Cooperative Relaying with Imperfect Channel State Information

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***Abstract*— We consider relay cooperation with imperfect channel state information (CSI) in the downlink of wireless networks. In particular, we consider a two-phase transmission where in the first phase the base station broadcasts information to the relays; the relays decode the data fully or partially depending on the transmission rate and the quality of their corresponding communication links. During the second phase, the relays cooperate by jointly beamforming information to multiple users given that channel mean and covariance are available at the transmitter side. The goal is to optimize the total network throughput (taking into account both transmission phases) by proper choice of the transmission rates, cooperation architecture and beamforming transmit vectors from the relays. The key contribution of this paper lies in the consideration of the impact of CSI imperfections in such a system. We first formulate the problem of finding the optimum throughput, which is not amenable to analytical solution. We therefore derive a suboptimum adaptive beamforming strategy that maximizes a derived upper bound on the average system throughput. Even though the relays have imperfect CSI, it is shown that relay cooperation can significantly improve the overall system throughput.**

I. Introduction

Relays have recently drawn great attention as a means for improving the range and spectral efficiency of cellular communications systems. They are of interest from a theoretical point of view [1], [2] and because of their use in standardized systems like the Wimax (802.16j) standard [3].

The performance of relay systems can be further improved by cooperation. Most of the literature on cooperative relays has focused on multiple relays cooperating to reach a single user or on one relay reaching multiple users [4], [5]. However, higher gains can be achieved if relays cooperate with each other for reaching multiple users.

In a recent paper [6], some of us investigated two-phase relay cooperation where in the first phase the base station (BS) broadcasts messages intended for different users to the relays. In a second phase the relays perform joint beamforming (linear precoding) in order to forward the information. It was shown that the overall system throughput can be significantly improved. However, this investigation assumed that the relays had perfect channel state information (CSI) for the links from the relays to the mobile stations (MSs), which is not easy to achieve in practice.

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Imperfections in channel knowledge at the transmit nodes arise from imperfect feedback, delay and latency, fast changing channels due to mobility, etc. [7]. *Imperfect transmit CSI*, i.e., knowledge about the CSI *statistics* only, has been considered in the literature in the context of point-to-point MIMO channels as well as MIMO broadcast channels (MIMO BC). Reflecting different amounts of channel knowledge, models assume knowledge of Channel Mean Information (CMI), Channel Covariance Information (CCI) and Random Vector Quantization (RVQ) [8]–[10].

The key contribution of our paper (and the key difference from [6]) is an investigation of the impact of such imperfect transmit CSI on two-phase cooperative relay schemes. Under this assumption, we propose an adaptive beamforming strategy that maximizes an upper bound on the overall system throughput by proper choice of transmit rates and cooperation architectures throughout the different phases of transmission. Our approach involves two major steps: First we derive an upper bound (UB) on the overall system throughput. The derived UB is a function of the expected transmission rates for phases 1 and 2. Second, we derive an upper bound on the expected transmission rates of phase 2 for general linear precoding strategies to further upper bound the derived total throughput expression. Our strategy is to select the scheme that maximizes the total UB from a set of candidate strategies. Even though suboptimal, it is shown that this strategy can improve the overall system throughput compared to non-cooperative schemes, and, as a matter of fact, gives a throughput that is fairly close to the optimum (genie-aided) throughput. We also emphasize the importance of asymmetric cooperation schemes, i.e., situations in which not all relays decode the signals intended for all users. While such asymmetric schemes were shown to be highly beneficial for the perfect CSI case in [6], it is remarkable that they retain their advantages for the imperfect CSI case.

The rest of the paper is organized as follows. In Section II we describe the problem setup and the fundamentals of asymmetric relay cooperation. We describe our proposed cooperative strategy in Section III. We also provide two upper bounds on the overall system throughput and the transmission rates of the second transmission phase for the considered configuration. Simulation results are presented in Section IV. Finally, conclusions are provided in Section V. We use the following notation: matrices and vectors are denoted by bold upper- and lower-case letters, respectively. Superscript T denotes transposition; superscript H Hermitian transposition; * complex conjugation; $tr(A)$ is the trace of a matrix A.

