Energy-Efficient Decentralized Cooperative Routing in Wireless Networks

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***Abstract*—Wireless *adhoc* networks transmit information from a source to a destination via multiple hops in order to save energy and, thus, increase the lifetime of battery-operated nodes. The energy savings can be especially significant in cooperative transmission schemes, where several nodes cooperate during one hop to forward the information to the next node along a route to the destination. Finding the best multi-hop transmission policy in such a network which determines nodes that are involved in each hop, is a very important problem, but also a very difficult one especially when the physical wireless channel behavior is to be accounted for and exploited. We model the above optimization problem for randomly fading channels as a decentralized control problem – the channel observations available at each node define the information structure, while the control policy is defined by the power and phase of the signal transmitted by each node.**

In particular, we consider the problem of computing an energy-optimal cooperative transmission scheme in a wireless network for two different channel fading models: (i) slow fading channels, where the channel gains of the links remain the same for a large number of transmissions, and (ii) fast fading channels, where the channel gains of the links change quickly from one transmission to another. For slow fading, we consider a factored class of policies (corresponding to local cooperation between nodes), and show that the computation of an optimal policy in this class is equivalent to a shortest path computation on an induced graph, whose edge costs can be computed in a decentralized manner using only locally available channel state information (CSI). For fast fading, both CSI acquisition and data transmission consume energy. Hence, we need to jointly optimize over both these; we cast this optimization problem as a large stochastic optimization problem. We then jointly optimize over a set of CSI functions of the local channel states, and a corresponding factored class of control policies corresponding to local cooperation between nodes with a local outage constraint. The resulting optimal scheme in this class can again be computed efficiently in a decentralized manner. We demonstrate significant energy savings for both slow and fast fading channels through numerical simulations of randomly distributed networks.

***Index Terms*—*Ad hoc* networks, channel state information (CSI), multiple input multiple output (MIMO) systems.**

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I. INTRODUCTION

WIRELESS communication channels have the characteristic that their amplitude and phase vary with time, i.e., the channel undergoes *fading*, which makes communication over links with low instantaneous channel gains infeasible. One way to overcome fading is to use *diversity*, i.e., take advantage of multiple signal paths distributed either in space or time (which fade independently) from the transmitter to the receiver. In this paper, we focus on communication techniques based on spatial diversity that exploit, in a distributed manner, the spatial separation between nodes in a wireless network. This is feasible because when the nodes are sufficiently apart, the links between them are likely to fade independently of each other [1, Ch. 13]. In cooperative communication networks, this is taken one step further by making the nodes cooperate with each other in transmitting each other's information [2]–[4]. It is a paradigm that promises significant gains in overall network throughput and network energy efficiency. In effect, cooperative communication achieves the proven gains of multiple input multiple output (MIMO) systems [1, Ch. 20], which require nodes that can transmit and/or receive with multiple antennas, using simpler, spatially separated single-antenna capable nodes. For a tutorial overview of cooperative communication, we refer the reader to [5] and [6], and the references therein. Many of the recently studied aspects include obtaining capacity scaling laws for random networks using cooperative communication [7], [8], designing protocols and codes for simple relay network topologies [4], design of practically implementable schemes where two mobile phones can cooperate [3], accounting for the overhead power consumption entailed in distributing the CSI required for cooperation [9], and the impact of available CSI on relay cooperation [10].

In this paper, we consider the problem of sending information packets from a source node to a destination node using a network of cooperating wireless relays to minimize the total energy consumption. While simple two-hop or three-hop relay topologies have received a lot of attention recently [9], [11]–[15], *ad hoc* networks often have tens or hundreds of nodes in practice. We therefore consider cooperative communications in wireless networks with arbitrary topologies. In our set up, we take the nodes to be of the decode-and-forward type [4], where a node which acts as a relay attempts to decode an entire message that it receives, and forwards it to the next node only if it can decode the message successfully. Furthermore, we assume that they are powered by batteries. Therefore, energy efficiency is of critical importance [16].

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A key and difficult problem in such networks with many nodes is finding an *energy optimal cooperative transmission policy* that guarantees delivery of information to the destination with a certain reliability (outage). In traditional, non-cooperative routing, the information is transmitted from the source via a sequence of relay links to a destination, where only two relay nodes communication on each relay link; many interesting results exist for this scenario, for example [17], [18], and [19]. When the data transmissions on different hops do not interfere with each other, the problem of finding a minimum energy route is equivalent to that of finding a minimum cost path on a graph where the cost for each edge corresponds to the average energy consumption in forwarding a message along the corresponding wireless link. The minimum cost path can be computed easily using the distributed Bellman-Ford algorithm [20]. In cooperative routing, on the other hand, multiple relays can now cooperate at the symbol-level and *together* forward the message in each hop. This immensely complicates the problem of finding the energy optimal cooperative transmission policy [21]. For example, each transmission in a cooperative route can be a broadcast, or a cooperative transmission from multiple relays to a given relay. In fact, this problem is computationally intractable in that it can be shown to be an NP-hard problem [22].

