Robust resource allocation in wireless localization networks

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***Abstract*—Reliable and accurate position estimation of "agent" nodes is essential for many wireless applications. Typically agents perform position estimation through ranging with respect to "anchor" nodes with known positions. In range-based localization techniques, system accuracy and energy efficiency are affected by both transmit power and signal bandwidth. We thus investigate the joint power and bandwidth allocation (JPBA) problems in wireless localization networks. We first formulate a general optimization model, which is proved to be non-convex. An approximate algorithm based on single condensation method is proposed to solve the problem. We then formulate the robust counterpart of the problem in the presence of uncertainty of the agents' positions. An low complexity method is also proposed to solve the robust JPBA problem. Numerical results validate the accuracy and robustness of the proposed algorithms.**

***Index Terms*—Network localization, resource allocation, robust optimization, geometric programming**

I. INTRODUCTION

In recent years, wireless services and applications based on position information have been widely investigated, such as cellular networks, search and rescue, navigation and intruder detection, etc [1]–[3]. Since the global position system (GPS) may fail under certain conditions such as indoors or in dense urban canyons, wireless network localization is a promising option in most GPS-challenged environments [4]–[6].

There are generally two kinds of nodes in location-aware networks: anchors with known positions and agents with unknown positions (see Fig. 1). In conventional range-based localization techniques, agent tries to determine its position based on the measurements from at least three different anchors (in 2-D localization). Depending on the positioning technique, the angle of arrival (AOA), the received signal strength (RSS) or time of arrival (TOA) information can be used to determine the location of a node [7].

Based on the equivalent Fisher information matrix (EFIM), the fundamental limits of wideband localization have been derived in terms of squared position error bound (SPEB) [8]. It is shown that the localization accuracy is determined by network topology, propagation channel conditions, signal waveforms and transmit power, etc. Since network topology and channel conditions are usually determined by external circumstances, appropriate joint power and bandwidth allocation (JPBA) among wireless nodes is the key tool for resource-restricted location-aware network design (e.g., wireless sensor networks).

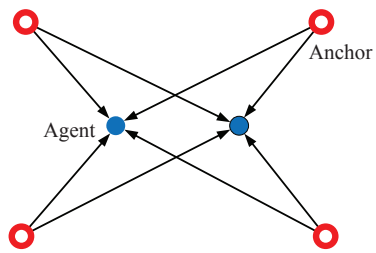

Fig. 1: Wireless localization networks: the agents are located by measurements from anchors

Some work has been carried out on power allocation optimization in wireless localization networks [9], [10]. However, to the best of the authors' knowledge, very limited work can be found about the bandwidth allocation in localization networks. We have proposed an optimal JPBA formulation for cooperative localization network [11], in which an iterative searching (IS) method is given to deal with the underlying non-convex problem. Another problem is that, in most existing power allocation optimization work, agents' positions are assumed to be known beforehand, which is usually unavailable in real localization systems. Therefore, robust resource allocation strategy is required. The authors in [12], [13] use robust optimization to deal with the uncertainties of channel parameters in power allocation problems. Another robust energy efficient method is proposed in [14], in which an asynchronous ranging/localization scenario is considered.

In this paper, we investigate the joint power and bandwidth allocation (JPBA) problems in TOA-based wireless localization networks. We first formulate two optimal JPBA problems, to maximize the localization accuracy and energy efficiency respectively. Then we propose an efficient approximation algorithm based on *single condensation* method to solve the non-convex optimal JPBA problems. We then formulate two robust JPBA counterparts to deal with the uncertainty in agents' positions. Optimal scheduling rules can be drawn based on the results obtained.

II. System model

A. Network settings

Consider a 2-D location-aware network consisting of N_{a} agents to be located and N_{b} anchors with known positions. Agents are able to determine their positions by TOA measurements with anchors. The sets of agents and anchors are represented by $\mathcal{N}_{\mathrm{a}} = \{1, 2, ..., N_{\mathrm{a}}\}$ and $\mathcal{N}_{\mathrm{b}} = \{N_{\mathrm{a}}+1, N_{\mathrm{a}}+2, ..., N_{\mathrm{a}}+N_{\mathrm{b}}\}$ respectively. The position of node k is denoted by $\mathbf{p}_k = [x_k, y_k]^{\mathrm{T}}, k \in \mathcal{N}_{\mathrm{a}} \cup \mathcal{N}_{\mathrm{b}}$. The distance between agent node k and anchor node j is denoted by

