbers is denoted by \(\mathbb{R}\) and \(\mathbb{R}^2\) denotes the \(xy\) plane.

C. List of acronyms

For convenience, a list of the most commonly encountered acronyms in this paper is provided in Table I, in alphabetic order.

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
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<tbody>
<tr>
<td>AoA</td>
<td>Angle-of-arrival</td>
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<tr>
<td>DP</td>
<td>Direct path</td>
</tr>
<tr>
<td>ED</td>
<td>Energy Detection</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
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<tr>
<td>FIM</td>
<td>Fisher Information Matrix</td>
</tr>
<tr>
<td>G-CRLB</td>
<td>Generalized Cramer-Rao Lower Bound</td>
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<tr>
<td>IP</td>
<td>Indirect path</td>
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<tr>
<td>MAP</td>
<td>Maximum a-posteriori</td>
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<td>MF</td>
<td>Matched filter</td>
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<tr>
<td>ML</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td>MPC</td>
<td>Multipath component</td>
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<tr>
<td>MSE</td>
<td>Mean squared error</td>
</tr>
<tr>
<td>(N)LoS</td>
<td>(Non) Line-of-sight</td>
</tr>
<tr>
<td>PF</td>
<td>Particle Filter</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>SPEB</td>
<td>Squared position error bound</td>
</tr>
<tr>
<td>ToA</td>
<td>Time of arrival</td>
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<tr>
<td>TDoA</td>
<td>Time difference of arrival</td>
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<tr>
<td>VA</td>
<td>Virtual anchor</td>
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TABLE I: Table of commonly used acronyms

II. PRELIMINARIES

A. Principles of ToA-based ranging

As mentioned in Section I, ToA-based ranging refers to the technique of determining the ranges between anchors and targets using the ToA of the ranging signal. For active localization, ranging can, in principle, be done by transmitting the ranging signal from an anchor (or target) and measuring the ToA at the target (or anchor). This is referred to as one-way ranging as it involves signal propagation over a one-way link. While conceptually simple, one-way ranging requires the clocks at the anchors and targets to be perfectly synchronized with respect to absolute time. However, due to oscillator imperfections, the local time at the anchors, \(T_A(t)\), differs from the absolute time, \(t\), and this variation is typically modeled as follows [20]:

\[
T_A(t) = (1 + \Delta_A)t + \mu_A, \tag{1}
\]

where \(\Delta_A > 0\) and \(\mu_A\) are known as the clock drift and offset, respectively (for an ideal clock, \(\Delta_A = \mu_A = 0\)). Similarly, let \(T_B(t) = (1 + \Delta_B)t + \mu_B\) denote the local time at a target. Thus, for one-way ranging, the errors in the measured ToAs (i.e.,
\[ |T_A(t) - T_B(t)| \] depend on \(|\mu_B - \mu_A|\), the difference between the clock offsets, as well as the clock drifts, \(\Delta_A\) and \(\Delta_B\). In particular, the errors due to the clock offsets are typically much larger in magnitude than those due to clock drifts [21]. This can be partially offset by considering the TDoA of the ranging signals (Fig. 2b) if the anchors are synchronized among themselves. In this case, a clock offset at a target does not lead to ranging errors, as it cancels out when the difference is taken. However, a drawback of TDoA-based ranging and localization is its relatively lower accuracy when compared to its ToA-based counterparts [11], along with the requirement of additional anchor nodes. Thus, in order to obtain the accuracy benefits of ToA-based localization, a more practical ranging solution is to employ two-way ranging. In this case, an anchor first transmits the ranging signal, which is received by the target. The target then waits for a pre-determined time, \(T_{\text{wait}}\), before transmitting an acknowledgment (ACK) signal back to the anchor. The time elapsed between the transmission of the ranging signal and the arrival of the ACK signal is equal to the signal propagation delay corresponding to twice the range plus \(T_{\text{wait}}\), from which the range can be estimated. This round-trip approach only requires the clocks to be frequency-synchronized, without agreeing on absolute time. Additionally, ranging for passive localization can be thought of as a special case of two-way ranging where \(T_{\text{wait}} = 0\) and the ACK signal is identical to the ranging signal. While the effects of clock-offset are mitigated by two-way ranging, the measured ToAs still differ from the true values by a factor equal to \((1 + \Delta_A)\) [21]. Therefore, clock synchronization at regular intervals is still necessary to minimize the errors in ToA estimation due to clock drift. For a summary of synchronization techniques for wireless networks, we refer the reader to [20].

### B. Multipath propagation

Multipath propagation is a fundamental feature of wireless signal transmission that distinguishes it from the wired case. Essentially, it refers to the phenomenon where signals propagate from a source to a destination by bouncing off various “interacting objects” (IOs) in the environment. Each such signal echo, taking a different propagation path from the source to the destination, is referred to as an MPC and the such signal echo, taking a different propagation path from the source to the destination, is referred to as an MPC and the propagation environment. In general, the channel impulse response due to multipath propagation at a particular time \(t\) can be written as

\[
h(t) = \sum_{i=1}^{L} a_i \delta(t - \tau_i),
\]

(2)

where \(a_i\) and \(\tau_i\) denote the complex amplitude and the delay of the \(i\)-th MPC \((i = 1, \ldots, L)\), respectively, and \(\delta(.)\) is a “pulse distortion” function. Ideally, \(\delta(.)\) should equal \(\delta(t)\), the Dirac-delta function, which is equivalent to each MPC being a scaled and delayed (but otherwise undistorted) copy of the transmit signal (or ranging signal for a localization application). However, \(\delta(.)\) might deviate from the ideal form, since the different frequency components of the transmit signal might experience different reflection/diffraction coefficients at the IOs. Furthermore, the antennas at the source and the destination might lead to a distortion of the transmit signal, which can be either modeled as a part of the transmit signal, or alternately, as part of the propagation channel. In either case, a distorted pulse waveform at the destination makes the determination of the signal ToA more difficult.

A large number of channel models have been established in the literature for the statistical characteristics of the MPC amplitudes, \(\{a_i\}\), and delays, \(\{\tau_i\}\); for shortage of space, we refer the reader to textbooks (e.g., [22]) for a more detailed presentation. However, we emphasize here that almost all existing channel models have been established for the purpose of modeling the impact of multipath on communication systems and not localization systems. While the propagation channel itself is independent of the application or the signal sent over it, it needs to be noted that channel models always involve a certain degree of simplification and the type of details that are discarded in this process does depend on the application. In particular, for communication systems, the correct modeling of the strongest MPC is the most important aspect, while for localization systems it is the first MPC that is the most important, even if it is relatively weak, since the first MPC is used as the basis of ranging. As an example, the propagation channel in industrial NLoS environments is characterized by a “soft onset”, where the MPC strength first increases with increasing delay, reaching a maximum at a delay of some \(50\)\(\mu s\) after the first MPC (corresponding to \(15\)\(\mu m\) excess distance), before decaying [23]. Therefore, as mentioned in Section I, ranging based on the ToA of the strongest MPC can cause large errors. For examples of multipath channel models suitable for localization applications, we refer the reader to [24]–[26].

### C. Estimation theory

In a typical estimation problem, the objective is to determine the values of one or more unknown parameters, denoted by a vector \(a \in \mathbb{R}^{N \times 1}\), from a collection of observations or measurements, \(r\), which are a random vector function of \(a\). For example, in ToA-based localization, the parameter \(a\) corresponds to the coordinates of a target, while \(r\) is the vector of noisy range measurements from different anchors. In this particular example, the random component is introduced by the noise. In general, the randomness in \(r\) is fully characterized by the conditional probability density function (pdf) of \(r\) given \(a\), denoted by \(f(r|a)\). As a function of \(r\), \(f(r|a)\) is a measure of how likely it is to observe \(r\), for a given value of \(a\). In particular, a high value of \(f(r|a)\) indicates that the values in the neighborhood of \(r\) are more likely to be observed. Hence, \(f(r|a)\) is also known as the likelihood function of \(r\), given \(a\). Under this interpretation, an intuitive estimate of \(a\) for a collection of observations \(r\), is the value that maximizes the likelihood function. This is known as the maximum likelihood (ML) estimate of \(a\) and can be expressed as follows:

\[
\hat{a}_{\text{ML}} = \arg \max_a f(r|a).
\]

Formally, \(f(r|a)\) is the probability of observing values lying in an \(\epsilon\)-sized neighborhood centered at \(r\), where \(\epsilon\) is an arbitrarily small positive constant.
For the special case where \( r \) is a Gaussian random vector with mean \( a \) and covariance matrix \( K \) (i.e., \( r \sim N(a, K) \)), the ML estimate of \( a \) reduces to solving the following optimization problem

\[
\hat{a}_{\text{ML}} = \arg \max_a \frac{\exp(- (r - a)^T K^{-1} (r - a))}{(2\pi)^{N/2}(\det(K))^{1/2}}
\]

(4)

\[
= \arg \min_a (r - a)^T K^{-1} (r - a).
\]

(5)

Furthermore, if \( K = \text{diag}(\sigma_1^2, \cdots, \sigma_2^2) \), then (5) reduces to the following weighted least-squares minimization problem:

\[
\hat{a}_{\text{ML}} = \arg \min_a \sum_{i=1}^N \frac{(r_i - a_i)^2}{\sigma_i^2},
\]

(6)

where \( r_i \) and \( a_i \) denote the \( i \)-th element of \( r \) and \( a \), respectively.

