Power Allocation in Asynchronous Location-aware Sensor Networks

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***Abstract*—Wireless localization systems based on determination of signal runtime (TOA/TDOA) are of great importance for a variety of applications. In many cases, synchronization of the clocks of the agent nodes to those of the network nodes (anchors) has to be performed together with the localization. The current paper investigates the fundamental accuracy limits of such a joint localization/synchronization. In particular we analyze the impact of allocating power to the different anchor nodes, and optimize this power allocation to maximize accuracy. Simulation results confirm the importance of proper power allocation; known special cases (TDOA localization, localization with already-synchronized clocks) are recovered from our general solution.**

I. Introduction

Recently, wireless sensor networks (WSNs) have received significant attentions due to their wide area of applications, such as environment monitoring, remote sensing and target tracking, etc. Network localization and synchronization are two main technical bases of WSNs. Operations such as resource management, data collection and transmission are all dependent to position information. Usually it is important to know not only the measurements themselves, but also where the measurements were obtained in WSNs [1], [2]. Synchronization is also challenging due to many factors, such as imperfect clock parameters (clock skew and offset) in each individual node implementations. Localization results rely on the timing accuracy directly. Even small timing differences among nodes cause significant localization errors.

Usually, location-aware networks consist of a limited number of anchors with known positions and many agents with unknown positions. Wireless localization refers to a process where the agents perform position estimation with respect to the anchors by different ways of measurements, such as received signal strength intensity (RSSI), angle of arrival (AOA), time of arrival (TOA), etc [3]. Time based metrics are obtained by measuring the signal propagation time between nodes. One-way and round-trip time of flight can be applied in synchronous and asynchronous networks, respectively. Alternatively, time difference of arrival (TDOA) provides another solution to asynchronous localization, in which synchronization is required among anchors, but not necessarily with the agents.

In [4] and [5], the fundamental limits of wideband localization have been derived in terms of Cramer Rao Lower Bound (CRLB). This, and related, work is mostly based on the assumptions that all nodes are perfectly synchronized, and one-way time of flight measurements are thus considered. Since time synchronization is challenging in wireless networks, TDOA is also attractive, even if there exists obvious performance degradation compared to the TOA based approaches [6].

The problem of network clock synchronization is, in itself, a challenging and relevant problem (see [7] and references therein). In most related papers, synchronization and localization are performed separately [8]. Some recent work has considered the use of TOA measurements to obtain joint localization and synchronization [9]. Ref [10] also shows that a better performance can be achieved by joint estimation rather than doing it separately.

In WSNs, transmit power is usually limited due to the battery capacity, regulatory constraints, and the created amount of co-channel interference. Power allocation has thus been widely applied for improving energy efficiency and network lifetime. Optimal and robust power allocation has been carried out in synchronous localization networks [11], [12], etc. However, to the best of the authors' knowledge, power allocation and optimization problems in asynchronous localization networks have not yet been addressed.

In this paper, we first derive the global equivalent Fisher information matrix (EFIM) of both localization and synchronization accuracy. After that, we establish general power allocation frameworks for asynchronous localization networks. We take the CRLBs of localization, synchronization and joint estimations as objective functions, respectively. All power allocation problems are proved to be convex. Optimal power allocation solutions can be achieved by the proposed frameworks, which are subject to global resource constraints.

II. System model

We consider a 2-D location-aware network of N_b anchors with known positions and N_a agents with unknown positions.[1] The sets of agents and anchors are represented by

[1]We only consider the 2-D scenario in this paper, while the 3-D localization problem can be solved straightforwardly.