$$d_{kj} = \|\mathbf{p}_k - \mathbf{p}_j\|_2 \tag{1}$$

The angle from node k to j is given by

$$\phi_{kj} = \arctan\left(\frac{y_k - y_j}{x_k - x_j}\right) \tag{2}$$

In our system, agents and anchors are assumed to be synchronized, so that one-way ranging is applied for range measurements. Therefore, only anchors are required to *broadcast* signals during localization, while agents are all set as quiet receivers. The power and bandwidth allocation results among anchors are represented as

$$\mathbf{x} := [x_1, x_2, ..., x_{N_{\mathrm{b}}}]^{\mathrm{T}}, \quad \boldsymbol{\beta} := [\beta_1, \beta_2, ..., \beta_{N_{\mathrm{b}}}]^{\mathrm{T}}$$

where x_i and β_i is the power and bandwidth allocated to anchor i, $i \in \mathcal{N}_{\mathrm{b}}$.

B. Positional error bound

As defined in [8], the SPEB is derived from the equivalent Fisher information matrix (EFIM). The definition of SPEB of agent k is

$$\mathbb{E}\{\|\hat{\mathbf{p}}_k - \mathbf{p}_k\|^2\} \geq \mathcal{P}(\mathbf{p}_k) \triangleq \mathrm{tr}\{\mathbf{J}_{\mathrm{e}}^{-1}(\mathbf{p}_k)\} \tag{3}$$

where $\mathbf{J}_{\mathrm{e}}(\mathbf{p}_k)$ is the EFIM of agent k's position obtained by measurements, which can be expressed as

$$\mathbf{J}_{\mathrm{e}}(\mathbf{p}_k) = \sum_{j \in \mathcal{N}_{\mathrm{b}}} \lambda_{kj} \mathbf{q}_{kj} \mathbf{q}_{kj}^{\mathrm{T}} \tag{4}$$

where $\mathbf{q}_{kj} = [\cos(\phi_{kj}), \sin(\phi_{kj})]^{\mathrm{T}}$. λ_{kj} is the range information intensity (RII) given by [8], [11]

$$\lambda_{kj} = \frac{\xi_{kj} x_j \beta_j^2}{d_{kj}^{\alpha}} \tag{5}$$

where ξ_{kj} is a positive coefficient determined by the channel properties, α indicates the pathloss coefficient.

According to the definition, SPEB can be expressed in a closed form as

$$\mathcal{P}(\mathbf{p}_k) = \frac{\sum_{j \in \mathcal{N}_{\mathrm{b}}} \lambda_{kj}}{\sum_{i=1}^{N_{\mathrm{b}}} \sum_{j>i}^{N_{\mathrm{b}}} \sin^2(\phi_{kj} - \phi_{ki}) \lambda_{ki} \lambda_{kj}} = \frac{f(\mathbf{x}, \boldsymbol{\beta})}{g(\mathbf{x}, \boldsymbol{\beta})} \tag{6}$$

SPEB characterizes the fundamental limit of localization accuracy. It can be used as a performance metric for localization-aware networks.

III. Optimal joint power and bandwidth allocation

In this section, we perform optimal JPBA formulations with full knowledge of the network parameters. Meaningful performance benchmarks can be drawn for system evaluations. Two different problem formulations are proposed in this section. Localization accuracy and power consumption are considered as the objective function respectively.

A. Localization accuracy maximization

In this part, localization accuracy is firstly considered as the objective function. The original problem can thus be formulated as

$$\mathscr{P}_1: \ \min. \quad \sum_{k \in \mathcal{N}_{\mathrm{a}}} \mathcal{P}(\mathbf{p}_k) \tag{7}$$

$$\text{s.t.} \quad 0 \leq x_i \leq x_0 \qquad i \in \mathcal{N}_{\mathrm{b}} \tag{8}$$

$$0 \leq \beta_i \leq \beta_0 \qquad i \in \mathcal{N}_{\mathrm{b}} \tag{9}$$

$$\sum_{i \in \mathcal{N}_{\mathrm{b}}} x_i \leq x_{\text{total}} \tag{10}$$

$$\sum_{i \in \mathcal{N}_{\mathrm{b}}} \beta_i \leq \beta_{\text{total}} \tag{11}$$

The objective function in (7) is to minimize the total SPEB. Constraints (8) and (9) show that each node has an upper limit on transmission bandwidth β_0 and peak power x_0 constraint due to the hardware design. (10) and (11) give the upper bound of total power x_{total} and bandwidth β_{total} that can be used in the whole network.