In ML estimation, the unknown parameter, \( a \), is treated as a fixed quantity. However, there are scenarios where it may be more appropriate to treat \( a \) as a random variable, taking on values which are governed by a marginal pdf \( f(a) \). This is known as the maximum a posteriori (MAP) estimate of \( a \) and can be expressed as follows:

\[
\hat{a}_{\text{MAP}} = \arg \max_a f(a | r)
\]

(7)

\[
= \arg \max_a \frac{f(r | a) f(a)}{f(r)}
\]

(8)

\[
= \arg \max_a f(r | a) f(a),
\]

(9)

where (8) follows from Bayes’ rule and (9) follows from the fact that the denominator in (8) is not a function of \( a \). A high value of \( f(a) \) implies that values in the neighborhood of \( a \) are more likely. Thus, in the context of (9), the MAP estimate skews the ML estimate towards a value of \( a \) that is more likely. As a result, \( f(a) \) is also referred to as the prior of \( a \), since it contains pre-existing information on the value of \( a \) that does not depend on the observations \( r \).

For the special case when \( a \) has a uniform prior (i.e., \( f(a) \) is the uniform pdf over the domain of \( a \)), the MAP and ML estimates of \( a \) coincide, which is intuitive since, in this case, all values of \( a \) are equally likely apriori and no particular value is more favored, regardless of \( r \).

In general, an estimate, \( \hat{a} \), of \( a \) is a function of \( r \) and is therefore, a random vector whose moments are well defined. In particular, \( \hat{a} \) is said to be an unbiased estimate of \( a \) if the following condition is satisfied

\[
E[\hat{a} - a] = 0.
\]

(10)

For an unbiased estimate \( \hat{a} \) of \( a \), its covariance matrix satisfies the following bound

\[
E[(\hat{a} - a)(\hat{a} - a)^T] \geq J^{-1}(a),
\]

(11)

where the quantity \( J(a) \) is known as the Fisher information matrix (FIM) of \( a \), which is defined in the following manner:

\[
J(a) = E \left[ \left( \frac{\partial}{\partial a} \ln f(r | a) \right) \left( \frac{\partial}{\partial a} \ln f(r | a) \right)^T \right].
\]

(12)

Intuitively, \( J(a) \) captures the information about \( a \) that can be extracted using only \( r \). The inequality in (11) is referred to as the Cramer-Rao lower bound (CRLB) and is met with equality by the ML estimate, \( \hat{a}_{\text{ML}} \), when the size of the observation vector, \( r \), tends to \( \infty \) [27]. Using simple linear algebra identities, the CRLB can also be expressed in the following manner, which may be useful in some cases:

\[
E[||\hat{a} - a||^2] \geq \text{tr}(J^{-1}(a)).
\]

(13)

Additionally, if \( a = [a_1^T a_2^T]^T \), where \( a_1 \in \mathbb{R}^{N_1 \times 1} \) and \( a_2 \in \mathbb{R}^{(N - N_1) \times 1} \), then the CRLB for the sub-parameter \( a_1 \) has the following form:

\[
E[||a_1 - a_1||^2] \geq \text{tr}((J^{-1}(a))_{N_1 \times N_1}),
\]

(14)

where \( \hat{a}_1 \) denotes the (unbiased) estimate of \( a_1 \).

When additional information on \( a \) is available in the form of a prior, \( f(a) \), the notion of FIM can be extended in the following manner:

\[
J_G(a) = E \left[ \left( \frac{\partial}{\partial a} \ln f(r | a) f(a) \right) \left( \frac{\partial}{\partial a} \ln f(r | a) f(a) \right)^T \right],
\]

(15)

where \( J_G(a) \) is known as the generalized FIM of \( a \), which captures the information on \( a \) that can be extracted from both \( r \) and the prior, \( f(a) \). For an unbiased estimate \( \hat{a} \), a generalized CRLB (G-CRLB) can be obtained by replacing \( J(a) \) with \( J_G(a) \) in the inequalities in (11), (13) and (14). Similarly, the G-CRLB is met with equality by the MAP estimate, \( \hat{a}_{\text{MAP}} \), when the size of \( r \) tends to \( \infty \).

**III. SIGNAL MODEL**

**A. Active Localization**

Due to its similarity in principle to passive localization, we assume two-way ranging throughout this paper for consistency of analysis and notation. Furthermore, we assume that the ACK transmitted by each target is unique (e.g., the communication protocol could require the transmission of identification information, such as device ID or MAC address etc.) and that the ACKs from multiple targets do not interfere with each other, for simplicity. As a result of these assumptions, the MPCs (i.e., both DPs and IPs) corresponding to each target are mutually distinguishable at each anchor. Therefore, the problem of jointly localizing \( M_t \) targets is equivalent to solving \( M_t \) independent instances of the single-target localization problem. Hence, without loss of generality, we consider the single target case for active localization.

Consider \( M_a \geq 3 \) anchors situated in \( \mathbb{R}^2 \), where the location of the \( i \)-th anchor is denoted by \( p_i = [x_i \ y_i]^T \). Similarly, let the target location be denoted by \( p = [x \ y]^T \) and let \( s_i^{ac}(t) \) denote the ranging signal transmitted by the \( i \)-th anchor. Assuming no pulse distortion (i.e., \( \xi(\cdot) = \delta_\epsilon(\cdot) \) in Section II-B), the return signal received at the \( i \)-th anchor, denoted by \( z_i^{ac}(t) \),

\[\text{We consider localization over a 2D plane throughout this work. The extension to the 3D case is straightforward.}\]
can be modeled as a superposition of a number of MPCs in the following manner:

\[ z_{ac}^{i}(t) = \sum_{l=1}^{L_{ac}^{i}} \alpha_{il}^{ac} s_{il}^{ac} (t - \tau_{il}^{ac}(p)) + \eta_{il}^{ac}(t), \quad (16) \]

where \( L_{ac}^{i} \) denotes the number of MPCs observed at the \( i \)-th anchor, \( \alpha_{il}^{ac} \) and \( \tau_{il}^{ac}(p) \) denote, respectively, the complex amplitude and the ToA of the \( l \)-th MPC at the \( i \)-th anchor, and \( \eta_{il}^{ac}(t) \) denotes the thermal noise, which is modeled as an additive white Gaussian random process with a two-sided power spectral density of \( N_0/2 \) (i.e., over a bandwidth \( B \)), \( \eta_{i}(t) \sim \mathcal{N}(0, N_0B) \) for each \( t \). Furthermore, the noise processes at different anchors are assumed to be mutually independent.

For a sufficiently high SNR, the first arriving MPC at the \( i \)-th anchor, with ToA \( \tau_{ac}^{i}(p) \), is a DP if and only if the optical LoS exists between the target and the \( i \)-th anchor. Otherwise, it is an IP. In general, the ToA of the \( l \)-th MPC can be modeled as follows:

\[ \tau_{il}^{ac}(p) = \frac{2}{c} \| p_i - p \|_2 + b_{il}^{ac}, \quad (17) \]

where \( c \) denotes the speed of light in free space and \( b_{il}^{ac} \) is a non-negative term that represents the excess delay of an IP, with respect to the DP round-trip propagation time, \( 2\| p_i - p \|_2/c \). Thus, \( b_{il}^{ac} = 0 \) corresponds to the case when the first MPC is a DP. The vector \( b_{i}^{ac} = [b_{1i}^{ac}, \ldots, b_{Mi}^{ac}]^T \) can either be treated as an unknown, fixed quantity or as a random vector with pdf \( f(b_{i}^{ac}) \).

Let \( z_{i}^{ac} \) denote the equivalent vector representation of \( z_{ac}^{i}(t) \), obtained by a Karhunen-Loeve expansion \[27\] and let \( z^{ac} = ([z_{1}^{ac}]^T \cdots [z_{M}^{ac}]^T)^T \). The unknown parameters, other than \( p \), can be represented as a vector, \( \theta^{ac} = ([\kappa_{1}^{ac}]^T \cdots [\kappa_{M}^{ac}]^T)^T \), where \( \kappa_{il}^{ac} = [\alpha_{il}^{ac}, b_{il}^{ac}, \ldots, \alpha_{il}^{ac}_{Mi}, b_{il}^{ac}_{Mi}]^T \) is the vector of MPC amplitudes and excess delays corresponding to \( z_{i}^{ac} \). From Section II-C, the likelihood function of \( z^{ac} \), given \( \theta^{ac} \) and \( p \), has the following expression:

\[ f(z^{ac}|p, \theta^{ac}) = \prod_{i=1}^{M} f(z_{i}^{ac}|p, \theta^{ac}), \quad (18) \]

\[ f(z_{i}^{ac}|p, \theta^{ac}) \propto \exp \left\{ \frac{2}{N_0} \int_{-\infty}^{\infty} z_{i}^{ac}(t) \sum_{l=1}^{L_{ac}^{i}} \alpha_{il}^{ac} s_{il}^{ac} (t - \tau_{il}^{ac}(p)) dt \right. \]

\[ - \frac{1}{N_0} \int_{-\infty}^{\infty} \left| \sum_{l=1}^{L_{ac}^{i}} \alpha_{il}^{ac} s_{il}^{ac} (t - \tau_{il}^{ac}(p)) \right|^2 dt \right\}, \quad (19) \]

where (18) results from the mutual independence of the collection of noise processes \( \{\eta_{il}^{ac}(t) : i = 1, \ldots, M_a\} \), and (19) follows from the fact that for each \( t \), \( \eta_{il}^{ac}(t) \) has a normal distribution.

\footnote{In some cases, optical LoS may be blocked but radio LoS may still exist if the ranging signal can penetrate through the blocking obstacle. However, such an MPC is still considered an IP as it would generally arrive at a later time, due to a slower propagation speed through the medium of the obstacle.}

The MAP estimate of the target location, denoted by \( \hat{p}_{MAP} \), can then be expressed as follows:

\[ \hat{p}_{MAP} = \arg \max_{p,\theta^{ac}} f(z^{ac}|p, \theta^{ac}) f(\theta^{ac}, p), \quad (20) \]

where the joint pdf \( f(\theta^{ac}, p) \) is the prior of \( \theta^{ac} \) and \( p \), which captures any a priori knowledge of \( p \) and \( \theta^{ac} \) through the joint pdf, \( f(b_{1}^{ac}, \ldots, b_{M}^{ac}) \) (e.g., floor plan information).