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II. System description and Problem Setup

Figure 1 shows the fundamental setup we consider. Relays receive data intended for different users from channels characterized by a channel matrix $\mathbf{G}$ whose entries are the complex amplitude gains from the BS antennas to the relay antennas. Depending on the transmission rates, transmission durations of the different messages and the channel matrix $\mathbf{G}$, some or all relays can decode the different messages. Decoded data are then transmitted to multiple users through joint beamforming (linear precoding) from the relays. For simplicity of exposition we assume single-antenna transmitters and receivers, 2 relays and 2 mobile stations. Extensions to multiple antenna systems as well as larger number of users is straightforward.

Let $\mathbf{H} = [\mathbf{h_1}\ \mathbf{h_2}]^T$ be the channel matrix between the relays and the users, where $\mathbf{h_i}$ denotes the channel vector from the relays to the i-th user. The received signal y_i at MS i can be written as:

$$y_i = \mathbf{h_i}^T \mathbf{x} + w \tag{1}$$

where $\mathbf{x}$ is a vector containing the transmit signals of the relays, and w is unit variance complex circularly symmetric AWGN, i.e. $w \sim CN(0, 1)$. In this work we consider linear precoding at the relay terminals so that the transmitted signal $\mathbf{x} = \sum_i \mathbf{T_i} b_i$, where $\mathbf{T_i}$ represents the beamforming vector and b_i the message intended for user i, respectively. We assume that the transmitted signal has to satisfy a total power constraint P, thus $tr(\mathbf{T}\mathbf{T}^H) = P$, where $\mathbf{T}$ is the beamforming matrix $\mathbf{T} = [\mathbf{T_1}\ \mathbf{T_2}]$.

As mentioned in the introduction, the key difference between [6] and the current paper is that [6] assumed that both BS and relays had full knowledge of the BS-to-relays and relays-to-mobiles channels. In the current paper, we assume full channel state information at the *receiving* nodes (CSIR). Furthermore, since the channel $\mathbf{G}$ is essentially static (relays are usually fixed)[1] we assume that $\mathbf{G}$ is perfectly known at the BS. However, due to high mobility of the MSs, limited availability of channel feedback and channel fluctuations, the channel matrix $\mathbf{H}$ is usually not fully available at the relays or the BS and only mean and covariance information is present. In this paper, the relays-to-mobiles channel is modeled as:

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{H_W} \tag{2}$$

where $\mathbf{H_W}$ is a complex white Gaussian random matrix with zero mean. Hence, $\mathbf{H} \sim CN(\bar{\mathbf{H}}, \mathbf{\Sigma})$, where $\bar{\mathbf{H}}$ and $\mathbf{\Sigma} = \alpha\mathbf{I}$ are the channel mean and covariance matrices, respectively. This model can account for various situations: outdated CSI, limited feedback with MMSE estimation at the TX and long term channel statistics only. Depending on the situation, $\bar{\mathbf{H}}$ and Σ have different interpretations [12]. Note also that for some interpretations, the channel shows two different types of randomness: random realizations of the channel, due to movement of physical objects like scatterers (this changes on a timescale determined by the Doppler frequency) and randomness of the CSI, e.g., due to noise, which changes on a timescale of the pilot symbol intervals. However, for the mathematical treatment of Sec. III, such a distinction is not necessary.