$\mathcal{N}_\mathrm{a} = \{1, 2, ..., N_\mathrm{a}\}$ and $\mathcal{N}_\mathrm{b} = \{N_\mathrm{a}+1, N_\mathrm{a}+2, ..., N_\mathrm{a}+N_\mathrm{b}\}$ respectively. The position of node k is denoted by $\mathbf{p}_k = [x_k, y_k]^\mathrm{T}, k \in \mathcal{N}_\mathrm{a} \cup \mathcal{N}_\mathrm{b}$. We assume the N_b anchors are *synchronized*, while there exists a clock offset υ_k between the agent k and anchors. The distance between agent k and anchor j is denoted by

$$d_{kj} = \|\mathbf{p}_k - \mathbf{p}_j\|_2 \tag{1}$$

where $||\cdot||_2$ denotes the ℓ-2 norm. The angle from node k to j is given by

$$\phi_{kj} = \arctan\left(\frac{y_k - y_j}{x_k - x_j}\right) \tag{2}$$

In this paper, anchors *broadcast* signals throughout the network, agents then perform range and position estimation by TOA estimation. The received signal at agent k from anchor j is

$$r_{kj}(t) = \alpha_{kj} s(t - \tau_{kj}) + z_{kj}(t) \tag{3}$$

where α_{kj} and τ_{kj} represent the amplitude and time delay on the link between k and j, respectively. $z_{kj}(t)$ is the measurement noise modeled as a zero-mean Gaussian random variable, and the power spectral density is $\frac{N_0}{2}$. The measured time delay τ_{kj} can be obtained as

$$\hat{\tau}_{kj} = \frac{1}{c} d_{kj} + \upsilon_k + n_k = \frac{1}{c}||\mathbf{p}_k - \mathbf{p}_j|| + \upsilon_k + n_k \tag{4}$$

where c is the propagation speed of signal. n_k is the estimation noise modeled as Gaussian distribution.

We use $\boldsymbol{\theta}$ to represent the position related parameter vector to be estimated.

$$\boldsymbol{\theta} = [\mathbf{p}^\mathrm{T}, d_0, \boldsymbol{\alpha}^\mathrm{T}]^\mathrm{T} \tag{5}$$

where $d_0 = c\upsilon_k$ is the equivalent distance bias caused by the clock offset at agent k. $\boldsymbol{\alpha}^\mathrm{T} = [\alpha_1, \alpha_2, ...\alpha_{N_b}]^\mathrm{T}$ denotes the amplitudes from N_b anchors, which are essentially nuisance parameters in our work.

III. Cramer-Rao Lower Bound

In this part, we consider a single agent scenario ($N_\mathrm{a} = 1$), while the general multiple agents scenario will be extended later.[2] The CRLB determines a lower bound on the performance (variance) of any unbiased estimator, which can be applied as the performance metrics of localization and synchronization. We first derive the global EFIM. After that, the corresponding EFIM of $\mathbf{p}$ and d_0 can be obtained accordingly.

Proposition 1: The global EFIM of position $\mathbf{p}$ and distance bias d_0 (equivalent to the clock offset) joint estimation can be represented as

$$\begin{aligned} \mathbf{J}_\mathrm{J}(\mathbf{p}, d_0) &= \sum_{j=1}^{N_b} \lambda_j \begin{pmatrix} \mathbf{q}_j\mathbf{q}_j^\mathrm{T} & \mathbf{q}_j \\ \mathbf{q}_j^\mathrm{T} & 1 \end{pmatrix} \\ &= \sum_{j=1}^{N_b} \lambda_j \mathbf{D}_{(\mathbf{p},d_0)}(\phi_j) \end{aligned} \tag{6}$$

[2]The subscript of k in section II is thus omitted for simplification.

where

$$\mathbf{D}_{(\mathbf{p},d_0)}(\phi_j) = \mathbf{v}_j\mathbf{v}_j^\mathrm{T} \tag{7}$$

$$\mathbf{v}_j = [\cos\phi_j, \sin\phi_j, 1]^\mathrm{T} \tag{8}$$

$$\mathbf{q}_j = [\cos\phi_j, \sin\phi_j]^\mathrm{T} \tag{9}$$

$$\lambda_j = \frac{8\pi^2\beta_j^2 \mathrm{SNR}_j}{c^2} \tag{10}$$

$$\beta_j \triangleq \sqrt{\frac{\int_{-\infty}^{+\infty} f^2|S(f)|^2 df}{\int_{-\infty}^{+\infty} |S(f)|^2 df}} \tag{11}$$

$$\mathrm{SNR}_j \triangleq \frac{|\alpha_j|^2 \int_{-\infty}^{+\infty} |S(f)|^2 df}{N_0} \tag{12}$$

where SNR_j is the signal to noise ratio of the transmitted signals from anchor j. $S(f)$ is the Fourier transform of $s(t)$, N_0 is the power spectral density of the additional white Gaussian noise (AWGN).