\textbf{B. Passive localization}

For passive localization, unless otherwise specified, we assume a transceiver model for the anchors, similar to the active localization case, instead of a distributed MIMO radar. We also retain the same anchor setup considered in Section III-A for the active localization case (i.e., \( M_a \) anchors with the \( i \)-th anchor location denoted by \([x_i, y_i]^T\)), while additionally considering \( M_L(>1) \) passive targets, all having identical radar signatures\footnote{By radar signature, we mean the scattering properties which are captured by the notion of the radar cross-section (RCS) \[28\]. The identical radar signature assumption implies that we are unable to distinguish between MPCs emanating from different targets based on any specific scattering characteristics, although the amplitudes of the reflected signals are, in general, different for the different targets.}, with the location of the \( j \)-th target denoted by \([x_j, y_j]^T\) (1 \( \leq j \leq M_L\)).

Let \( s_{pa}^{i}(t) \) denote the ranging signal transmitted by the \( i \)-th anchor. Unlike the active localization case, the received signal at any anchor typically contains other MPCs that are neither DPs nor IPs corresponding to any target (e.g., the MPC due to the anchor \( \rightarrow \) obstacle \( \rightarrow \) anchor path in Fig. 1). These MPCs, referred to as \textit{background clutter}, do not contain any information about the target location(s), as they do not reflect off any target. Furthermore, the clutter MPCs generally tend to drown out the DPs and IPs as they have a much higher SNR, in comparison. Thus, it is important to eliminate the background clutter in order to accurately detect the MPCs that reflect off targets. This process is referred to as \textit{background subtraction}. An obvious approach is to first measure the background response by transmitting \( s_{pa}^{i}(t) \) when no target is present and then, subtracting it from the response obtained when one or more targets are present \[29\]. However, this method may not be suitable for all applications as it may be difficult in some applications to ensure that no targets are present when measuring the background response. Moreover, the effectiveness of this technique is limited to static or quasi-static environments that do not change much over the course of both measurements. Other background subtraction techniques that do not require a \textit{targetless} background measurement are reported in \[30\], \[31\]. These techniques are effective for time-varying environments, as well.

After background subtraction, let \( z_{pa}^{i}(t) \) denote the received signal at the \( i \)-th anchor due to all the target(s), which is modeled as follows, similar to (16):

\[ z_{pa}^{i}(t) = \sum_{l=1}^{L_{pa}^{i}} \alpha_{il}^{pa} s_{il}^{ac} (t - \tau_{il}^{pa}(q)) + \eta_{il}^{pa}(t), \quad (21) \]

where \( L_{i}^{pa} \) denotes the number of observed MPCs, \( \alpha_{il}^{pa} \) and \( \tau_{il}^{pa}(q) \) respectively denote the complex amplitude and the ToA.
of the \( l \)-th MPC, where \( \mathbf{q} = [\mathbf{q}_1^T \ldots \mathbf{q}_{M_L}^T]^T \), and \( \eta_{\alpha}(t) \) denotes the additive noise, which has the same properties as \( \eta_{ac}(t) \). Unlike the active localization case, it is not obvious which MPC corresponds to which target. In particular, an MPC with a later ToA could be a DP corresponding to a target that is far away, while an earlier arriving MPC could be an IP corresponding to a nearby target. Thus, it is not necessary for all the DP ToAs to be smaller than the IP ToAs for the multitarget case. Therefore, it is important to identify the MPCs corresponding to a particular target, in order to localize it. This process is known as data association and to do this, we define the following decision variables [32], [33]:

\[
\begin{align*}
    k_{ilj} &= \begin{cases} 
        1, & \text{if } \tau_{il}^{pa}(\mathbf{q}) \text{ is a DP to the } j \text{-th target} \\
        0, & \text{else}
    \end{cases}, \\
    g_{ilj} &= \begin{cases} 
        1, & \text{if } \tau_{il}^{pa}(\mathbf{q}) \text{ is an IP associated with the } j \text{-th target} \\
        0, & \text{else}
    \end{cases}. 
\end{align*}
\]

(22)

(23)

Due to finite bandwidth, which induces a finite time resolution, two or more closely separated MPCs may overlap with one another and not be resolvable (see Fig. 4). Therefore, it is possible for an observed MPC to be associated with more than one target. Thus, if the \( l \)-th MPC at the \( i \)-th anchor is associated with the \( j \)-th target, then

\[
\tau_{il}^{pa}(\mathbf{q}) = \begin{cases} 
    2|\mathbf{p}_i - \mathbf{q}_j|_2/c, & \text{if } k_{ilj} = 1 \\
    2|\mathbf{p}_i - \mathbf{q}_j|_2/c + b_{ilj}^{pa}, & \text{if } g_{ilj} = 1,
\end{cases}
\]

(24)

where \( b_{ilj}^{pa} \) is a positive term that represents the excess delay of the IP relative to the DP round-trip propagation time, \( 2|\mathbf{p}_i - \mathbf{q}_j|_2/c \). For all other cases, \( b_{ilj}^{pa} = 0 \). The vector \( \mathbf{b}_{ilj}^{pa} = [b_{ilj}^{pa}, \ldots b_{ilj}^{pa,M_L}]^T \) can either be treated as an unknown, fixed quantity or as a random vector with pdf \( f(b_{ilj}^{pa}) \).

Let \( z_{ilj}^{pa}(t) \) denote the equivalent vector representation of \( z_{il}^{pa}(t) \) and let \( \mathbf{z}_{pa} = [(z_{11}^{pa})^T \ldots (z_{M_L}^{pa})^T]^T \). The unknown parameters, other than \( \mathbf{q} \), can then be represented as a vector, \( \theta_{pa} = [(\kappa_{1pa}^{pa})^T \ldots (\kappa_{M_Lpa}^{pa})^T]^T \), where \( \kappa_{il}^{pa} = [\alpha_{il1}^{pa}, b_{il1}^{pa}, g_{il1}^{pa}, \ldots \alpha_{il1}^{pa}, b_{il1}^{pa,M_L}, g_{il1}^{pa,M_L}]^T \) is the vector of MPC amplitudes, excess delays and target associations corresponding to \( z_{ilj}^{pa} \). Similar to the active localization case, the likelihood function of \( \mathbf{z}_{pa} \), given \( \mathbf{q} \) and \( \theta_{pa} \), has the following expression:

\[
f(\mathbf{z}_{pa}|\mathbf{q}, \theta_{pa}) = \prod_{i=1}^{M_L} f(z_{ilj}^{pa} | \mathbf{q}, \theta_{pa})\)

\[
f(z_{ilj}^{pa} | \mathbf{q}, \theta_{pa}) \propto \exp \left\{ \frac{2}{N_0} \int_{-\infty}^{\infty} \sum_{l=1}^{L_{il}} \alpha_{il}^{pa} s_{il}^{pa}(t - z_{il}^{pa}(q)) dt \right\}
\]

\[
- \frac{1}{N_0} \int_{-\infty}^{\infty} \left[ \sum_{l=1}^{L_{il}} \alpha_{il}^{pa} s_{il}(t - z_{il}^{pa}(q)) \right]^2 dt
\]

(25)

(26)

The joint MAP estimates of all the target locations, denoted by \( \mathbf{q}_{\text{MAP}} \), can then be expressed as follows:

\[
\hat{\mathbf{q}}_{\text{MAP}} = \arg \max_{\mathbf{q}, \theta_{pa}} f(\mathbf{z}_{pa} | \mathbf{q}, \theta_{pa}) f(\theta_{pa}, \mathbf{q})
\]

(27)

where the joint pdf \( f(\theta_{pa}, \mathbf{q}) \) is the prior of \( \theta_{pa} \) and \( \mathbf{q} \).

C. Tracking

For tracking applications, the MAP estimation framework can be extended to include moving targets as well, by incorporating a time component. For simplicity, we consider a single target and hence, both active and passive tracking reduce to the same problem after suitable background subtraction for the passive case. Let \( \mathbf{p}(t) = [x(t), y(t)]^T \) denote the location of the target at time \( t \) and let \( \theta(t) \) denote the other unknown parameters, related to the MPCs. Similarly, let \( \mathbf{z}(t) = [z_1(t)^T \ldots z_{M_L}(t)^T]^T \) denote the vector representation of the received signal at time \( t \). Assuming slotted time, let \( \mathbf{z}(1:t) \) denote the collection of measurements up to, and including, time \( t \). The MAP estimate of \( \mathbf{p}(t) \) can then be formulated as follows:

\[
\hat{\mathbf{p}}_{\text{MAP}}(t) = \arg \max_{\mathbf{p}(t)} f(\mathbf{p}(t) | \mathbf{z}(1:t))
\]

(28)

where, in general, all the measurements up to, and including, time \( t \) can be used to estimate the current target location. Typically, the target’s movement is governed by a motion model, which, if known, can also be incorporated as a prior in the MAP estimation problem. A popular assumption in many applications is a Markov motion model, where \( \mathbf{p}(t) \) (and \( \theta(t) \)) depends only on \( \mathbf{p}(t-1) \) (and \( \theta(t-1) \)) via a conditional pdf, denoted by \( f(p(t), \theta(t)|p(t-1), \theta(t-1)) \). In this case, (28) can be decomposed into two recursive steps: prediction and update, given by

Prediction: \( f(p(t), \theta(t)|z(1:t-1)) = \int f(p(t), \theta(t)|p(t-1), \theta(t-1)) f(p(t-1), \theta(t-1)|z(1:t-1)) dp(t-1) \)

(29)

Update:

\[
\hat{\mathbf{p}}_{\text{MAP}}(t) = \arg \max_{\mathbf{p}(t)} f(z(t)|\mathbf{p}(t), \theta(t)) f(\mathbf{p}(t), \theta(t)|z(1:t-1))
\]

(30)

Intuitively, the prediction step in (29) involves estimating the current target location with the help of all previous location estimates and measurements (i.e., excluding the current measurement, \( z(t) \)), while the update step in (30) is a refinement of the predicted target location using \( z(t) \).