Next we describe the transmission strategy in greater detail.
Phase 1: During this phase, the data is transmitted from the BS to the relays. One or two messages might be transmitted, and each relay might be able to decode no, one, or all messages depending on their channel states, the duration of the phase and the transmission rates. For example, if the transmission rate for phase 1, $R^{(1)}$, is equal to the capacity of the stronger relay's link, then the weaker relay will not be able to decode the transmitted message. This results in different possible transmission strategies for phase 1 since different relays can know certain messages but not others. For more details we refer the reader to [6]. Note that asymmetry of message knowledge at the relays, even though suboptimal in the second transmission phase, can possibly lead to overall-throughput gains. This is true since, for certain channel conditions (bad channel from the BS to one of the relays), symmetric knowledge might require such a long phase-1 transmission that the overall throughput is negatively impacted.
Phase 2: In the second transmission phase, the relays jointly beamform the decoded data to the MSs. Beamforming is done based on different message knowledge at the relays. In this work, we use MMSE beamforming noting that other criteria could also be used for choosing second-phase transmit vectors [13]. Note that MSE beamforming in our case is different from conventional beamforming if relays have asymmetric message knowledge. We derive MSE beamforming vectors, with perfect and imperfect CSI, for both symmetric and asymmetric cooperation architectures in the appendix.

With proper combinations of symmetric/asymmetric and single/dual messaging, different transmission strategies are possible. Let $\mathcal{S}$ be the set of all available strategies. If we denote by $|\mathcal{S}|$ the cardinality of the set $\mathcal{S}$, then $|\mathcal{S}| = 8$ in case of a 2-relay, 2-MS system [2].

Let $R_k^{(j)}$ denote the transmission rate to user k in the j-th transmission phase, $j = 1 \ldots 2$. Thus, if n_k represents the total number of bits transmitted to the k-th user, then the overall system throughput, defined as total number of transmitted bits divided by the total time spent in transmission can be written as [6]:

$$th = \frac{n_1 + n_2}{\frac{n_1}{R_1^{(1)}} + \frac{n_2}{R_2^{(1)}} + \max\{\frac{n_1}{\log(1+\gamma_1)}, \frac{n_2}{\log(1+\gamma_2)}\}} \tag{3}$$

where γ_k represents the receive signal-to-interference-plus-noise ratio $SINR$ at user k in the second transmission phase. Obviously, γ_k and the transmission rates $R_k^{(j)}$ depend on the transmission strategy and hence on the beamforming transmit

[1] Changes in the BS-to-relay channel occur only due to movement of scatterers like cars, leaves, etc. The power of the time-variant components is typically much smaller than the power in the time-invariant components [11]

[2] Note that if in addition to MSE, we also include other optimization criteria for the choice of the beamforming matrix T, then $|\mathcal{S}| > 8$.

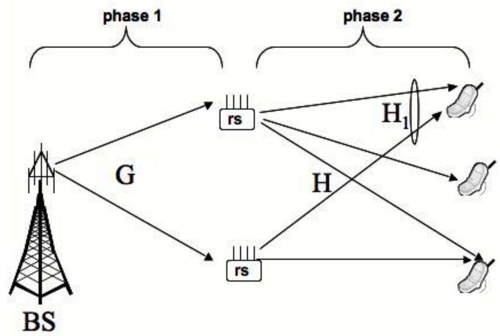

Fig. 1: System architecture

vectors. Note also that for a given channel realization, the throughput is a random variable, where different realizations correspond to different realizations of the CSI. This is in contrast to the case of perfect CSI in [6], where the throughput is a deterministic quantity for a given channel state.

Since the channel matrix $\mathbf{H}$ is not fully known to the transmitting nodes (BS and relays) our goal is to maximize the expected value of throughput $\max E[th]$, where the maximization is over the choice of n_k, $R_k^{(j)}$ and the transmission strategy $s \in \mathcal{S}$ (i.e. single vs dual and symmetry vs asymmetry) subject to power constraints at the BS and relays. The expectation is taken w.r.t. to the randomness in the channel matrix $\mathbf{H}$ conditioned on the available channel statistics. This optimization problem is hard to solve because it is non-convex. We therefore propose an alternative suboptimal strategy that examines the set of possible combinations (i.e. the set $\mathcal{S}$), and picks the best, where "the best" refers to the strategy that maximizes a derived UB on the expected system throughput. Hence, the random variations of throughput are taken into account by optimizing an UB on the expected value of the system throughput which we derive in the following section.