Proof: Omitted due to space constraints. See [5] and [13] and references therein.

Based on the global EFIM in (6), we try to extract the individual EFIMs for position ($\mathbf{J}_\mathrm{L}(\mathbf{p})$) and clock offset ($\mathbf{J}_\mathrm{S}(d_0)$) errors according to the definition of EFIM in [5], which is the Schur complement of the original FIM.

$$\mathbf{J}_\mathrm{L}(\mathbf{p}) = \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j\mathbf{q}_j^\mathrm{T} - \frac{1}{\sum_{j=1}^{N_b}\lambda_j} \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j^\mathrm{T} \tag{13}$$

$$\mathbf{J}_\mathrm{S}(d_0) = \sum_{j=1}^{N_b} \lambda_j - \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j^\mathrm{T} \Big(\sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j\mathbf{q}_j^\mathrm{T}\Big)^{-1} \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j \tag{14}$$

As indicated in [14], [15], λ_j is termed "ranging information intensity (RII)", which can be used to describe the ranging performance. RII is the inverse of the CRLB of TOA estimation in additional white Gaussian noise (AWGN), which can be generally simplified as

$$\lambda_j = \xi_j \frac{w_j\beta_j^2}{d_j^\varrho} \tag{15}$$

where ξ_j indicate the channel gain of the link between agent and anchor j. w_j and β_j are transmit power and effective bandwidth of anchor j. ϱ is the pathloss coefficient.

Remarks 1: As shown in (6), $\mathbf{J}_\mathrm{J}(\mathbf{p}, d_0)$ is a 3×3 matrix, which shows that each anchor adds ranging information for joint estimation of $\mathbf{p}$ and d_0 weighted by λ_j.

Remarks 2: The EFIM of position estimation in (13) is exactly the same as the results obtained in TDOA [6]. It implies that, if there are uncertainties in the clock information, asynchronous TOA localization is equivalent to TDOA localization.

Remarks 3: If the network were perfectly synchronized, $\mathbf{J}_\mathrm{J}(\mathbf{p}, d_0)$ will reduce to a 2×2 EFIM proposed in [5]. Obviously, the unknown clock offset reduces the ranging information, and CRLB of position estimation will increase with respect to the clock offset estimation errors.

If there are multiple agents inside the network, the corresponding global EFIM is a $3N_{\rm a} \times 3N_{\rm a}$ matrix block diagonal matrix. The $i^{\rm th}$ element of the matrix is $\mathbf{J}_{\rm J}(\mathbf{p}_i, d_0)$ in (6).

IV. Optimal power allocation problems

In this section, we perform the optimal power allocation problem with different objective functions. To simplify the notations, we consider the single-agent scenario, the multiple-agent problem can be solved analogously.

A. Power allocation for synchronization

We first consider the timing synchronization problem, which is important in wireless networks. Clock offset estimation error is tried to be optimized by properly allocating power resources among anchors.

The EFIM of clock offset estimation is given in (14), by which the CRLB can be achieved as

$$\mathbb{E}\{||\hat{d}_0 - d_0||^2\} \geq \mathcal{P}_{\rm S}(d_0) \triangleq \text{tr}\{\mathbf{J}_{\rm S}^{-1}(d_0)\} \tag{16}$$

We set the error bound of clock offset estimation as the objective function to be minimized. The corresponding problem can be formulated as

$$\begin{aligned} \mathscr{P}_1:\ \min. \quad & \mathcal{P}_{\rm S}(d_0;\mathbf{w}) & (17)\\ \text{s.t.} \quad & 0 \leq w_j \leq w_0 \qquad j \in \mathcal{N}_{\rm b} & (18)\\ & \sum_{j\in\mathcal{N}_{\rm b}} w_j \leq w_{\rm total} & (19) \end{aligned}$$

In $\mathscr{P}_1$, we use $\mathbf{w} = [w_1, ...w_{N_{\rm b}}]^{\rm T}$ to indicate the power allocation vector among anchors. Constraint (18) shows that each node has a peak power constraint w_0 due to the hardware design, e.g., power amplifier saturation. (19) gives the upper bound of the total power ($w_{\rm total}$) that can be used in the whole network.