B. Power consumption minimization

In cost and resources restricted wireless networks, we usually want to minimize the energy consumption, which will benefit the system lifetime. An energy efficient problem is formulated to minimize total transmit power, subject to a predefined accuracy requirement and bandwidth constraint.

$$\mathscr{P}_2: \ \min. \quad \sum_{i \in \mathcal{N}_{\mathrm{b}}} x_i \tag{12}$$

$$\text{s.t.} \quad 0 \leq \beta_i \leq \beta_0 \qquad i \in \mathcal{N}_{\mathrm{b}} \tag{13}$$

$$\sum_{i \in \mathcal{N}_{\mathrm{b}}} \beta_i \leq \beta_{\text{total}} \tag{14}$$

$$\mathcal{P}(\mathbf{p}_k) \leq \mathcal{P}_0 \qquad k \in \mathcal{N}_{\mathrm{a}} \tag{15}$$

The objective function (12) is the sum of the transmit power. Constraints (13) and (14) are bandwidth related conditions which are similar to $\mathscr{P}_1$. Constraints (15) indicates the localization accuracy requirement ($\mathcal{P}_0$) that all agents inside the network should meet.

C. An efficient solution based on single condensation method

Unfortunately, problems $\mathscr{P}_1$ and $\mathscr{P}_2$ are both non-convex due to the SPEB formulation in (6). Therefore, approximate algorithms with high accuracy are required to solve the problems. In this section, we take problem $\mathscr{P}_1$ as the example, by which $\mathscr{P}_2$ can be solved accordingly.

Since all the constraints in $\mathscr{P}_1$ are convex, we only need to focus on the SPEB related objective function (7). In (6), it's clear that λ_{kj} and $\sin^2(\phi_{kj}-\phi_{ki})$ are nonnegative ($k \in \mathcal{N}_\mathrm{a}, j \in \mathcal{N}_\mathrm{b}$). Therefore, SPEB is the ratio between two posynomials which is termed as an intractable NP-hard Complementary Geometric Programming (CGP) problem in [15], [16]. One key idea to this problem is to turn CGP into geometric programming (GP) by proper relaxations. An efficient *single condensation* (SC) method is proposed in [16] by which a solution that satisfies the Karush-Kuhn-Tucker (KKT) conditions of the original problem can be achieved. In SC method, CGP can be turned into GP by approximating the denominator of the ratio of posynomials, $g(\mathbf{z})$, with a monomial $\tilde{g}(\mathbf{z})$, but leaving the numerator $f(\mathbf{z})$ as a posynomial.

Lemma 1: Let $g(\mathbf{z}) = \sum_i \mu_i(\mathbf{z})$ be a posynomial. Then

$$g(\mathbf{z}) \geq \tilde{g}(\mathbf{z}) = \prod_i \left(\frac{\mu_i(\mathbf{z})}{\kappa_i}\right)^{\kappa_i} \tag{16}$$

If, in addition, $\kappa_i = \mu_i(\mathbf{z}_0)/g(\mathbf{z}_0)$, $\forall i$, for any fixed positive $\mathbf{z}_0$, then $\tilde{g}(\mathbf{z}_0) = g(\mathbf{z}_0)$, and $\tilde{g}(\mathbf{z}_0)$ is the best local monomial approximation to $g(\mathbf{z}_0)$ near $\mathbf{z}_0$ in the sense of first order Taylor approximation.

Proof: See [16].

According to *Lemma 1*, the denominator in (6) could be written as

$$g(\mathbf{x}, \boldsymbol{\beta}) \geq \tilde{g}(\mathbf{x}, \boldsymbol{\beta}) = \prod_i \left(\frac{\mu_i(\mathbf{x}, \boldsymbol{\beta})}{\kappa_i}\right)^{\kappa_i} \tag{17}$$

where $\kappa_i = \frac{\mu_i(\mathbf{x}^{(j)}, \boldsymbol{\beta}^{(j)})}{g(\mathbf{x}^{(j)}, \boldsymbol{\beta}^{(j)})}$. $\mathbf{x}^{(j)}$ and $\boldsymbol{\beta}^{(j)}$ are the power and bandwidth vectors in j^{th} iteration respectively. An approximate form of $\mathscr{P}_1$ can be obtained by replacing $g(\mathbf{x}, \boldsymbol{\beta})$ with $\tilde{g}(\mathbf{x}, \boldsymbol{\beta})$ in (6).