III. Cooperation with imperfect CSI

Our proposed optimization strategy selects the transmission parameters (rates, number of bits, architectures) that maximize the upper bound on the expected system throughput. We show that this strategy can provide significant improvements in the overall system throughput even with imperfect CSI.

A. Upper bound on overall mean system throughput:

Theorem 3.1: For the system shown in Fig. 1 and $x = \frac{n_1}{n_2}$, an upper bound on the expected system throughput is given by:

$$E[th] \leq \frac{x+1}{ax + b + \max\{\frac{x}{E[R_1^{(2)}]}, \frac{1}{E[R_2^{(2)}]}\}}, \quad \forall x \tag{4}$$

with a and b representing the inverse of transmit rates of phase 1 messages 1 and 2, respectively. $E[R_k^{(2)}],\ k = 1, 2$ denotes the expected transmission rates for users 1 and 2 in phase 2.

To prove Theorem 3.1 we first observe that the expected throughput can be written as a convex combination of concave functions. This is followed by the use of Jensen's inequality and careful handling of the derived upper bound for different values of x. Due to space limitations the details of the proof are relegated to [13].

Using arguments similar to [6], it is not hard to show that, in order to maximize the UB, the rates of phase 1, $R^{(1)}$, take values from the set $\{0, C_{BR_1}, C_{BR_2}\}$ (which determines the corresponding values for a and b), where C_{BR_i} denotes the capacity of the link between the BS and relay i. Since the function above is either concave or convex over the intervals $[0, x^*], [x^*, \infty]$, with x^* defined as the ratio of the expected values $x^* = \frac{E[R_1^{(2)}]}{E[R_2^{(2)}]}$, then to maximize the UB we only look at the set of points $x \in \{0,\ \infty,\ x^*\}$.

The algorithm is summarized as follows:

- Compute the MSE beamforming matrices $\mathbf{T}$ for the set of candidate strategies $s \in \mathcal{S}$ consisting of symmetric/asymmetric, single/dual messaging according to the Appendix such that:
  1) $x = \frac{E[R_1^{(2)}]}{E[R_2^{(2)}]}$ for dual messaging
  2) $x = 0$ or ∞ for single user messaging
- The BS chooses the strategy that maximizes an upper bound on the expected throughput (Eq. 4) from the finite set $\mathcal{S}$.

Note that we also replace exact values of the expected rates of phase 2 with corresponding upper bounds in terms of the mean and covariance; which again leads to an upper bound on the expected system throughput. Next we provide a derivation of upper bounds on phase 2 expected transmission rates.

B. Upper bound on phase 2 expected transmission rates $E[R_k^{(2)}]$:

Conditioned on a given channel mean $\bar{\mathbf{H}}$ and a covariance matrix $\mathbf{\Sigma}$ available at the transmitting nodes (BS and relays) the expected value of the rate $R_k^{(2)}$ for user k in the second transmission phase is:

$$E[R_k^{(2)}] = E\left[\log\left(1 + \frac{|\mathbf{h_k}^H \mathbf{T_k}(\bar{\mathbf{H}}, \mathbf{\Sigma})|^2}{1 + \sum_{j \neq k} |\mathbf{h_k}^H \mathbf{T_j}(\bar{\mathbf{H}}, \mathbf{\Sigma})|^2}\right) | \bar{\mathbf{H}}, \mathbf{\Sigma}\right] \tag{5}$$

where K is the total number of users (here for simplicity $K = 2$). This equation assumes "simple" receivers that are affected by interference in the same way as by noise. To simplify notation, we drop the conditioning as it should be clear from the context. Now the expected value of the rate R_k can be simplified as:

$$E[R_k] = E\Big[\log(1 + |\mathbf{h_k}^H \mathbf{T_k}(\bar{\mathbf{H}}, \mathbf{\Sigma})|^2 + \sum_{j \neq k} |\mathbf{h_k}^H \mathbf{T_j}(\bar{\mathbf{H}}, \mathbf{\Sigma})|^2) - \log(1 + \sum_{j \neq k} |\mathbf{h_k}^H \mathbf{T_j}(\bar{\mathbf{H}}, \mathbf{\Sigma})|^2)\Big] \tag{6}$$

Given the channel model in Eq.(2), the first term in the r.h.s. of Eq.(6) can be simply bounded using Jensen's inequality as:

$$\text{Term1} \leq \log\Big(1 + \mathbf{T_k}^H(\bar{\mathbf{h}}_\mathbf{k}\bar{\mathbf{h}}_\mathbf{k}^H + \alpha\mathbf{I})\mathbf{T_k} + \sum_{j\neq k}\mathbf{T_j}^H(\bar{\mathbf{h}}_\mathbf{k}\bar{\mathbf{h}}_\mathbf{k}^H + \alpha\mathbf{I})\mathbf{T_j}\Big) \tag{7}$$

Next we compute a lower bound on the 2nd term on the r.h.s. of Eq.(6) :

$$\text{Term 2} = E[\log(1 + \sum_{j\neq k}|\mathbf{h}_k^H\mathbf{T_j}|^2)] = E[\log(1+y)] \tag{8}$$

Using the Markov inequality,

$$\begin{aligned} E[\log(1+y)] &\geq \max_{a\geq 0} a\Pr[y \geq 2^a - 1] \\ &= \max_{a\geq 0} a[1 - F_y(2^a - 1)] \end{aligned} \tag{9}$$

It is not hard to show that y is the sum of non central χ_2^2 RVs with non-centrality parameter $s_j^2 = |\bar{\mathbf{h}}_\mathbf{k}^H\mathbf{T_j}|^2$, where the variance of the generating Gaussian RVs is $\sigma^2 = \frac{\alpha}{2}\mathbf{T_j}^H\mathbf{T_j}$, that is $\sigma = \sqrt{\frac{\alpha}{2}tr(\mathbf{T_j}\mathbf{T_j}^H)}$.

For the special case where $K = 2$, the distribution of y is a chi-square with 2 degrees of freedom, χ_2^2, i.e.:

$$f_Y(y) = \frac{1}{2\sigma^2}e^{-\frac{s^2+y}{2\sigma^2}}I_0\left(\sqrt{y}\frac{s}{\sigma^2}\right) \tag{10}$$

where $I_0(.)$ is the modified 0-th order Bessel function of the first kind [14]. Hence the CDF of y is:

$$F_Y(y) = 1 - Q_1\left(\frac{s}{\sigma}, \frac{\sqrt{y}}{\sigma}\right) \tag{11}$$

where $Q_1(.,.)$ is the generalized Marcum Q function [14]. Thus, we can lower bound Term 2 as:

$$E[\log(1+y)] \geq \max_{a>0} a\left[Q_1\left(\frac{|\bar{\mathbf{h}}_\mathbf{k}^H\mathbf{T_j}|}{\sqrt{\frac{\alpha}{2}tr(\mathbf{T_j}\mathbf{T_j}^H)}}, \frac{\sqrt{2^a-1}}{\sqrt{\frac{\alpha}{2}tr(\mathbf{T_j}\mathbf{T_j}^H)}}\right)\right] \tag{12}$$

One could solve for a numerically or alternatively we could further lower bound the expression above using known bounds on the Marcum function in [15].
Inserting (7) and (12) into (6) we obtain an UB on the expected phase-2 transmission rate. This UB, together with Eq. (4), provides the desired UB on the mean system throughput.