For linear Gaussian measurement and motion models\(^\text{g}\), the optimal solution for (29)-(30) is the well-known Kalman filter. However, for ToA-based localization, the relationship between \( \mathbf{p}(t) \) and \( \mathbf{z}(t) \) is non-linear (see (16)-(17)) and hence, a Kalman filter is, in general, not optimal. The most common solutions used for non-Gaussian systems are the extended Kalman filter (EKF) [34] and particle filters (PFs) [35], which are described in more detail in Section VII-A in the context of multipath exploiting tracking algorithms. Both solutions were shown to be special cases of a paradigm called belief condensation filtering [36].

\(^\text{g}\)The measurement model is said to linear Gaussian if \( \mathbf{z}(t) = \mathbf{H}(t) \mathbf{p}(t) + \mathbf{n}_1(t) \), for some matrix, \( \mathbf{H}(t) \), and Gaussian random vector, \( \mathbf{n}_1(t) \). Similarly, the motion model is said to be linear Gaussian if \( \mathbf{p}(t) = \mathbf{F}(t-1) + \mathbf{n}_2(t) \), for some matrix \( \mathbf{F}(t) \) and Gaussian random vector, \( \mathbf{n}_2(t) \).
For the MAP estimates, \( \hat{p}_{\text{MAP}} \), \( q_{\text{MAP}} \) and \( p_{\text{MAP}}^{(i)} \), the localization accuracy is commonly characterized by the mean-square error (MSE), which is equal to \( \mathbb{E}[\|p_{\text{MAP}} - p\|^2] \), \( \mathbb{E}[\|q_{\text{MAP}} - q\|^2] \) and \( \mathbb{E}[\|p_{\text{MAP}}^{(i)} - p^{(i)}\|^2] \) for the active localization, passive localization and tracking cases, respectively. In the next section, we analyze the impact of multipath on the MSE and provide a lower bound for the achievable MSE in the presence of multipath.

IV. FUNDAMENTAL LIMITS OF WIDEBAND TOA-BASED LOCALIZATION

In order to analyze the impact of multipath on the localization accuracy, we consider a common notational framework for both active and passive localization, based on the signal model described in Sections III-A and III-B. For a target at \( \mathbf{p} = [x \ y]^T \), let \( \mathbf{z} = [z_1^T \cdots z_{M_a}^T]^T \) denote the received signal vector, where \( z_i \) is the equivalent vector representation of the received signal, \( z_i(t) = \sum_{l=1}^{L} \alpha_{il} s_i(t - \tau_{il}(\mathbf{p})) + \eta_i(t) \), corresponding to a ranging signal \( s_i(t) \) transmitted by the \( i \)-th anchor. Similarly, let \( \theta \) denote the vector of MPC amplitudes and excess delays. For this setup, the MSE of any unbiased estimate, \( \mathbf{p} \), of \( \mathbf{p} \) can be bounded using the G-CRLB defined in Section II-C, in the following manner:

\[
\mathbb{E}[\|\hat{\mathbf{p}} - \mathbf{p}\|^2] \geq \text{tr}([J_G^{-1}(\Theta)]_{2 \times 2}), \tag{31}
\]

where \( \Theta \triangleq [\mathbf{p}^T \ \theta^T]^T \) and \( J_G(\Theta) \) is the generalized FIM of \( \Theta \), given by:

\[
J_G(\Theta) \triangleq \mathbb{E} \left[ \left( \frac{\partial}{\partial \Theta} \ln(f(\mathbf{z} | \Theta) f(\mathbf{z} | \Theta)) \right) \left( \frac{\partial}{\partial \Theta} \ln(f(\mathbf{z} | \Theta) f(\mathbf{z} | \Theta)) \right)^T \right]. \tag{32}
\]

In the high SNR regime, the bound in (31) is met with equality by the MAP estimate of \( \mathbf{p} \), obtained by solving (20) and (27) for active and passive localization, respectively. In localization terminology, the quantity on the RHS of (31) is referred to as the squared position error bound (SPEB) [37]. Intuitively, \( J_G(\Theta) \) captures the target location information contained in the DPs and IPs and for closed-form expressions of \( J_G(\Theta) \), we refer the reader to [38]. While we have assumed non-cooperative localization in our analysis so far (i.e., the targets do not communicate with each other and therefore, each target is localized using only its ranges to the anchors), significant improvements in both accuracy and coverage area can be achieved through cooperative localization, where the ranges between targets are also used for localization [39]–[41]. The generalized FIM framework can be extended to include cooperative localization as well, and the corresponding SPEB was derived in [42]. Similarly, the generalized FIM can also be extended by including a time component to characterize the fundamental limits of tracking [43].

For simplicity and without loss of generality, we restrict our focus to non-cooperative localization, and consider the following special cases of \( J_G(\Theta) \), which encompass a majority of practical scenarios and correspond to the extremes with respect to knowledge of the prior, \( f(\Theta) \):

(a) **No prior IP information available**: This scenario typically arises when the distribution of the obstacle locations that give rise to the observed MPCs is unknown, which corresponds to a uniform pdf for \( f(\Theta) \) over the domain of \( \Theta \). In this case, \( J_G(\Theta) \) can be decomposed as a sum of rank one matrices in the following manner [38]:

\[
J_G(\Theta) = \sum_{i=1}^{M_a} \delta_i(\mathbf{p}) \lambda_i \mathbf{u}(\phi_i) \mathbf{u}(\phi_i)^T, \tag{33}
\]

where \( \delta_i(\mathbf{p}) = \begin{cases} 1, & \text{if } i \text{-th anchor has LoS to } \mathbf{p} \\ 0, & \text{else} \end{cases} \)

(b) **Complete IP knowledge available**: In contrast to the previous case, this scenario usually arises when all the obstacle locations are known exactly (e.g., in indoor environments where the floor plan may be available [45]), which corresponds to the case when \( \Theta \) is deterministic. In this case, the SPEB also depends on the location information contained in the IPs, in addition to the range information intensity (RII), \( \lambda_i \), which has the following expression:

\[
\lambda_i = 8\pi^2 \beta^2 (1 - \chi_i) \text{SNR}_i/c^2, \tag{35}
\]

where \( \beta = \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |S_i(f)|^2 df \right)^{1/2} \). \tag{36}

In (35)-(36), \( S_i(f) \) and \( \beta \) and \( \text{SNR}_i \) denote the Fourier transform and the effective bandwidth of \( s_i(t) \), respectively, while \( \text{SNR}_i \) denotes the SNR of the DP signal component at the \( i \)-th anchor. In particular, the impact of a large bandwidth on ranging accuracy is captured by \( \beta \). The term \( \chi_i \in [0,1] \) in (35) determines the impact of MPC overlap due to finite time resolution (see Fig. 4).

In particular, \( \chi_i = 1 \) corresponds to the case when the DP cannot be resolved and thus, no range information is available from the \( i \)-th anchor. On the other hand, \( \chi_i = 0 \) corresponds to the case where the DP does not overlap with any other MPC, which results in the most accurate range estimate. Apart from the ranging accuracy, the anchor geometry also impacts \( J_G(\Theta) \) and the SPEB [44], which is captured in (33) by the outer product \( \mathbf{u}(\phi_i) \mathbf{u}(\phi_i)^T \) and the unit vector in the direction of the \( i \)-th anchor, relative to \( \mathbf{p} \) (i.e., \( \mathbf{u}(\phi_i) = \begin{bmatrix} \cos \phi_i & \sin \phi_i \end{bmatrix} \)).

\[10\]The reciprocal of \( \lambda_i \) is the CRLB for an unbiased estimate of the range, \( \| \mathbf{p} - \hat{\mathbf{p}} \|_2 \). In other words, for an unbiased estimate, \( \hat{r}_i \), of \( \| \mathbf{p} - \hat{\mathbf{p}} \|_2 \), \[\sigma_{\hat{r}_i}^2 \geq 1/\lambda_i \), where \( \sigma_{\hat{r}_i}^2 \) denotes the variance of \( \hat{r}_i \).
information present in the DPs. To illustrate this, suppose the target location is estimated using only the ranges obtained from the DPs. Using this estimate in conjunction with the obstacle locations, the propagation paths of the feasible IPs at each anchor can be determined, in principle, by ray-tracing. The difference between the resulting collection of signals, denoted by \( \{ z_i^k : i = 1, \ldots, M_a \} \) and the observed signals, \( \{ z_i : i = 1, \ldots, M_a \} \), can then be used as the basis for refining the estimate of the target location.

In summary, the SPEB provides a benchmark for evaluating the performance of algorithms that solve \((20)\) and \((27)\). Although the localization problem and the SPEB were formulated and derived directly from the received waveforms, it can be shown that localization can be decomposed into the following two \textit{independent} steps, without any loss in optimality \([46]\): (a) the ML estimation of the MPC ToAs from the received waveforms, and (b) MAP estimation of the target location using the ToAs/ranges estimated in (a). As a result, there has been a lot of research that has focused on solving each of these problems individually. In the next section, we provide an overview of ToA estimation techniques in the presence of multipath.

Subsequently, for MAP estimation of the target location, we have seen above that if no prior IP information is available, then multipath is an impediment to localization, as (a) no further reduction in the SPEB is possible from the ToAs of the IPs, and (b) the accuracy of the DP ToA estimation can be corrupted by MPC overlap. Since it is sufficient to use only the ToAs of DPs for localization, it is important to identify the anchors having LoS to the target. This has resulted in a number of techniques being developed for LoS detection for both active and passive localization, which are reviewed in Section VI. On the other hand, if complete IP information is available in the form of an environment map, then multipath is a blessing as the propagation path of each IP can, in principle, be reconstructed. Specifically, each IP can be converted to a virtual DP using simple ray optics principles, as discussed in Section VII. Consequently, each IP contributes an additional term to the summation in \((33)\), which lowers the SPEB.