$$\begin{aligned} \mathscr{P}_3: \ \min. \quad & \sum_{k \in \mathcal{N}_\mathrm{a}} \tilde{\mathcal{P}}(\mathbf{p}_k) \\ \text{s.t.} \quad & (8) - (11) \end{aligned} \tag{18}$$

where the $\tilde{\mathcal{P}}(\mathbf{p}_k)$ in (18) is an approximated posynomial obtained by *Lemma 1*. Problem $\mathscr{P}_3$ is thus a geometric programming problem, and can be solved efficiently by lots of off-the-shelf solvers. The details of SC method are summarized in **Algorithm 1**, by which problem $\mathscr{P}_2$ can also be solved. Note that the JPBA problems are essentially nonconvex, the SC method may possibly converge to local optimal points. Therefore, the starting point selection is important. In **Algorithm 1**, we chose uniform resource allocation strategy as the initial point, i.e.,

$$x_i^{(0)} = \frac{x_\text{total}}{N_\mathrm{b}}, \beta_i^{(0)} = \frac{\beta_\text{total}}{N_\mathrm{b}}, i \in \mathcal{N}_\mathrm{b}$$

IV. Robust Power and Bandwidth Allocation

In practical scenarios, localization systems do not have access to perfect network parameters. Instead, we consider the estimates (subject to uncertainty) of the network parameters is obtained by initial (and rough) measurements. Applying them directly into problem $\mathscr{P}_1$ and $\mathscr{P}_2$ may lead to unreliable or even unfeasible solutions. In this section, we formulate the robust JPBA counterparts to deal with the uncertainties.

Algorithm 1 SC method for optimal JPBA problems

Initialization. Let $\mathbf{x}^{(0)} = \mathbf{x}_0$ and $\boldsymbol{\beta}^{(0)} = \boldsymbol{\beta}_0$ and $m = 1$.
while convergence not satisfied.
  Calculate κ_i with $\mathbf{x}^{(m-1)}$ and $\boldsymbol{\beta}^{(m-1)}$.
  Substitute κ_i into $\tilde{g}(\mathbf{x}^{(m-1)}, \boldsymbol{\beta}^{(m-1)})$. Obtained $\tilde{\mathcal{P}}(\mathbf{p}_k)$.
  Solve the problem $\mathscr{P}_3$. **Output**: $\mathbf{x}^{(m)}$ and $\boldsymbol{\beta}^{(m)}$.
  Convergence checking, if $||\mathbf{x}^{(m)} - \mathbf{x}^{(m-1)}|| \leq \epsilon_\mathbf{x}$ and $||\boldsymbol{\beta}^{(m)} - \boldsymbol{\beta}^{(m-1)}|| \leq \epsilon_{\boldsymbol{\beta}}$ are satisfied.
end
Output: resource allocation results $\mathbf{x}^\star$ and $\boldsymbol{\beta}^\star$.

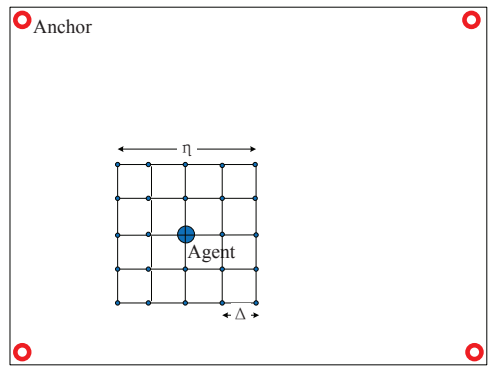

Fig. 2: Robust localization system with uncertainties

A. Robust resource allocation model

As shown in Fig. 2, the blue dot is the position estimate result of the agent by initial measurements subject to uncertainty. The actual position of the agent may be located inside the $\eta \times \eta$ continuous area $\mathcal{C}$. The uncertain parameter η is decided by the ranging techniques and channel environments. The key idea of the robust consideration is that, we take all the uncertain area into account rather than the single position obtained by measurements. In light of this approach, the resource allocation strategy works for the whole uncertain area $\mathcal{C}$. The robust JPBA problem (to minimize the localization error) can be formulated as

$$\begin{aligned} \mathscr{P}_4: \ \min. \quad & \sum_{\mathbf{p}_k \in \mathcal{C}} \mathcal{P}(\mathbf{p}_k) \\ \text{s.t.} \quad & (8) - (11) \end{aligned} \tag{19}$$