IV. Simulations

In the following simulations we adopt the CMI model of [8] with noisy analog feedback. The receivers feedback their channel vectors through a noisy feedback channel:

$$\mathbf{H_{FB}} = \sqrt{\beta SNR}\ \mathbf{H} + \mathbf{W} \tag{13}$$

where β controls the quality of the feedback channel. Then the channels are estimated at the transmitters resulting in the CMI model of Eq. (2) with $\bar{H}$ representing the channel estimate and α representing the mean square estimation error.

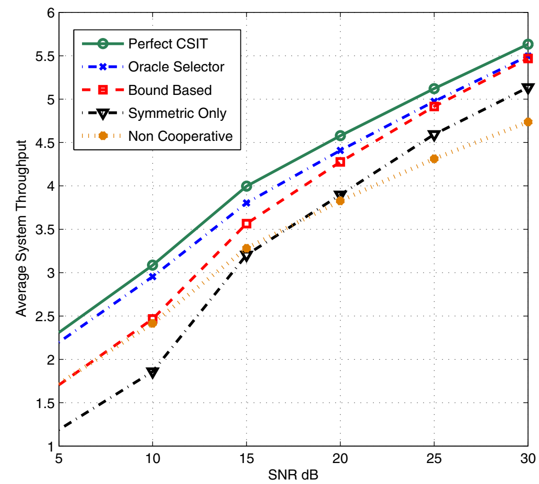

Fig. 2: Average system throughput with different transmission methods

The following figures compare the system throughput of our UB-based adaptive beamforming with two benchmarking strategies: The perfect CSIT (the beamforming vectors are computed based on the true channel realizations and the base station selects an optimal transmission scheme) and the oracle selector strategy (only channel statistics are available for computing the beamforming coefficients, but the BS is genie aided and somehow knows which among the available strategies will achieve the highest throughput based on the true instantaneous CSI, not the statistics). The reason we also consider this oracle strategy is that, even with the statistics based beamforming coefficients, maximizing an UB on the average system throughput does not necessarily mean that the selected strategy would be the best one to actually use. Thus, this oracle strategy provides a tighter UB on the achievable system throughput and hence serves as a benchmarking tool. We also plot (i) the proposed bound-based adaptive beamforming CMI strategy, (ii) a strategy that only employs symmetric cooperation (all messages are known to all relays) and (iii) a non cooperative round robin transmission scheme which alternates between users 1 and 2 while selecting the relay with best channel to the MS. In Fig.2 we compare the achievable throughput of the different aforementioned strategies. The empirical CDF of the system throughput of these strategies is plotted in Fig. 3. Figure 5 shows how often single/dual messaging as well as symmetric/asymmetric cooperation architectures are optimum. Finally, Fig. 4 demonstrates the effect of feedback quality on the total system throughput. From these figures we observe the following:

- Our proposed adaptive technique is able to closely ap-

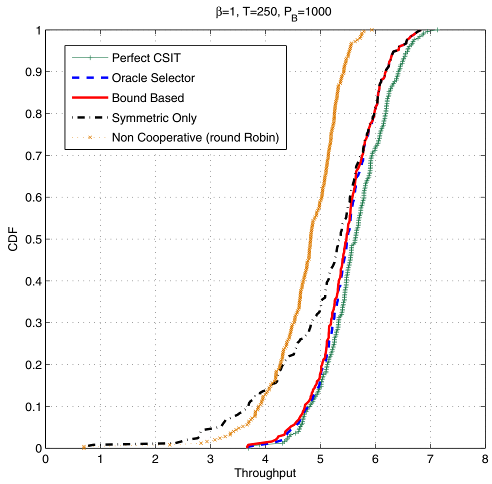

Fig. 3: Cumulative Distribution Function for $\beta = 1$, $SNR = 20\ dB$

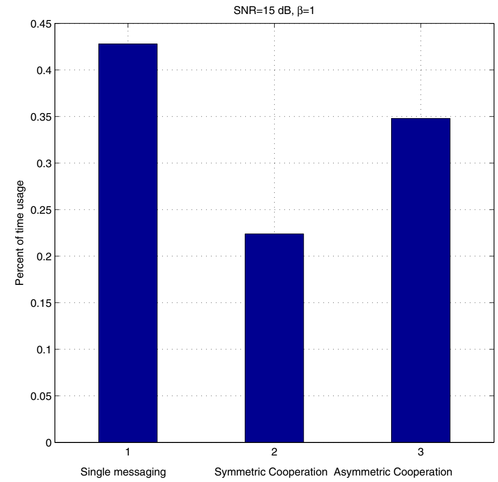

Fig. 5: Usage of various architectures $\beta = 1$

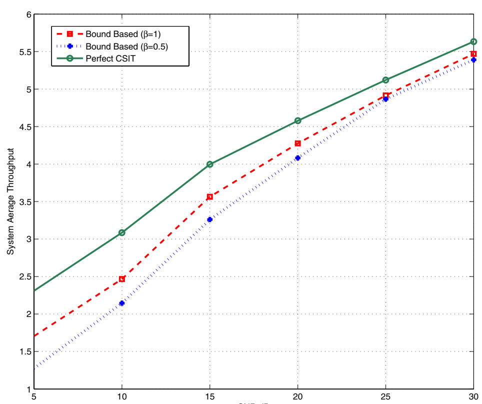

Fig. 4: Throughput for different feedback quality

proach the performance of the perfect-CSI case[3] and is significantly better than the conventional round-robin scheme (Fig. 2 and 3).

- As long as the SNR of the CSI is as good as the SNR of the data (corresponding to $\beta = 1$), the impact of imperfect CSI on the overall throughput is small. This fact can be used as a guideline to optimize feedback for given performance guarantees.
- Even though suboptimal for the second phase of transmission, asymmetric cooperation is important from a total system throughput standpoint. This insight was first gained in [6], and it is remarkable that is remains valid for the imperfect CSI case.

[3]We would like to point out that, since the choice of the beamforming vectors here is suboptimally optimized over a finite set, the perfect CSI case considered here is different from [6] where the beamforming vectors were chosen from a continuous domain to maximize the total system throughput expression. More results on closing this gap can be found in [13].

V. Summary and Conclusions

In this paper we derived an upper bound on the overall mean system throughput for a 2-phase transmission system (BS to relays to users) under the assumption of imperfect CSI. We used this bound to optimize an adaptive beamforming strategy that achieves a performance that, under many practical circumstances, is close to the ideal (perfect-CSI) throughput.

As components of this strategy, we proposed and derived MMSE beamforming filters for the imperfect CSI case and for the asymmetric cooperation architecture (see Appendix) and a transmission strategy that approximately maximizes the sum throughput of the system.

The results derived in this paper show that relay cooperation is a useful strategy even if only imperfect CSI is available. Furthermore, they allow to investigate the impact of the quality of the feedback channel on the system throughput, and thus trade off overhead for the feedback channel with throughput during the actual data transmission.

VI. Acknowledgements

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Appendix

In this appendix we derive the symmetric and asymmetric MMSE linear precoding vectors for perfect and imperfect CSI. We assume that the relays can fully cooperate and have perfect synchronization.