Proposition 2: The problem $\mathscr{P}_1$ is convex in $\mathbf{w}$.

Proof: Since the constraints (18) and (19) are affine, we only need to prove the objective function (17) is convex in $\mathbf{w}$. According to (14), $\mathbf{J}_{\rm S}(d_0)$ can be rewritten as

$$\mathbf{J}_{\rm S}(d_0) = \sum_{j=1}^{N_{\rm b}} \lambda_j - f(\boldsymbol{\lambda}) \tag{20}$$

where

$$\begin{aligned} f(\boldsymbol{\lambda}) &= \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j^{\rm T} \Big(\sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j \mathbf{q}_j^{\rm T} \Big)^{-1} \sum_{j=1}^{N_b} \lambda_j \mathbf{q}_j \\ &= \mathbf{B}^{\rm T}\mathbf{A}^{-1}\mathbf{B} \end{aligned} \tag{21}$$

(21) is a scalar function of $\boldsymbol{\lambda}$, and furthermore, $\boldsymbol{\lambda}$ is also a linear function of $\mathbf{w}$. Note that $\mathcal{P}_{\rm S}(d_0) = \frac{1}{J_{\rm S}(d_0)}$. Therefore, we only need to prove that (21) is convex in $\boldsymbol{\lambda}$, the proof of this proposition is thus complete.

Given any $\gamma \in [0,1]$, $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$, we can prove the convexity of (21) according to the definitions of convex optimization, i.e.,

$$f(\gamma\boldsymbol{\lambda} + (1-\gamma)\boldsymbol{\lambda}') \leq \gamma f(\boldsymbol{\lambda}) + (1-\gamma) f(\boldsymbol{\lambda}')\} \tag{22}$$

The details of (22) is omitted here due to the space constraints.

According to proposition 2, optimal power vector $\mathbf{w}_{\rm S}^\star$ can be achieved by solving $\mathscr{P}_1$ through many of the shelf solvers, such as SDPT3 and Mosek, etc.

B. Power allocation for localization

Position information is what we really care in location-aware networks. Similar to the previous problem, the performance metric is defined as

$$\mathbb{E}\{||\hat{\mathbf{p}} - \mathbf{p}||^2\} \geq \mathcal{P}_{\rm L}(\mathbf{p}) \triangleq \text{tr}\{\mathbf{J}_{\rm L}^{-1}(\mathbf{p})\} \tag{23}$$

where $\mathbf{J}_{\rm L}(\mathbf{p})$ is shown in (13).

We then can formulate a problem that minimizing the localization error.

$$\begin{aligned} \mathscr{P}_2:\ \min. \quad & \mathcal{P}_{\rm L}(\mathbf{p};\mathbf{w}) & (24)\\ \text{s.t.} \quad & (18)-(19) & (25) \end{aligned}$$

Proposition 3: Problem $\mathscr{P}_2$ is convex in $\mathbf{w}$.

Proof: See Appendix.

Therefore, we can achieve the global optimal solution ($\mathbf{w}_{\rm L}^\star$) of $\mathscr{P}_2$ according to proposition 3.

C. Power allocation for joint synchronization and localization

In this part, we try to formulate a power allocation problem for joint synchronization and localization. Since $\mathscr{P}_1$ and $\mathscr{P}_2$ are both convex, the power allocation problem is formulated as

$$\begin{aligned} \mathscr{P}_{3a}:\ \min \quad & \mathcal{P}_{\rm L}(\mathbf{p};\mathbf{w}) & (26)\\ \text{s.t.} \quad & \mathcal{P}_{\rm S}(d_0;\mathbf{w}) \leq \varepsilon & (27)\\ & (18)-(19) \end{aligned}$$

in which we set the objective function as the CRLB of position error. Constraint (27) shows that there exists a threshold for synchronization accuracy. Apparently, $\mathscr{P}_{3a}$ is a convex problem and can be solved easily. We can similarly formulate another convex synchronization-optimized problem $\mathscr{P}_{3b}$,

$$\begin{aligned} \mathscr{P}_{3b}:\ \min \quad & \mathcal{P}_{\rm S}(d_0;\mathbf{w}) & (28)\\ \text{s.t.} \quad & \mathcal{P}_{\rm L}(\mathbf{p};\mathbf{w}) \leq \delta & (29)\\ & (18)-(19) \end{aligned}$$