Similarly, the robust counterpart of $\mathscr{P}_2$ can be written as

$$\mathscr{P}_5: \ \min. \quad \sum_{k \in \mathcal{N}_\mathrm{b}} x_k \tag{20}$$

$$\text{s.t.} \quad \mathcal{P}(\mathbf{p}_k) \leq \mathcal{P}_0 \qquad \mathbf{p}_k \in \mathcal{C} \tag{21}$$

$$(13) - (14)$$

Constraint (21) indicates that any agent inside the uncertain set $\mathcal{C}$ is required a preset localization accuracy requirement. The rest constraints are the same as $\mathscr{P}_2$.

The robust counterparts can be treated as combinations of multiple optimal JPBA problems, which can be solved by

Algorithm 1. Therefore, the main difficulty in solving $\mathscr{P}_4$ and $\mathscr{P}_5$ lies in the fact that $\mathcal{C}$ is a continuous area. There are infinite points inside it which makes the solution extremely time exhausting.

One idea to deal with the continuous uncertain area is to approximate $\mathcal{C}$ with discrete grid set $\mathcal{G} = \{g_n\}_{n=1}^{N}$. Suppose $\mathcal{G}$ that is generated by sampling uniformly in both vertical and horizontal directions with a spacing Δ. Ideally, $\triangle$ should approach to zero as close as possible. However, it will make the problem too complicated to be solved directly. Therefore, a reasonable value for Δ to make $\mathcal{G}$ a sufficient subset of $\mathcal{C}$ is required. Here we propose a practical method based on the related work in [14], which helps us to find the optimal size of $\triangle$.

Lemma 2: Define $\mathcal{S}$ as a small continuous $\Delta \times \Delta$ sub area obtained from $\mathcal{C}$ by sampling. g_c is the central point of $\mathcal{S}$. Define $d_{\min}(g_c) = \min_{m \in \mathcal{N}_b}\{d_{m,g_c}\}$ as the closest distance from the agent to anchors. If we have $\Delta \ll d_{\min}(g_c)$, SPEB of agent k can be approximated as a convex function of $\mathbf{p}_k \in \mathcal{S}$.

Proof: See Appendix A.

Remark: Based on *Lemma 2*, we could conclude that

$$\max \mathcal{P}(\mathbf{p}_k)_{\mathbf{p}_k \in \mathcal{S}} = \max \mathcal{P}(\mathbf{p}_{g_n}), n = 1, \cdots, 4 \tag{22}$$

where g_n is the corner point of $\mathcal{S}$. (22) implies that, inside a small $\Delta \times \Delta$ area, we can get the largest SPEB on the four corner points.

According to the conclusion above, if $\Delta \ll d_{\min}(g_c)$ is satisfied, we can sample the continuous area $\mathcal{C}$ with Δ that makes $\mathcal{G} \approx \mathcal{C}$. The robust JPBA solution is presented as **Algorithm 2**.

Algorithm 2 Robust JPBA solution

1: Initialization. Decide the uncertain size η and initial agent position g_c.
2: Calculate the minimum distance $d_{\min}(g_c)$, then set the sampling size $\Delta \ll d_{\min}(g_c)$.
3: Sampling $\mathcal{C}$ uniformly in both vertical and horizontal directions with $\triangle$, output the discrete discrete grid set $\mathcal{G} = \{g_n\}_{n=1}^{N}$.
4: Solve the problem $\mathscr{P}_4$ or $\mathscr{P}_5$ with $\mathcal{G}$ by **Algorithm 1**.
5: **Output**: robust result $\mathbf{x}^\star$ and $\boldsymbol{\beta}^\star$.

V. Numeric Results

A. Simulation backgrounds

In this section, we present numerical results to evaluate the performance of the proposed JPBA problems. The global power x_{total} and bandwidth β_{total} for localization are normalized. Peak power and bandwidth constraints on each node are $x_0 = 0.4$ ($\mathscr{P}_1$ and $\mathscr{P}_4$) and $\beta_0 = \beta_{\text{total}} = 1$. Channel gain ξ_{kj} is set as $\xi_{kj} = 10^3$ here. Only path loss effects are considered as the channel gains. A simple localization network example is applied in our results where N_a agents are randomly deployed in a squared area, i.e., $U([0, D] \times [0, D])$ and four anchors are prelocated at the corners of the square. The thresholds for the convergence check step in **Algorithm 1** are set as $\epsilon_{\mathbf{x}} = \epsilon_{\boldsymbol{\beta}} = 10^{-3}$.