### V. ToA Estimation

In this section, we describe the impact of multipath on ToA estimation. To begin with, consider the simple problem of estimating the ToA of a signal, \( s(t) \), between two points, in the presence of noise and the absence of multipath. In this case, the received signal, \( y(t) \), can be modeled as follows:

\[
y(t) = s(t - \tau) + n(t),
\]

where \( \tau \) is the ToA and \( n(t) \) is the noise process described in Section III. In the presence of Gaussian noise, the ML estimate of \( \tau \) can be expressed as follows:

\[
\hat{\tau}_{\text{ML}} = \arg \max_\tau \left| \int_{-\infty}^{\infty} y(t) s^*(t - \tau) dt \right|.
\]  

The above expression can be interpreted as aligning \( y(t) \) with the template \( s(t) \) and returning the location of the best match, determined by the maximum absolute value of the cross-correlation function, \( C_{ys}(\tau) = \int_{-\infty}^{\infty} y(t)s^*(t - \tau) dt \), as the optimal estimate of \( \tau \). For this reason, \((38)\) is popularly known as the matched filter (MF) or correlation receiver.

For the multipath signal, \( z_i(t) \), in Section IV, let \( \tau_i(p) = [\tau_{1i}(p) \cdots \tau_{Li}(p)]^T \) and \( \alpha_i = [\alpha_{1i} \cdots \alpha_{Li}]^T \) denote the vector of MPC ToAs and amplitudes, respectively. Then, the ML estimate of \( \tau_i(p) \) and \( \alpha_i \) can be expressed as follows \([47]\):

\[
\hat{\tau}_{i,\text{ML}}(p) = \arg \max_{\tau} y(\tau)^H R_i^{-1}(\tau) y(\tau), \tag{39}
\]

and \( \hat{\alpha}_{i,\text{ML}} = R_i^{-1}(\hat{\tau}_{i,\text{ML}}(p)) y(\hat{\tau}_{i,\text{ML}}(p)) \),

\[
\text{where } y(\tau) \triangleq \begin{bmatrix} \int_{-\infty}^{\infty} z_i(t) s_i^*(t - \tau_{1i}) dt \\ \vdots \\ \int_{-\infty}^{\infty} z_i(t) s_i^*(t - \tau_{Li}) dt \end{bmatrix}.
\]

\[
R_i(\tau) = \begin{bmatrix} R_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{Li} \end{bmatrix},
\]

and \( R_{ib} = \int_{-\infty}^{\infty} s_i(t - \tau_a) s_i^*(t - \tau_b) dt, \quad a, b \in \{1, \ldots, Li\} \).

The expression in \((39)\) is the extension of the MF receiver for the multipath case. The matrix, \( R_i(\tau) \), in \((42)\) is known as the autocorrelation matrix of \( s_i(t) \), corresponding to \( \tau_i \), and is, in general, not diagonal, due to MPC overlap. As a result, the estimation of all the ToAs are coupled and the solution for the optimal \( \tau_i \) involves searching over a multi-dimensional space, which is computationally intensive. However, if \( R_i(\tau_i) \) is diagonal, then the coupling across the MPC ToAs no longer exists and each ToA value can be estimated independently. For this to occur, the following condition must be satisfied:

\[
\int_{-\infty}^{\infty} s_i(t) s_i^*(t - \tau) dt = 0, \quad \forall \tau \neq 0. \tag{44}
\]

However, \((44)\) is only satisfied if the width of \( s_i(t) \) is infinitesimally small, which is equivalent to \( s_i(t) \) having infinite bandwidth. Since this is not practical, a suitable alternative is to choose \( s_i(t) \) such that the integral in \((44)\) evaluates to a very small quantity, for all non-zero \( \tau \). Signals that satisfy this condition are said to have low auto-correlation and there exists a huge body of work on the design of such signals for a variety of applications, including localization. However, the mathematical principles involved in the design of these signals is beyond the scope of this paper and the reader is referred to \([48]\) for a detailed exposition.

In spite of using a ranging signal with low auto-correlation, the implementation of the MF receiver in the form given by \((39)\) is not always practical since the number of MPCs, \( Li \),
may not be known beforehand. To overcome this challenge, the MF receiver can be implemented in an iterative manner [49], where at first, the ToA and the amplitude of the MPC with the highest SNR is estimated in the following manner:

$\hat{\tau}_{i1}(p) = \arg \max_{\tau} \int_{-\infty}^{\infty} z_i(t)s^*(t-\tau)dt,$ \hspace{1cm} (45)

$\hat{\alpha}_{i1} = \int_{-\infty}^{\infty} z_i(t)s^*(t-\hat{\tau}_{i1}(p))dt,$ \hspace{1cm} (46)

where $l^i_1$ denotes the index of the MPC with the highest SNR at the $i$-th anchor. In LoS conditions, the strongest MPC typically corresponds to the DP; however, this is generally not the case in NLoS situations. Therefore, ranging based on the ToA of the strongest MPC produces large errors [50] and the resulting in NLoS situations. Therefore, ranging based on the ToA of the $i$-th anchor. In LoS conditions, the strongest MPC typically corresponds to the DP; however, this is generally not the case in NLoS situations. Therefore, ranging based on the ToA of the strongest MPC produces large errors [50] and the resulting localization MSE is far from the SPEB. The ToAs of the other MPCs can be estimated using successive interference cancellation; for instance, the ToA of the second strongest peak can be estimated in the following manner, by subtracting the strongest MPC from $z_i(t)$:

$\hat{\tau}_{i2}(p) = \arg \max_{\tau} \int_{-\infty}^{\infty} [z_i(t) - \hat{\alpha}_{i1} s(t-\hat{\tau}_{i1}(p))]s^*(t-\tau)dt,$ \hspace{1cm} (47)

$\hat{\alpha}_{i2} = \int_{-\infty}^{\infty} [z_i(t) - \hat{\alpha}_{i1} s(t-\hat{\tau}_{i1}(p))]s^*(t-\hat{\tau}_{i2}(p))dt,$ \hspace{1cm} (48)

where $l^i_2$ denotes the index of the MPC with the second highest SNR at the $i$-th anchor. This iterative process can be continued until all the MPCs are extracted.\footnote{This is the principle behind the channel estimation algorithm, CLEAN [51]. Typically, the iterative process stops when the SNR of an extracted MPC falls below a certain fraction of the SNR of the strongest MPC. It can be further generalized in the SAGE [52] and RiMax [53] algorithms, which provide ML estimates of all MPC ToAs and amplitudes and even angles of arrival and departures, when multiple observations from different antenna elements in an array are available.}

As seen from Section IV, when there is no prior IP information, it is sufficient to estimate the ToA of only the first MPC and then either use or discard the corresponding range value for location estimation, depending on whether it is a DP or not. In this case, a further reduction in the complexity of the iterative MF receiver can be achieved by restricting the upper limit of the integral in (47) to $\hat{\tau}_{i1}(p)$, which results in only the detection of any MPCs that arrive earlier than the strongest MPC [54].

The MF receiver involves sampling the received signal at Nyquist rates, which may require expensive clock circuitry for wideband ranging signals. A popular sub-Nyquist ToA estimator is based on the energy detection (ED) receiver, which computes the energy of $z_i(t)$ every $T_b$ seconds (where $T_b$ is less than the Nyquist rate $1/(2\beta)$, for $\beta$ given by (36)) and a MPC is detected if the energy in a delay bin exceeds a threshold, $\eta_{\text{ED}}$ [55]–[58]. A detailed analysis of ToA estimation techniques for ED receivers can be found in [59].

Alternatively, a threshold version of the MF receiver can be implemented, wherein an MPC is detected whenever the magnitude of the cross-correlation function, $C_{z_is}(\tau)$, given in (49), exceeds a threshold. These threshold-based ToA estimators are attractive due to their simple implementation and their performance, which is threshold dependent, is analyzed in [60].

$C_{z_is}(\tau) = \int_{-\infty}^{\infty} z_i(t)s^*(t-\tau)dt.$ \hspace{1cm} (49)

VI. LoS Detection

In the absence of any prior IP information, the SPEB is solely a function of the DPs, as seen in Section IV. In this case, the MAP and ML estimates of the target location coincide and from (6), target localization can be reduced to a least-squares minimization problem in the following manner:

$\hat{p} = \arg \min_p \sum_{i=1}^{M_a} \delta_i(p)(c_t_{i1}(p) - \|p - p_i\|^2), \hspace{1cm} (50)$

where $\delta_i(p)$ denotes the estimate of $\delta_i(p)$, defined in (34). The importance of LoS/NLoS detection is evident in (50) and in this section, we provide a summary of the techniques investigated in the literature to solve this problem.