V. Numerical results and discussions

A. Network settings

In this section, we present numerical results for performance evaluation. The total power for all nodes is normalized, i.e., $w_{\rm total} = 1$. A simple network with $N_{\rm a}$ agents and $N_{\rm b}$ anchors are deployed inside a square area i.e., $U([0,10]\times[0,10])$ (see Fig. 1). The channel gain is determined by the free-space pathloss only, and the pathloss coefficient is set as 2 [11], [15]. The channel parameters is given by $\xi_{kj}/d_{kj}^2 = 10^4/d_{kj}^2$. c is the light speed in free space, i.e., 3×10^8m/s. The proposed power allocation problems are all solved by the standard solver package CVX [16].

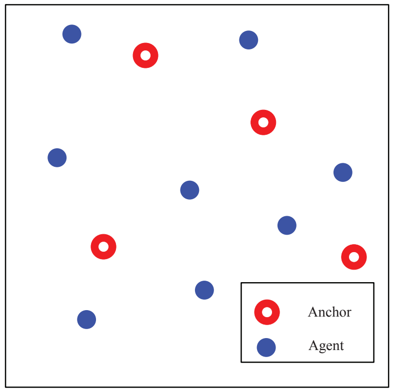

Fig. 1. The location-aware network with N_b anchors and N_a agents

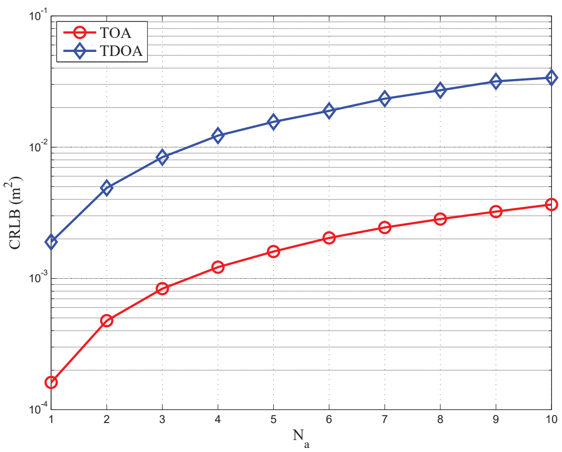

Fig. 2. Position estimation results between TOA and TDOA

B. Position estimation results

We first compare the localization accuracy of TOA and TDOA with optimal power allocation results. $N_b = 8$ anchors are prelocated, while multiple agents are randomly deployed inside the area.

Second, we compare the TDOA localization results between optimal and uniform power allocation results. We consider two different scenarios in this part. One is the multiple-agent with $N_b = 8$ prelocated anchors, the other is only one agent deployed at the center of the area, multiple anchors are uniformly distributed within the area. We can see the following phenomenons from the results.

- In Fig. 2, localization accuracy in asynchronous TOA (essentially TDOA) decreases obviously compared to synchronous TOA. The main reason for the degradation is due to the lack of clock offset information.
- In Fig. 3, the CRLB of position estimation decreases in both strategies with increasing the number of anchors. Furthermore, the results of optimal strategy decreases faster than the uniform strategy. The reason is that, according to the conclusions drawn in [17], [18], the optimal power allocation vector is sparse. Therefore, only a small number of "best" anchors are essentially active during localization. However, power resources are uniformly allocated into each anchor in the uniform strategy, which makes this solution not as power efficient as the optimal one.
- In Fig. 4, the CRLB of position estimation increases almost linearly in both power allocation strategies. It agrees to the intuition that when more agents are involved inside the network, a larger total localization error will be achieved due to the fixed total power constraint (w_{total}). Similar conclusions are drawn in [15] and [17], etc. However, the mean error of each agent stays at the same level due to the *broadcast* strategy of anchors, which is different from cooperative localization scenario. It can also be inferred that, optimal power allocation will benefit the localization accuracy.