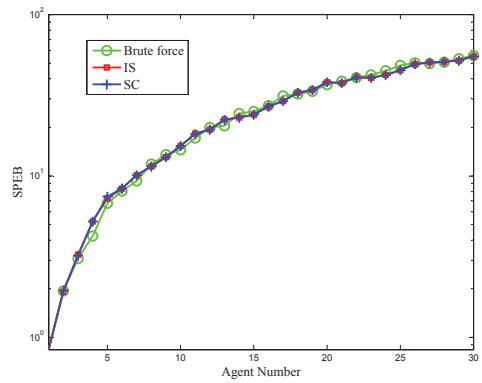

Fig. 3: Accuracy of IS and SC w.r.t the number of agents

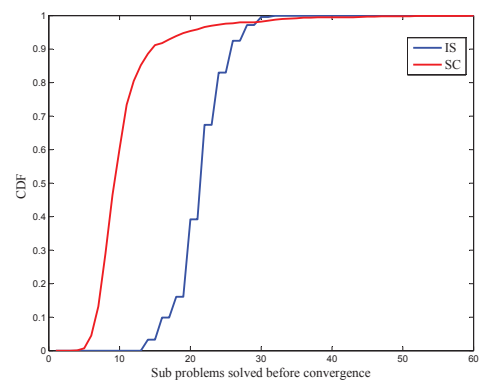

Fig. 4: Complexity comparison between SC and IS (with one agent)

B. Performance evaluation of SC

In Fig. 3, SC is compared with a high accuracy iterative searching (IS) method proposed in [11] and the time consuming brute force search method. SPEB results w.r.t the number of agents are shown. It can be seen that, all the three methods are able to achieve close results, which implies that both SC and IS are likely to find the global optimal solution in the investigated cases.

Complexity performance of SC is evaluated by comparing to IS, which is also a low complexity method. In Fig. 4, the cumulative probability functions (CDF) of sub problems to be solved in SC and IS are illustrated. It is notable that, the unsmooth CDF of IS is due to the fact that there are two subproblems in each iteration of IS. From the results, it is suggested that SC is more time efficient than IS. SC tends to converge by solving 15 sub problems, while nearly 30 sub problems are required in IS.

C. Performance analysis in optimal JPBA strategies

In this part, results obtained by optimal JPBA strategies from problems $\mathscr{P}_1$ and $\mathscr{P}_2$ are presented. Two different resource allocation strategies are applied for performance evaluation. The first one is the uniform allocation strategy, in which all power and bandwidth are equally allocated onto N_b

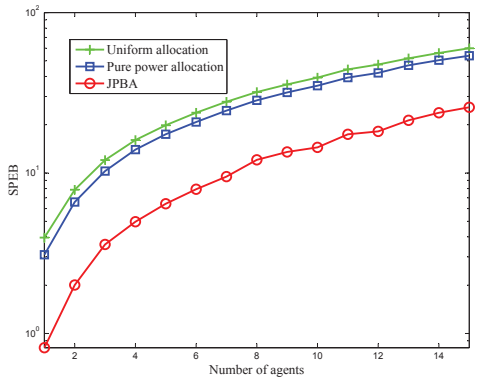

Fig. 5: SPEB as a function of the number of agents

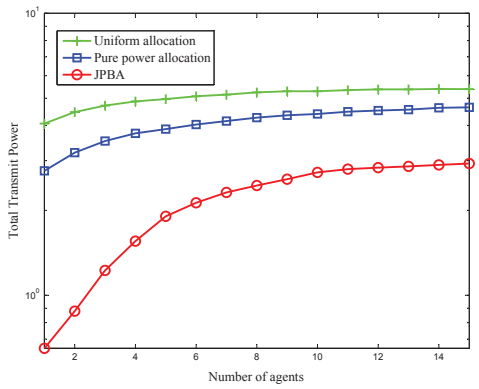

Fig. 6: Total transmit power as a function of the number of agents

anchors. It's the simplest but least efficient option compared to other optimized allocation strategy. The second one is the pure power allocation strategy, which has been investigated in [9], [12], [13]. Power allocation is optimized by SDP, while the bandwidth, however, is only uniformly allocated.