A. Active Localization

After estimating the MPC ToAs and amplitudes, the channel impulse response as seen by the $i$-th anchor can be approximated as follows:

$\hat{h}_{ic}(t) \approx \sum_{l=1}^{L_{ac}} \delta_{il} \delta_D(t - \tau_{il}(p)). \hspace{1cm} (51)$

The statistical properties of many features of $\hat{h}_{ic}(t)$ (e.g., total received power, mean excess delay etc.) vary depending on the existence (or the lack thereof) of LoS between the $i$-th anchor and the target; for instance, the total received power is relatively lower in NLoS scenarios, in general. This is the basis of a number of LoS/NLoS detection techniques reported in the literature. In general, let $\gamma_i = [\gamma_{i1} \cdots \gamma_{iN}]$ denote a vector of features extracted from $\hat{h}_{ic}(t)$. Then, the following tools can be used to detect LoS/NLoS:

1. **Hypothesis testing:** In this case, the LoS/NLoS detection problem can be cast as a likelihood ratio test in the following manner [61]:

$f(\gamma_{i1}|\delta_i(p) = 1) \overset{\text{LoS}}{\gtrsim} \epsilon, \hspace{1cm} (52)$

where $\epsilon$ is a suitable threshold (e.g., for Bayesian hypothesis testing, $\epsilon = \mathbb{P}(\delta_i(p) = 0)/\mathbb{P}(\delta_i(p) = 1)$, where $\mathbb{P}(\delta_i(p) = 1)$ and $\mathbb{P}(\delta_i(p) = 0)$ denote the prior LoS distribution between the $i$-th anchor and $p$ [27]). Typically, the conditional pdfs of $\gamma_i$ under LoS and NLoS conditions are obtained by fitting a suitable distribution to measurement/simulation data. However, since multidimensional pdfs are hard to estimate, a common, but unrealistic, assumption in the literature is to assume
that the different features are statistically independent, in which case (52) reduces to the following rule:
\[
\prod_{j=1}^{N} \frac{f(\gamma_{ij}|\delta_i(p) = 1)}{f(\gamma_{ij}|\delta_i(p) = 0)} \approx \frac{\text{LoS}}{\text{NLoS}}. \tag{53}
\]

2. Non-parametric techniques: In this case, a portion of the measurement data is used to train a non-parametric classifier, such as a support vector machine (SVM) or an artificial neural network (ANN), for detecting LoS/NLoS. An advantage of this approach is that the statistical dependence among the features is inherently captured.

Some of the common signal features that have been used for LoS/NLoS detection are:

a) Mean excess delay: It is an indicator of how quickly the signal decays after the MPC corresponding to the LoS path. A small value is indicative of LoS.\[
\tau_m = \left( \int_{-\infty}^{\infty} t \hat{h}_{ac}^i(t)^2 dt \right) / \left( \int_{-\infty}^{\infty} \hat{h}_{ac}^i(t)^2 dt \right).\tag{54}
\]

b) Root mean square (RMS) delay spread: This metric is the standard deviation of the excess delay, relative to the LoS component and like \( \tau_m \), a small value is a strong indicator of LoS.\[
\tau_{rms} = \left( \int_{-\infty}^{\infty} (t - \tau_m)^2 \hat{h}_{ac}^i(t)^2 dt \right) / \left( \int_{-\infty}^{\infty} \hat{h}_{ac}^i(t)^2 dt \right).\tag{55}
\]

c) Amplitude kurtosis: This is a measure of the peakiness of \( \hat{h}_{ac}^i(t) \), with a large value being indicative of LoS conditions.\[
\kappa_{[\hat{h}_{ac}^i]} = \frac{\text{E}[(\hat{h}_{ac}^i(t) - \mu_{[\hat{h}_{ac}^i]})^4]}{\sigma_{[\hat{h}_{ac}^i]}^4},\tag{56}
\]
where \( \mu_{[\hat{h}_{ac}^i]} \) and \( \sigma_{[\hat{h}_{ac}^i]} \) denote the mean and standard deviation of \( \hat{h}_{ac}^i(t) \).

d) Total received power: \[
P_{ac} = \int_{-\infty}^{\infty} \hat{h}_{ac}^i(t)^2 dt.\tag{57}
\]

e) Maximum signal amplitude: \[
h_{ac}^{max} = \max |\hat{h}_{ac}^i(t)|. \tag{58}
\]

f) Rise time: A small rise time is strongly correlated with the presence of LoS.\[
t_{rise} = t_H - t_L, \tag{59}
\]
where \( t_L = \min \{ t : |\hat{h}_{ac}^i(t)| \geq \mu \sigma_n \} \) \tag{60}
and \( t_H = \min \{ t : |\hat{h}_{ac}^i(t)| \geq \mu H h_{ac}^{max} \}. \tag{61}
\]

For suitably chosen scaling factors, \( \mu_L (> 1) \) and \( \mu_H (< 1) \), \( t_L \) denotes the earliest instant when the magnitude of \( \hat{h}_{ac}^i(t) \) exceeds the noise floor (given by the standard deviation of the noise, \( \sigma_n = \sqrt{N_0 \beta} \)) by a factor of \( \mu_L \). Similarly, \( t_H \) denotes the earliest instant when the magnitude of \( \hat{h}_{ac}^i(t) \) reaches a fraction, \( \mu_H \), of the maximum signal amplitude, \( h_{ac}^{max} \).

g) ToA of the first MPC: \[
\hat{\tau}_{1f}^ac(p) = \frac{2}{c} ||p - p_i|| + n_i, \text{ where } n_i \sim N(0, N_0 \beta). \tag{62}
\]

We now present a summary of the significant results on the LoS/NLoS detection problem:

1) Hypothesis Testing:
- In [62], [63], \( \kappa_{[\hat{h}_{ac}^i]} \), \( \tau_{rms} \) and \( \tau_m \) were used as features and their pdfs were found to be well approximated by the log-normal distribution, with different parameters for the LoS and NLoS cases, based on the histograms generated from realizations of the IEEE 802.15.4a channel model. Similarly, in [64], \( \kappa_{[h_{ac}]} \), \( \tau_{rms} \) and \( h_{ac}^{max} \) were each approximated to a log-normal distribution, based on measurements conducted in an indoor residential environment.
- In [65], the features used were \( \tau_m \), \( P_{ac} \) and a hybrid metric, \( \hat{\xi}_{hyb} = -P_{ac} \tau_m \). Based on extensive simulations in an indoor office environment, \( \tau_m \) was modeled as a normal random variable, with a different mean and variance for the LoS and NLoS cases, while the pdfs of \( P_{ac} \) and \( \hat{\xi}_{hyb} \) were each approximated by a Weibull distribution, again with different parameters for the LoS and NLoS cases.
- In [66], \( P_{ac} \), \( \tau_{rms} \) and \( \hat{\tau}_{1f}^ac(p) \) were considered. Based on an extensive measurement campaign, \( \hat{\tau}_{1f}^ac(p) \) was modeled as a Gaussian random variable for the LoS case, and as a Gaussian-exponential mixture for the NLoS case (this model for \( \hat{\tau}_{1f}^ac(p) \) has also been considered in [62], [63], [67]), while the pdfs of \( P_{ac} \) and \( \tau_{rms} \) were approximated by a log-normal and a one-sided Gaussian distribution, respectively, with different parameters for LoS and NLoS cases.

In particular, the authors claim that \( \tau_{rms} \) is relatively more effective at detecting LoS/NLoS than the other two features.
- In [68], LoS detection techniques for the ED receiver were investigated. For a collection of energy samples, features such as sample variance, sample skewness and maximum sample amplitude were found to be highly correlated to the existence of LoS.

2) Non-parametric Techniques:
- In [69], \( \tau_{rms} \) and \( \kappa_{[\hat{h}_{ac}^i]} \) were used for LoS/NLoS detection, but instead of fitting their distributions to a parametric pdf, a portion of the measured signals was used as training data to obtain empirical pdfs for \( \tau_{rms} \) and \( \kappa_{[\hat{h}_{ac}^i]} \) under both LoS and NLoS conditions, which were then used to implement the likelihood ratio test, given in (52)-(53).

- Similarly, in [70], the pdf of \( \hat{\tau}_{1f}^ac(p) \) was empirically obtained using Parzen window estimation and its distance to a normal distribution, measured in terms of Kullback-Leibler (KL) divergence, was used as a criterion for determining LoS/NLoS.
- In [71], SVMs were trained for LoS identification using \( \kappa_{[\hat{h}_{ac}^i]} \), \( \tau_{rms} \), \( \tau_m \), \( P_{ac} \), \( t_{rise} \), \( h_{ac}^{max} \) and \( \hat{\tau}_{1f}^ac(p) \) as the features, and in [72], regression based on SVMs and Gaussian processes were used on the same features to
Additionally estimate the excess delay, $h_{ac}^{e}$, in the case of NLoS and mitigate the ranging error.

- In [73], $\tau_m$, $P_r^{ac}$ and a hybrid metric, $\xi_{hyb} = -P_r^{ac} \tau_m$ were used as features, similar to [65]. However, instead of a likelihood ratio test, LoS/NLoS detection was performed using an ANN where the inputs were the likelihood ratios of the features.

In addition to the above techniques, the weighted-least squares formulation in (50) has also been used as the basis for an alternate class of LoS/NLoS detection techniques involving $\hat{\tau}_{ac}^i(p)$. It is easily seen that (50) is a special case of the following problem:

$$\hat{p}_{wls} = \arg \min_p \sum_{i=1}^{M_a} w_i (c^2_{\tau,ac}(p) - \|p - p_i\|^2),$$

(63)

where $w_i$ is the weight assigned to the ToA measurement from the $i$-th anchor. In particular, if all the weights are equal to one and the $i$-th anchor has LoS to the target, then the $i$-th residue, $(c^2_{\tau,ac}(p) - \|p_{wls} - p_i\|^2)$, is a random variable with a central chi-squared distribution. On the other hand, if the $i$-th anchor does not have LoS to the target (i.e., $\hat{\tau}_{ac}^i(p)$ corresponds to an IP), then the $i$-th residue has a non-central chi-squared distribution. This principle was used in [74] to jointly identify the LoS anchors and localize the target. Similarly, in [75], the median of the residues was used as a metric to identify the NLoS anchors. Rather than using (52) and (53) to make a hard decision on the presence of LoS/NLoS, the likelihood ratios can alternately be used as weights in (63). As a result, anchors having LoS to the target have larger weights than those that are in NLoS condition. This approach was considered in [62], [63] and [76], where instead of likelihood ratios, the ratio between the energy of the first MPC (i.e., $|\hat{a}_{ac}^1|^2$) and that of strongest MPC (i.e., $|h_{max}^{ac}|^2$) was used for the weights.