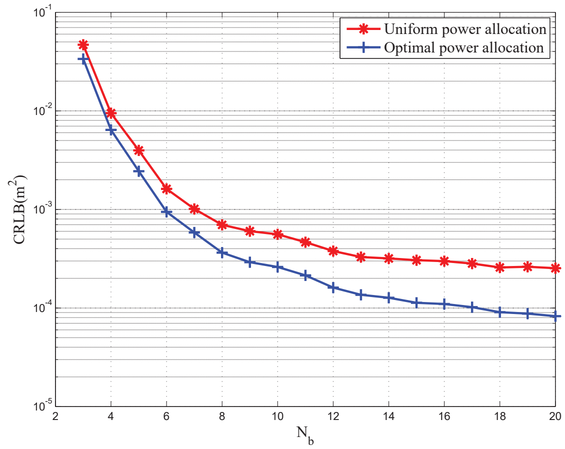

Fig. 3. Position estimation results between optimal and uniform power allocation strategies - single agent

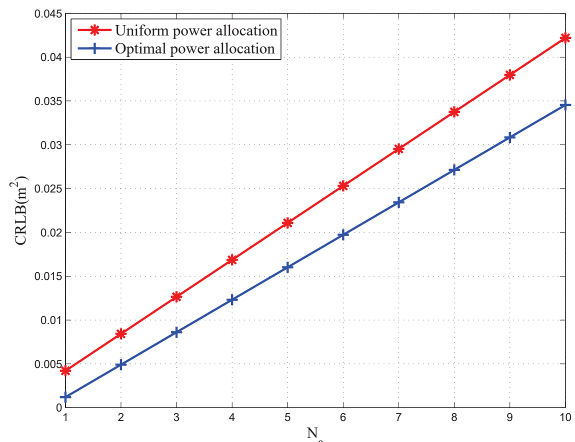

Fig. 4. Position estimation results between optimal and uniform power allocation strategies - multiple agents

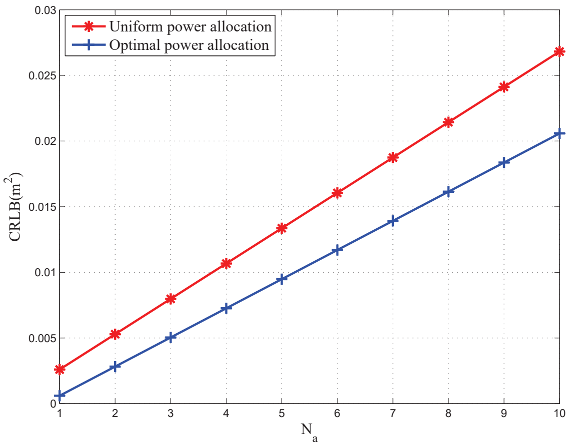

Fig. 5. Clock offset estimation results between optimal and uniform power allocation strategies - multiple agents