In Fig. 5, SPEB results achieved by the three strategies are illustrated. From the results we can see that,

- Total SPEB increases almost linearly w.r.t the number of agents. The main reason for that is the fixed global constraints on power and bandwidth (x_{total} and B_{total}). Similar conclusions are drawn in [12].
- JPBA outperforms the uniform resource allocation and pure power allocation strategy. It agrees to our intuition that, proper bandwidth allocation is important to the TOA-based localization system.

In the second part, global power consumption among the three strategies are compared. A given accuracy threshold in $\mathscr{P}_2$ is set as $\mathcal{P}_0 = 1$. In Fig. 6 we can see that, similar to the accuracy results, JPBA is the most energy efficient while the uniform allocation is the worst one. It suggests that proper bandwidth allocation will help the agents in energy efficiency as well. In the pure power allocation strategy, however, the only way to make the position estimation results good enough is to increase the transmit power.

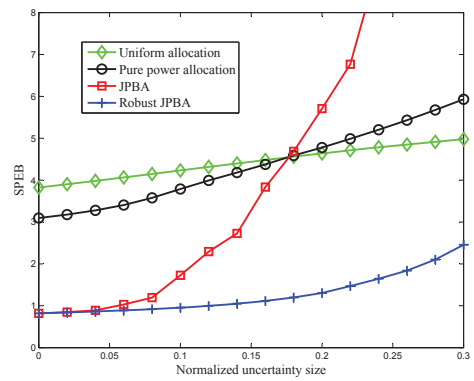

Fig. 7: SPEB as a function of normalized uncertainty size

D. Accuracy results in robust JPBA strategies

In this section, accuracy results obtained by robust JPBA strategies from $\mathscr{P}_4$ and $\mathscr{P}_5$ are also presented. As shown in Fig. 2, we take the initial position by rough measurements as the center of the uncertain area $\mathcal{C}$. Uniform sampling is performed as described in **Algorithm 2**. The sampling spacing $\triangle = \delta d_{\min}(g_{\text{c}})$. δ is set as 0.1 to meet the tradeoff between accuracy and computation complexity. The normalized uncertainty size is defined as

$$\zeta = \frac{\eta}{D} \tag{23}$$

where η is the uncertainty size and D is the area side.

In Fig. 7, SPEB results achieved by the four strategies are illustrated. From the results we can see that,

- When ζ is small enough, the robust JPBA is equivalent to the optimal one. The accuracy results achieved by these two strategies are quite close, and much better than uniform and pure power allocation strategies.
- The error in the optimal JPBA becomes large as ζ increases. It performs even worse than the uniform allocation (e.g., $\zeta > 0.2$). We can conclude that optimal JPBA strategy does not work well with inaccurate position information of agents.
- The robust JPBA performs best among all the four strategies even though the existence of large localization error, which agrees to our intuition.

In Fig. 8, the results show the power consumption of uniform allocation, pure power allocation and robust JPBA w.r.t the uncertainty size respectively. It can be concluded that robust JPBA is the most energy efficient resource allocation strategy among the three.

VI. CONCLUSION

In this paper, we have investigated the resource (power and bandwidth) allocation problem in wireless localization systems. We formulated two optimal JPBA problems to maximize the localization accuracy, and to minimize the power consumption during localization. Since they are essentially non-convex, we give high accuracy and efficient approximation methods to solve them. To deal with the uncertainty of the

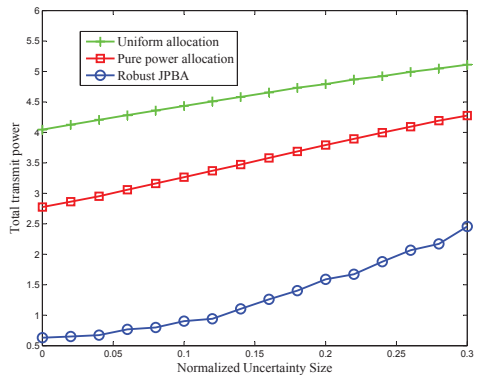

Fig. 8: Total transmit power as a function of normalized uncertainty size

agent's position, we formulate the robust JPBA counterparts to provide insight to practical resource allocation strategies. Several useful conclusions can be drawn by numeric results: (i) optimal JPBA gives an upper bound of the system accuracy and efficiency; it outperforms the uniform allocation and pure power allocation strategies in both accuracy and energy efficiency; (ii) when the agent's position is subject to large uncertainty, optimal JPBA will lead to unreliable results, while robust JPBA is a better option; (iii) due to the uncertainty of agent's position, a more balanced resource allocation strategy is preferred (which is different to the sparse allocation results in the optimal JPBA scenario [11]). These intuitive results, and the algorithms developed in this paper, can form the basis for practical deployment of wireless localization networks.