The extent of collaboration between nodes in a route depends critically on the channel state information (CSI) that is available at the relay nodes. Hence, the CSI acquisition processes and their energy efficiencies must be considered when selecting the optimal route [9]. For example, if the transmitting nodes know the channel gains on their outgoing links, they can adjust their transmission power as a function of this CSI. Hence, the availability of CSI increases the choice of the cooperative transmission schemes. We explicitly model the energy cost of acquiring CSI, and optimize among the various choices of cooperation and CSI acquisition that they require.

Another related aspect that has received relatively little attention is the need to distinguish between slow fading and fast fading channels when building energy-efficient routes. Slow fading channels do not change for a long time, and an optimum collaborative route stays optimum for many messages. Thus, the relative cost for obtaining and communicating CSI per unit time is negligible. Fast fading channels, on the other hand, change from one transmission to another. This can be due to fast moving objects in the environment or due to the movement of the nodes themselves. Thus, the CSI at the transmitter node has to be refreshed often. The significant cost for this process must be factored into the total cost calculation. Moreover, since the channel states change quickly, obtaining global CSI about many nodes in the network in a time-bound manner is practically difficult. Therefore, nodes and, consequently, cooperative transmission routes should use instantaneous CSI of only *local links*.

For a wireless network with slow fading, the problem of computing an optimal cooperative route (with broadcasts and beamforming) was first considered in [21] and shown to be NP-hard in [22]. Both these papers also suggested heuristics for computing a collaborative route and implicitly assumed slow fading. These heuristics do not extend to the fast fading scenario due to the difficulties described above. Cooperative routing in a network with fast fading channels using a limited class of routing schemes over a specific network topology was considered in [23]. However, the cost of acquiring CSI at the transmitting nodes was not factored in, and the heuristics developed were not decentralized.

In this paper, we consider large networks with cooperative routing that exploit broadcasts and distributed beamforming. We model the problem of computing the energy optimal path as a stochastic network optimization problem. The goal of the optimization problem is to jointly optimize over (i) the information structure, i.e., the CSI available at each node, and (ii) the transmission policy, i.e., the transmission power and phase of the signal transmitted by each node as a function of the available CSI. Thus, in effect, we model the system as a feedback control system for which we would like to compute an optimal scheme, i.e., both the information structure and the transmission policy, that minimizes the average energy consumption in the network. In this setting, we make the following novel contributions:

A. Slow Fading Channels

We propose a sequence of optimization problems to compute a minimal-energy cooperative route in different sub-classes of all cooperative transmission scheme. These inherently contain traditional point-to-point routes. We show how to solve these optimization problems in a decentralized manner, with different problems in the sequence requiring different degrees of decentralization. The solutions to this sequence of problems do converge to the optimal (but NP-hard to compute) solution. Our approach thus provides a systematic way to trade-off computational complexity and decentralization with energy-efficiency.

B. Fast Fading Channels

For fast fading channels, we formulate the problem of computing a route as a decentralized control problem described by a Markov decision process (MDP). We consider a class of cooperative routing schemes and explicit mechanisms (and their costs) to acquire the required local CSI. We compute, in a scalable and distributed manner, an optimal cooperative scheme in a sub-class of cooperative schemes where local nodes form cooperative relay sub-networks, and data is routed from one cooperative relay subset to another. We do not consider cooperation between *different* relay subsets due to the high energy cost and complicated coordination mechanisms required to acquire CSI. The resulting scheme adapts the cooperative route at the time-scale of changes in shadowing, while the cooperation scheme within a relay sub-network adapts to the instantaneous channel states. We note that while we consider only a sub-class of information structures and policies, our problem formulation is generic enough to allow for the investigation of other information structures and transmission policies as well.

The remainder of the paper is organized as follows: Section II describes the system model and the physical constraints. Sections III and IV consider the cooperative routing problem for slow fading and fast fading channels, respectively. Section V presents our conclusions and discusses future research directions.