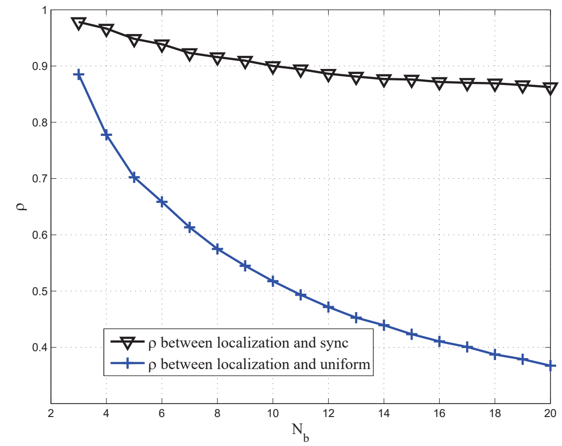

Fig. 7. Correlation between different power vectors

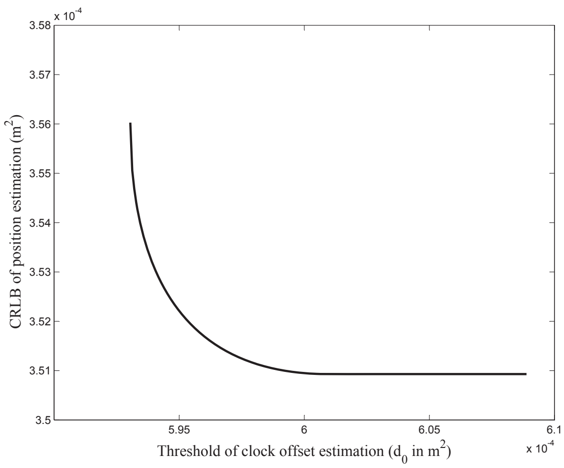

Fig. 6. Optimal tradeoffs between localization and synchronization

C. Synchronization accuracy and tradeoffs between localization and synchronization

In Fig. 5, we show the clock offset estimation errors with respect to the agent number. Similar to the position errors, the synchronization error also increases linearly with respect to the agent number, which can also be explained through the fixed power constraint aspect.

We then investigate the tradeoff between localization and synchronization accuracy from $\mathscr{P}_{3a}$ and $\mathscr{P}_{3b}$. Several remarks can be drawn from the numeric results in Fig. 6.

- There exists a dynamic tradeoff range between the localization and synchronization accuracy. Position estimation and synchronization are competing for the limited resources within this dynamic range. We can achieve a better localization accuracy by sacrificing the synchronization performance, and vice versa.
- It is clear that, if the accuracy threshold is set beyond the ability to be achieved, problem $\mathscr{P}_{3a}$ and $\mathscr{P}_{3b}$ become infeasible. According to Fig. 6, if we want an unreasonable synchronization accuracy ($\sigma^2_{d_0} < 5.92\times10^{-4}$), the CRLB of position estimation will become extremely large, which is impractical.
- On the other hand, if we relax one constraint (e.g., synchronization) enough, we can achieve the best result on the other one. It implies that, if we do not pay attention to one rating, the other one could achieve the optimal result, which agrees to our intuition well.

Fig. 6 shows that, we can not achieve both performance limits in localization and synchronization simultaneously.

D. Discussions of optimal power results

We study the similarity between $\mathbf{w}^\star_{\mathrm{L}}$ and $\mathbf{w}^\star_{\mathrm{S}}$ in this part. The correlation coefficient ρ is applied here, which is defined as

$$\rho(\mathbf{w}_1, \mathbf{w}_2) = \frac{\mathbf{w}_1 \cdot \mathbf{w}_2}{||\mathbf{w}_1|| \cdot ||\mathbf{w}_2||} \tag{30}$$

In Fig. 7, we first compare the correlation between $\mathbf{w}^\star_{\mathrm{L}}$ and uniform power allocation results $\mathbf{w}_{\text{uni}}$. It can be seen that $\rho(\mathbf{w}^\star_{\mathrm{L}}, \mathbf{w}_{\text{uni}})$ decreases with the increasing number of anchors, that is also due to the sparsity of optimal power vector. We can also see that, despite a slow reduction of the correlation between $\mathbf{w}^\star_{\mathrm{L}}$ and $\mathbf{w}^\star_{\mathrm{S}}$, $\rho(\mathbf{w}^\star_{\mathrm{L}}, \mathbf{w}^\star_{\mathrm{S}})$ is relative large (greater than 0.85 in all investigated cases). It implies the power distributions for synchronization and localization are similar, but not the same.

VI. Conclusion

In this paper, we have investigated the power optimization problems in asynchronous localization networks. We first gave the fundamental limits of position estimation and synchronization in terms of CRLBs, respectively. We then proved the power allocation problems are all convex with respect to the objective functions. Numeric results are then achieved and compared to the sub-optimal solutions such as uniform power allocation strategy. We can conclude that, (i) asynchronous TOA localization is equivalent to TDOA in sense of FIM and CRLB, (ii) the performance advantage of optimal power allocation is higher when the anchor number is large, (iii) optimal power allocation results for localization and synchronization are quite similar, but not the same. The meaningful results highlight the fundamental performance analysis of asynchronous localization networks, and are useful for heuristic localization and synchronization algorithms design and development.