For equally weighted measurements, an efficient solution to (50) or (63) that is robust to IPs and does not require prior NLoS detection was proposed in [77], [78], based on the principle of projecting on to convex sets, while an alternative approach based on semi-definite programming (SDP) was investigated in [79]. On the other hand, if LoS detection is done beforehand, then the NLoS measurements can be used to restrict the feasible region for the target location, as demonstrated in [80], where (63) was cast as a linear program, by replacing the sum of squared errors in the objective function with the sum of absolute errors instead. For the special case when the target lies within the convex hull of the anchors, a distributed algorithm based on Caratheodory’s theorem [81] was proposed in [82]. For a comprehensive survey of the techniques developed to solve (50), we refer the reader to [83].

B. Passive Localization

For the passive localization of a single target, the aforementioned LoS/NLoS detection techniques can still be used. However, these techniques are not very effective when multiple targets are present, since the estimated impulse response, $h_{ac}^{en}(t)$, can have both LoS and NLoS characteristics, depending on the number of targets in LoS and NLoS conditions, with respect to the $i$-th anchor. Since the number of targets is typically unknown (e.g., number of intruders in a building), passive localization involves the following steps:

- Identifying the number of targets from $\{\hat{z}_{r_{ac}}^i(t) : 1 \leq i \leq M_a\}$.
- If multiple targets have been detected, then identifying the MPCs corresponding to each target (i.e., data association).
- Detecting the presence of LoS/NLoS for each target-anchor pair.
- Localizing each target using the DPs corresponding to it.

Fortunately, these steps can be carried out jointly. The main principle that is exploited is that the DPs corresponding to a particular target intersect at a point (i.e., the target location), whereas the IPs, in general, do not. The following two cases have been investigated in the literature, which are described below:

1. For a distributed MIMO radar, where each TX and RX is equipped with an antenna array [84]:

   In this case, the angle-of-departure (AoD) at the RX and the AoA at the TX can be estimated, in addition to the ToA, for each MPC. It is easily seen that an estimate of the target location can be obtained using any two of these three quantities, as shown in Fig. 5. This gives rise to three location estimates, denoted by $(\hat{x}_{ToA}/\hat{y}_{ToA}/\hat{AoA})$, $(\hat{x}_{AoA}/\hat{y}_{ToA}/\hat{AoD})$ and $(\hat{x}_{AoD}/\hat{y}_{AoA}/\hat{AoA})$, where the notation indicates the pair of quantities used in obtaining each estimate. If these three points are close together, then it is highly likely that the MPC in question is a DP. Otherwise, the MPC is a likely to be an IP. For a DP detected in this manner, an estimate of the target location can be obtained by a weighted linear combination of $(\hat{x}_{ToA}/\hat{y}_{ToA}/\hat{AoA})$, $(\hat{x}_{ToA}/\hat{y}_{ToA}/\hat{AoD})$ and $(\hat{x}_{AoD}/\hat{y}_{AoA}/\hat{AoA})$.

2. Each anchor is equipped with only one antenna [32], [85]:

   In this case, the angular information contained in an MPC cannot be estimated unambiguously and therefore, only the ToA values are used for joint LoS detection and localization. This is done in an iterative manner, as illustrated in Fig. 6. Suppose, for Anchors 1 and 2,
the received signals $y_1^{pa}(t)$ and $y_2^{pa}(t)$ contain two and one MPCs, respectively. Initially, all circle intersections corresponding to pairs of MPCs from the first two anchors are considered to be prospective target locations (e.g., $q_1$ through $q_4$ in Fig. 6a), and at each such point $q$, we define a blocking likelihood based on the joint distribution of $\{\delta_i(q) : i = 1, 2\}$, which captures the likelihood that there exists a target at $q$ and the circles passing in its vicinity represent DPs to it. The blocking likelihood is an environment specific prior that represents probabilistic multipath information; specifically, regarding the existence of LoS/NLoS, which can be obtained either from measurements or mathematical models [86], [87], or a combination of both. Suppose, for Anchor 3, $y_3^{pa}(t)$ also contains a solitary MPC and its ToA circle passes through the vicinity of $q_1$. In this case, we do not immediately conclude that a target exists at $q_1$, but merely update the blocking likelihood of $q_1$, as shown in Fig. 6b. Similarly, the presence of a target at $q_2$ is not ruled out, but its blocking likelihood is updated to reflect the hypothesis that the DP from the third anchor is blocked. In addition, the newly created circle intersection points (e.g., $q_5$ through $q_8$) are also treated as potential target locations, and their blocking likelihoods are computed as well. This iterative process continues until the MPCs from all the anchors have been taken into account. During this process of storing and updating the likelihoods of all circle intersections, a point is eliminated as a target location if its blocking likelihood exceeds a threshold, $\mu$, at any stage. The points that survive in the end are the estimated target locations and the circles that pass through their vicinity represent the DPs, from which the existence of LoS/NLoS for each anchor-target link can be inferred.

Finally, although the techniques discussed in this subsection were developed specifically for passive multi-target localization, it is evident that they are applicable for active localization as well.

VII. EXPLOITING MULTIPATH

We now discuss how IPs can be used to improve the localization accuracy, by means of a simple example [88], [89]. Consider an environment consisting of a single anchor having LoS to a target, as shown in Fig. 7. In addition to the DP, suppose that the anchor also receives four IPs corresponding to single-bounce reflections off each of the four walls enclosing the anchor and the target. In this case, the target cannot be localized using the DP alone. However, it can be observed that the IP via Wall 1 has the same ToA as that of a DP from a virtual anchor (VA) on the other side of Wall 1. On repeating the same process for the remaining walls, it is easily seen that there are a sufficient number of (real or virtual) anchors to localize the target unambiguously. The positions of the VAs can be easily obtained if the location of the walls are known.

While we only considered single-bounce specular IPs for the sake of simplicity, this principle can be extended to other kinds of IPs, involving multiple reflections as well as diffraction off corners [31][90] and scattering off rough surfaces [91]. In general, if the map of the environment is known (including the material properties of the obstacles), then the propagation path of any IP can, in principle, be determined, which in turn, renders it equivalent to a DP. As a consequence, it is possible to accurately localize and track targets in NLoS conditions, as well [92], [93].

Although the motivation in [89] was to localize a target using only one real anchor, the VA principle is useful even when there are enough real anchors available for localization, since each virtual DP adds an additional term to the summation in (33), thereby increasing the overall ranging information [94]–[96]. To illustrate this, consider a $10m \times 10m$ square room, where a single target is distributed uniformly and anchors are distributed according to a homogeneous Poisson point process [97], independent of the target location. For a given realization, we consider the real anchors, as well as
VAs due to single bounce reflections, as shown in Fig. 7. To cover the spectrum from none to full multipath knowledge, we model the uncertainty in the VA locations as a zero-mean Gaussian random variable with variance $\sigma^2_{VA}$. Thus, $\sigma^2_{VA} = 0$ represents the case when the map of the environment is known exactly, while $\sigma^2_{VA} = \infty$ corresponds to the case when no multipath information is available. From (35), the RII, $\lambda$, of a real anchor can be expressed as $SNR/\sigma^2$, where $\sigma = c\sqrt{1-\chi^2}/(2\sqrt{2}\beta)$ can be interpreted as the standard deviation of the ranging error at unit SNR. Using this interpretation, we model the RII of a virtual anchor as $\lambda = SNR/(\sigma^2 + \sigma^2_{VA})$. For the SNR, we assume an inverse-square law pathloss model, i.e., $SNR = (d_0/d)^2$, where $d_0 = 1m$ is a reference distance and $d$ denotes the range. For this model, the cumulative distribution function (cdf) of the SPEB is plotted in Fig. 8, for varying $\sigma_{VA}$. We observe that even for $\sigma_{VA} = 3\sigma$, the 80th percentile of the SPEB reduces by a factor of 1/2, relative to the case where multipath is not exploited (i.e., $\sigma_{VA} = \infty$). While the gains due to multipath exploitation depend on a number of factors, such as the number of anchors deployed, environment geometry, SNR etc., this toy example serves to illustrate the potential benefits of even partial multipath information. In general, ray tracing can be used to predict the performance of multipath-assisted localization for a given environment geometry.

A. Multipath assisted Tracking Algorithms

In recent years, there has been a lot of progress in the development of localization and tracking algorithms that exploit multipath information [90], [94], [99]–[106]. Many of these algorithms follow a common template, given by Algorithm 1, which we briefly describe using the example in Fig. 7.

**Algorithm 1 Template of Multipath assisted Tracking**

Define state vector, state evolution and the measurement model
Initialize state vector at $t = 0$.

for each $t$ do
Perform data association
Update target position(s) using the EKF or a PF
end for

**1) State vector, State evolution and the measurement model:** A state-space approach is commonly used for navigation and tracking problems, where we distinguish between two distinct cases that influence the structure of the state space:

- **Map of environment known:** In this case, the state vector, denoted by $x^{(t)}$, typically consists only of the variables capturing the target trajectory, i.e., $x^{(t)} = [p^{(t)} \dot{p}^{(t)}]$, where $\dot{p}^{(t)} = [v_x^{(t)} v_y^{(t)}]$ denotes the instantaneous velocity at time slot $t$.

- **Map of environment unknown:** In this case, the VA locations can also be included in the state vector, along with the target related variables. The estimation of VA locations is akin to inferring the map of the environment, which is the well-known simultaneous localization and mapping (SLAM) problem [107]–[109]. A number of recent works have focussed on multipath-assisted SLAM [90], [100], [102], [103], [106], [110], [111].