A. MMSE beamforming, asymmetric, perfect CSI

The goal is to minimize the MSE between the received users' signals and the transmitted messages. The result for the symmetric case is similar to [16]. Here we provide the analysis for the asymmetric case where we have to add a set of constraints to force some of the entries of some of the beamforming vectors to zero. The optimization problem can be written as:

$$\begin{aligned} &\min_{\mathbf{T_j}, j=1:K} \sum_{j=1}^{K} MSE_j \\ &\text{s.t.} \;\; tr\left(\sum_{i=1}^{K} \mathbf{T_i}^H \mathbf{T_i}\right) = P \\ &\text{and} \;\; \mathbf{e_l}^T \mathbf{T_j} = 0; \; l \in \mathcal{L}, \; j = n+1 \ldots K \end{aligned} \quad (14)$$

where $\mathbf{e_l}$ is binary vector with ones at the entries that are supposed to be nulled from $\mathbf{T_j}$ since the l-th relay does not have a message to the j-th user. For simplicity let us assume that whenever this situation arises l is only one index, i.e. only 1 relay does not have a message, i.e. $|\mathcal{L}| = 1$.

Writing the Lagrangian and taking the derivative w.r.t. $\mathbf{T_j}^*$ we can show that: for a user k receiving data from all relays (we have n of these) the optimal beamforming vector is given by:

$$\mathbf{T_k} = (\sum_{i=1}^{K} \mathbf{h_i}^* \mathbf{h_i}^T + \lambda_1 \mathbf{I})^{-1} \mathbf{h_k}^* \quad (15)$$

where λ_1 is a Lagrangian multiplier chosen to satisfy the total power constraint. For the users that have entries forced to zero (message not known at some relay station) the beamforming vector becomes:

$$\mathbf{T_j} = (\sum_{i=1}^{K} \mathbf{h_i}^* \mathbf{h_i}^T + \lambda_1 \mathbf{I})^{-1} (\mathbf{h_j}^* - \lambda_j^* e) \quad (16)$$

where the Lagrange multiplier corresponding to the j-th transmit vector λ_j has to satisfy:

$$\lambda_j = -\frac{(\mathbf{A} + \lambda_1 \mathbf{I})_l^{-1} \mathbf{h_j}^*}{(\mathbf{A} + \lambda_1 \mathbf{I})_{l,l}^{-1}} \quad (17)$$

with $\mathbf{A} = \sum_{i=1}^{K} \mathbf{h_i}^* \mathbf{h_i}^T$. We solve for λ_1 numerically and refer the reader to [13] for further details.

B. MMSE beamforming, Imperfect CSI

For the CMI case we minimize the *expected* MSE as follows:

$$\begin{aligned} &\min E_H[MSE] \quad s.t. \quad tr(\mathbf{T}^H \mathbf{T}) = P \\ &\text{and} \;\; \mathbf{e_l}^T \mathbf{T_j} = 0 \;\; \forall \;\; l \in \mathcal{L} \;\; \text{if} \;\; \mathcal{L} \neq \Phi \end{aligned} \quad (18)$$

with $\mathbf{H} \sim N(\bar{\mathbf{H}}, \mathbf{\Sigma})$. For the symmetric architecture, the expected MSE can be written as:

$$\begin{aligned} E_H[MSE] = &tr(2KI) + tr(T^H E[H^H H] T) \\ &- 2Re(tr(E(HT))) \end{aligned} \quad (19)$$

After writing the Lagrangian and taking the derivative w.r.t. $\mathbf{T_j}^*$ we obtain:

$$\mathbf{T_j} = \left(\sum_{i=1}^{K} \bar{\mathbf{h_i}}^* \bar{\mathbf{h_i}}^T + \lambda \mathbf{I} + K\alpha \mathbf{I} \right)^{-1} \bar{\mathbf{h_j}}^* \quad (20)$$

To satisfy the power constraint we can show that λ is the solution of the equation:

$$P = \sum \frac{\gamma_j}{(\gamma_j + \lambda)^2} \quad (21)$$

where $\gamma_j, j = 1 \ldots M$ are the eigenvalues of the hermitian matrix $\mathbf{A}^T = \sum_{i=1}^{K} \bar{\mathbf{h_i}} \bar{\mathbf{h_i}}^H + K\alpha \mathbf{I}$

The result can be easily extended to the asymmetric case by adding the extra zero forcing constraints, for details see [13].

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