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APPENDIX: PROOF OF PROPOSITION 3

According to the definition of CRLB, we have

$$\begin{aligned}\mathcal{P}_{\mathrm{L}}(\mathbf{p}) &= \mathcal{P}_{\mathrm{L}}(\mathbf{x}) + \mathcal{P}_{\mathrm{L}}(\mathbf{y}) \\ &= \mathbb{E}\{||\hat{x}-x||^2\} + \mathbb{E}\{||\hat{y}-y||^2\}\end{aligned} \tag{31}$$

Therefore, if $\mathcal{P}_{\mathrm{L}}(x)$ and $\mathcal{P}_{\mathrm{L}}(y)$ are convex in $\mathbf{w}$, the proof is thus complete.

Similar to the EFIM of clock offset estimation in (14), we can also derive the EFIM for $\hat{x}$ and $\hat{y}$ respectively. Without loss of generality, we take $\mathbf{J}_{\mathrm{L}}(x)$ for example here, while $\mathbf{J}_{\mathrm{L}}(y)$ can be obtained accordingly.

We first rewrite $\mathbf{J}_{\mathrm{J}}(\mathbf{p}, d_0)$ in (6) as

$$\mathbf{J}_{\mathrm{J}}(\mathbf{p}, d_0) = \begin{bmatrix} \mathbf{L} & \mathbf{H}^{\mathrm{T}} \\ \mathbf{H} & \mathbf{S} \end{bmatrix} \tag{32}$$

where

$$\begin{aligned}\mathbf{L} &= \sum_{j=1}^{N_b} \lambda_j \cos^2\phi_j \\ \mathbf{H} &= [\sum_{j=1}^{N_b} \lambda_j \cos\phi_j \sin\phi_j \quad \sum_{j=1}^{N_b} \lambda_j \cos\phi_j]^{\mathrm{T}} \\ &= \sum_{j=1}^{N_b} \lambda_j \cos\phi_j \mathbf{h}_j \\ \mathbf{S} &= \begin{bmatrix} \sum_{j=1}^{N_b} \lambda_j \sin^2\phi_j & \sum_{j=1}^{N_b} \lambda_j \sin\phi_j \\ \sum_{j=1}^{N_b} \lambda_j \sin\phi_j & \sum_{j=1}^{N_b} \lambda_j \end{bmatrix} \\ &= \sum_{j=1}^{N_b} \lambda_j \mathbf{h}_j \mathbf{h}_j^{\mathrm{T}}\end{aligned}$$

where $\mathbf{h}_j = [\sin\phi_j \ 1]^{\mathrm{T}}$. We can thus get the EFIM of x as

$$\begin{aligned}\mathbf{J}_{\mathrm{L}}(x) &= \mathbf{L} - \mathbf{H}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{H} \\ &= \sum_{j=1}^{N_b} \lambda_j \cos^2\phi_j - \\ &\sum_{j=1}^{N_b} \lambda_j \cos\phi_j \mathbf{h}_j^{\mathrm{T}} \big(\sum_{j=1}^{N_b} \lambda_j \mathbf{h}_j \mathbf{h}_j^{\mathrm{T}}\big)^{-1} \sum_{j=1}^{N_b} \lambda_j \cos\phi_j \mathbf{h}_j\end{aligned} \tag{33}$$

Then we have

$$\mathcal{P}_{\mathrm{L}}(x) = \mathrm{tr}\{\mathbf{J}_{\mathrm{L}}(x)^{-1}\} = \frac{1}{J_{\mathrm{L}}(x)} \tag{34}$$

where $\mathcal{P}_{\mathrm{L}}(x)$ can be proved convex in $\mathbf{w}$ by proposition 2.

Similarly, $\mathcal{P}_{\mathrm{L}}(y)$ can be also proved convex in $\mathbf{w}$. Therefore, $\mathcal{P}_{\mathrm{L}}(\mathbf{p})$ is convex in $\mathbf{w}$. The proof is thus complete.

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