For both cases, a linear Gaussian Markov model is a common assumption for the evolution of $x^{(t)}$ with respect to $t$, which can be described as follows:

$$x^{(t)} = Fx^{(t-1)} + n^{(t)}, \quad (64)$$
where $n^{(t)} \sim \mathcal{N}(0, Q^{(t)})$ denotes the observation noise vector at time $t$. For the example in Fig. 7, $F$ has the following structure, assuming constant velocity between successive time slots:

$$ F = \begin{bmatrix} I_2 & \Delta t I_2 \\ 0_2 & I_2 \end{bmatrix}, $$

where $\Delta t$ is the slot duration. Similarly, in multipath-assisted SLAM problems, the VA locations are typically stationary; hence, the submatrix in $F$ governing the evolution of the VA locations equals the identity matrix\(^{13}\). For the measurement model, let $N_s \geq M_s$ denote the total number of anchors, where $p_{M_s+1}, \ldots, p_{N_s}$ denote the VA locations. The measured signal, $z_i(t)$, at the $i$-th anchor ($i = 1, \ldots, N_s$) can be simplified as follows:

$$ z_i(t) = \alpha_i s_i \left( t - \frac{2}{c} \| p_i - p^{(t)} \| \right) + \eta_i(t). \quad (66) $$

Since $z_i(t)$ is a non-linear function of $p^{(t)}$, the measurement vector $z^{(t)} = [z_1^{(t)} \cdots z_{N_s}^{(t)}]^T$, consisting of the vector representations $z_i^{(t)}$ of $z_i(t)$, can be expressed in terms of the state vector, $x^{(t)}$, by some non-linear vector function $h(\cdot)$, as follows:

$$ z^{(t)} = h(x^{(t)}) + w^{(t)}, \quad (67) $$

where $w^{(t)} \sim \mathcal{N}(0, R^{(t)})$ denotes the additive noise vector at time $t$.

2) Data Association: Before estimating $p^{(t)}$ from (64) and (67), it is important to map the received MPCs to the corresponding VAs. This is especially challenging in SLAM problems, where the VA locations also need to be estimated as part of the state vector. Further, the total number of anchors (real and virtual) may not always equal the number of observed MPCs, due to blockage or low SNR. Apart from the iterative method described in Section VI-B, additional data association techniques have been studied in [112]–[115].

3) Location updates using the EKF or a PF: As shown in (28), estimating $x^{(t)}$ involves the estimation of the posterior distribution, $f(x^{(t)}|z^{(1:t)})$. For linear Gaussian state and measurement models, $f(x^{(t)}|z^{(1:t)})$ is also Gaussian and thus, completely characterized by its mean vector, $\hat{m}^{(t)}$, and covariance matrix, $\hat{P}^{(t)}$, which are iteratively estimated by the Kalman filter. However, since the measurement model given by (67) is generally non-linear in $x^{(t)}$, $f(x^{(t)}|z^{(1:t)})$ need not be Gaussian. However, the EKF approximates $f(x^{(t)}|z^{(1:t)})$ as a Gaussian, with mean $\hat{m}^{(t)}$ and covariance matrix $\hat{P}^{(t)}$, by linearizing (67) about $\hat{x}_i$ using a first-order Taylor series expansion [116], as given below:

$$ \hat{H}(x^{(t)}) = \begin{bmatrix} \frac{\partial}{\partial x_{i1}} h(x^{(t)}) & \cdots & \frac{\partial}{\partial x_{iN}} h(x^{(t)}) \end{bmatrix}, \quad (68) $$

where the columns of $\hat{H}(x^{(t)})$ are the partial derivatives $h(x^{(t)})$ with respect to the components of $x^{(t)}$. The recursive update equations for $\hat{m}^{(t)}$ and $\hat{P}^{(t)}$ are provided in Algorithm 2.

On the other hand, a PF approximates $f(x^{(t)}|z^{(1:t)})$ by a discrete distribution, supported on $N_s$ particles $\{\hat{x}_i^{(t)} : i = 1, \ldots, N_s\}$, as follows [116]:

$$ f(x^{(t)}|z^{(1:t)}) \approx \sum_{i=1}^{N_s} w_i^{(t)} \delta(x^{(t)} - \hat{x}_i^{(t)}), \quad (69) $$

where $\delta(.)$ denotes the discrete delta function and $\sum_{i=1}^{N_s} w_i^{(t)} = 1$. The accuracy of (69) is a function of $N_s$, the choice of particles $\{\hat{x}_i^{(t)} : i = 1, \ldots, N_s\}$ and their weights (probabilities) $\{w_i^{(t)} : i = 1, \ldots, N_s\}$. A number of PF variants have been analyzed in the literature, which vary in their choice of particles and weights. A relatively simple variant for ToA-based tracking is the sequential importance sampling (SIS) PF, which is described in Algorithm 3. For a more detailed mathematical treatment of other PF variants along with implementation issues, we refer the reader to [116].

Algorithm 2 EKF for ToA-based tracking

```plaintext
for each $t$ do
    $m^{(t|t-1)} = F \hat{m}^{(t-1)} \quad \triangleright \text{Predicted value of } \hat{m}^{(t)}$
    $P^{(t|t-1)} = Q^{(t-1)} + FP^{(t-1)}F^T \quad \triangleright \text{Predicted value of } \hat{P}^{(t)}$
    $\hat{m}^{(t)} = m^{(t|t-1)} + K^{(t)}(z^{(t)} - h(m^{(t|t-1)})) \quad \triangleright \text{Update step to obtain } \hat{m}^{(t)}$
    $\hat{P}^{(t)} = P^{(t|t-1)} - K^{(t)}H(m^{(t|t-1)})P^{(t|t-1)} \quad \triangleright \text{Update step to obtain } \hat{P}^{(t)}$
    where $K^{(t)} = P^{(t|t-1)}(H(m^{(t|t-1)}))^T(S^{(t)})^{-1}$ and $S^{(t)} = H(m^{(t|t-1)})(H(m^{(t|t-1)}))^T + R^{(t)}$
end for
```

Algorithm 3 PF for ToA-based tracking

```plaintext
for each $t$ do
    for $i = 1 : N_s$ do
        Generate $\hat{x}_i^{(t)} \sim f(x^{(t)}|z^{(1:t)}) \sim \mathcal{N}(F\hat{x}_i^{(t-1)}, Q^{(t)})$
        $w_i^{(t)} = f(z^{(t)}|\hat{x}_i^{(t)})$
        $w_i^{(t)} = \exp[-(z^{(t)} - h(\hat{x}_i^{(t)}))^T(R^{(t)})^{-1}(z^{(t)} - h(\hat{x}_i^{(t)}))]/(2\pi)^{N/2}|\det(R^{(t)})|^{1/2}$
    end for
    $w_i^{(t)} = w_i^{(t)}/\sum_{i=1}^{N_s} w_i^{(t)}$
end for
```

The benefits of multipath exploitation are not restricted to ToA-based localization alone. The VA principle was employed in [117], [118] to analyze the accuracy gain for AoA-based localization, while a similar analysis was carried out for joint ToA/AoA based localization in [119], [120]. Multipath-assisted RF fingerprinting was investigated in [121], [122], while the feasibility of localization using the phase of the MPCs (as opposed to just their delays) was studied...
Finally, the concept of multipath exploitation has found widespread use in radar imaging as well, which can be viewed as a special case of joint AoA/ToA-based localization. In imaging applications, a beam is transmitted by an antenna array (real or virtual) in a number of directions and the presence as well as proximity of a target in a given direction is determined by the round-trip time of the DP component, which is mapped to a pixel value in the radar image. In through-the-wall radar imaging (TWRI) applications, apart from the DP component, there are IPs from reflections off other walls as well, which manifest themselves as ghost images around the vicinity of the walls, leading to high false alarms. Since the location of a ghost image is a function of the target and anchor positions, as well as the surrounding geometry, a number of recent works have focused on techniques to associate ghost images with the true target locations, which serves the dual purpose of eliminating false alarms and improving the signal to clutter ratio by signal averaging [124]–[135].

Finally, while specular MPCs contain useful location information, the received signal at an anchor may also contain diffuse MPCs, which are typically caused by scattering of rough surfaces [112], [113]. Unlike specular MPCs, the diffuse MPCs are, in general, not resolvable and act as interference in estimating the ToAs of the specular MPCs. Diffuse MPCs can be treated as the superposition of multiple irreversible independent and identically distributed signal returns; hence, the central limit theorem can be invoked to model them as a Gaussian distribution. A shadowing model for diffuse MPCs is used to model the multipath statistics. In particular, in the absence of any IP information, it was shown that only the DPs contain useful location information and multipath is in fact, a hindrance to accurate localization. On the other hand, if complete IP information is available (i.e., the propagation paths of all MPCs are known), then each IP is equivalent to a direct path from a VA and hence, multipath is a blessing, as the spatial diversity it offers can be exploited to improve the localization accuracy. However, as seen in Section VI-B and Section VII, even partial multipath information, whether in terms of the blocking likelihood or an uncertainty in the VA locations, can still be useful in deriving better algorithms and performance bounds for localization systems.

VIII. CONCLUSION

In this paper, we provided a survey of the existing literature on the impact of multipath on the accuracy of ToA-based localization. To provide a common framework in which to understand the different works, we first formulated target localization as an MAP estimation problem, where the distribution of the amplitudes and delays of the IPs acts as a prior. Under this framework, we defined the SPEB, which provided a lower bound on the achievable accuracy and depended, among other things, on the extent of prior knowledge about the multipath statistics. In particular, in the absence of any IP information, it was shown that only the DPs contain useful location information and multipath is in fact, a hindrance to accurate localization. On the other hand, if complete IP information is available (i.e., the propagation paths of all MPCs are known), then each IP is equivalent to a direct path from a VA and hence, multipath is a blessing, as the spatial diversity it offers can be exploited to improve the localization accuracy. However, as seen in Section VI-B and Section VII, even partial multipath information, whether in terms of the blocking likelihood or an uncertainty in the VA locations, can still be useful in deriving better algorithms and performance bounds for localization systems.

REFERENCES