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Appendix A

Here we prove *Lemma 2*. Firstly, the SPEB of point $\mathbf{u}$ ($\mathbf{u} \in \mathcal{G}$) could be represented as

$$\mathcal{P}(\mathbf{u}) = \frac{\sum_{i=1}^{N_\mathrm{b}} \lambda_{ki}}{\sum_{i=1}^{N_\mathrm{b}} \sum_{j>i}^{N_\mathrm{b}} \left|\mathbf{v}_{ij}^\mathrm{T}\mathbf{u} - \mathbf{v}_{ij}^\mathrm{T}\mathbf{p}_i\right|^2 \frac{\lambda_{ki}\lambda_{kj}}{d_{ki}^2 d_{kj}^2}} \tag{24}$$

where $\mathbf{u} = [x_k, y_k]^\mathrm{T}$, $\mathbf{p}_i = [x_i, y_i]^\mathrm{T}$, and $\mathbf{v}_{ij} = [y_i - y_j, x_j - x_i]^\mathrm{T}$, $i, j \in \mathcal{N}_\mathrm{b}$.

As shown in Fig. 2, $\mathbf{g}_\mathrm{c}$ is the center point of the uncertain area $\mathcal{S}$. When $\Delta \ll d_{\min}(g_\mathrm{c})$, we have $\|\mathbf{u} - \mathbf{g}_\mathrm{c}\|_2 \ll \|\mathbf{g}_\mathrm{c} - \mathbf{p}_i\|_2$, then $d_{ki} \approx d_{\mathbf{g}_\mathrm{c} i}$. Therefore, $\lambda_{\mathbf{g}_\mathrm{c} i} \approx \lambda_{ki}$, where $\lambda_{\mathbf{g}_\mathrm{c} i} = \frac{\xi_{ki} x_i \beta_i^2}{d_{\mathbf{g}_\mathrm{c} i}^\alpha}$. Hence the SPEB of point $\mathbf{u} \in \mathcal{G}$ in (24) could be approximated as

$$\begin{aligned}\mathcal{P}(\mathbf{u}) &\approx \frac{\sum_{i=1}^{N_\mathrm{b}} \lambda_{g_\mathrm{c} i}}{\sum_{i=1}^{N_\mathrm{b}} \sum_{j>i}^{N_\mathrm{b}} \left|\mathbf{v}_{ij}^\mathrm{T}\mathbf{u} - \mathbf{v}_{ij}^\mathrm{T}\mathbf{p}_i\right|^2 \frac{\lambda_{\mathbf{g}_\mathrm{c} i}\lambda_{\mathbf{g}_\mathrm{c} j}}{d_{\mathbf{g}_\mathrm{c} i}^2 d_{\mathbf{g}_\mathrm{c} j}^2}} \\ &\approx \frac{\sum_{i=1}^{N_\mathrm{b}} \lambda_{g_\mathrm{c} i}}{\sum_{i=1}^{N_\mathrm{b}} \sum_{j>i}^{N_\mathrm{b}} \left|\mathbf{F}_{ij}(\mathbf{u})\right| \frac{\lambda_{\mathbf{g}_\mathrm{c} i}\lambda_{\mathbf{g}_\mathrm{c} j}}{d_{\mathbf{g}_\mathrm{c} i}^2 d_{\mathbf{g}_\mathrm{c} j}^2}}\end{aligned} \tag{25}$$

where $\mathbf{F}_{ij}(\mathbf{u}) = \mathbf{v}_{ij}^\mathrm{T}(\mathbf{g}_\mathrm{c} - \mathbf{p}_i)(\mathbf{g}_\mathrm{c} - \mathbf{p}_i)^\mathrm{T} + 2\mathbf{v}_{ij}^\mathrm{T}(\mathbf{g}_\mathrm{c} - \mathbf{p}_i)\mathbf{v}_{ij}^\mathrm{T}(\mathbf{u} - \mathbf{p}_i)$. Since $\mathbf{F}_{ij}(\mathbf{u})$ is an affine function of $\mathbf{u}$, SPEB $\mathcal{P}(\mathbf{u})$ tends to be convex when the condition $\Delta \ll d_{\min}(g_\mathrm{c})$ holds.

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