II. Problem Formulation

A. System Model

We consider a network with a set of nodes, V, that communicate with each other using wireless links. Time is divided into frames of length less than or equal to the coherence time of wireless links in the network, which is the time for which the channel gain on the links remain constant (see, for example, [1]). Frame level synchronization is assumed between the transmitters; analysis to take into account the effect of imperfect synchronization or the overhead that synchronization entails is beyond the scope of this paper. Each frame is split into a time of T_d symbol durations for data transmission, and a time of T_c symbol durations for exchange of network control packets (including acquisition and distribution of CSI). The channel between nodes i and j during frame[1] t has an exponentially distributed power gain $H_{ij}(t)$, with a mean $\bar{H}_{ij}$, which need not be the same for all links. Thus, if at time t, node i transmits a message at power $P_i(t)$, the message is received at node j with power $H_{ij}(t)P_i(t)$. To model the gain, we assume the standard Rayleigh fading model, where the received complex baseband signal has real and imaginary parts that are Gaussian distributed and independent. The Rayleigh model physically arises when the received signal consists of a large number of multipaths whose attenuation and phase are independent of each other [1]. Also, the channel gains are assumed to be independent ergodic stochastic processes.[2] We denote the corresponding matrix in $\mathbb{R}^{|V|\times|V|}$ by $H(t)$. The sequence of random variables, $H_{ij}(t), t=1,\ldots,N$, has some arbitrary higher order statistics. The channels are assumed to be reciprocal, i.e., $H_{ij}(t)=H_{ji}(t)$, which is often the case, for example, in time division duplex systems [1]. Also, for computations we assume the standard path loss model in which the mean channel gain, $\bar{H}_{ij}$, between nodes i and j, is inversely proportional to a power of the distance between the two nodes. Note, however, that the methods in this paper apply for any arbitrary path loss model. In general, the mean channel gains can change slowly due to *shadowing* – the computations in this paper will need to be repeated at this slow time-scale. A more detailed treatment on the standard channel model used in this paper can be found in [1].

For the computation of the transmission power required on a specific link, we make the following assumptions. All transmissions in the network occur at an instantaneous data rate of r bits/s/Hz using a bandwidth B Hz. The bandwidth is sufficient so that the transmissions do not interfere significantly with each other; the power spectral spectral density of interference plus noise equals N_0. A message is transmitted within one frame, so that no coding across frames occurs. A received signal can be successfully decoded at a receiver if its power exceeds a power threshold, γ. For purposes of illustration, we use the Shannon capacity formula to determine the following relationship between the threshold and r:$\gamma = N_0B(2^r-1)$; this implies the use of strong codes and a large number of bits in a single frame [1]. Similar threshold formulas exist for MFSK and MQAM modulation constellations with or without error correction coding [24]. The methods in this paper do not depend on the specific form of the (monotonic increasing) function that maps r to γ.

We model the wireless network as an undirected graph $G=(V,E)$, where E is the set of links. A link between nodes i and j exists if $\bar{H}_{ij}(t)$ is greater than a small pre-defined threshold.[3]

A node can only decode transmissions from its neighboring nodes. Let $\mathcal{N}_h(i)$ denote the set of h-hop neighbors of i, i.e., there exists a path of at most h hops from node i to every node in $\mathcal{N}_h(i)$ in the graph $G(V,E)$. The class of transmission policies, denoted by $\mathcal{P}$, is such that every policy is a sequence of transmissions, where each transmission is of one of the following two types. Note that the same class was considered in [21], [22], but for slow fading channels only.

1) Broadcast: Wireless environments inherently have the broadcast advantage, i.e., using a single transmission, a node can forward a message to multiple nodes, thus saving power. In particular, a message is transmitted from a single node i with power $P_i(t)$. More than one of its neighbors (in the set $\mathcal{N}_1(i)$) may be able to decode the message. As mentioned, a node $k\in\mathcal{N}_1(i)$ can successfully decode the message if $P_i(t)H_{ik}(t) \geq \gamma$. Direct (point-to-point) transmission, in which the transmission power is such that at most one node decodes the packet, is a special case. Thus, to forward a message from node i to a set of nodes $\mathcal{S}$, the minimum broadcasting energy needed is $T_d\gamma/\min_{k\in\mathcal{S}} H_{ik}(t)$, while forwarding the message one at a time consumes energy $T_d\gamma\sum_{k\in\mathcal{S}} 1/H_{ik}(t)$.

2) Cooperative Beamforming: If multiple neighboring nodes of a node, j, have a message which needs to be forwarded to node j, they can phase-align and scale their transmit signals so that they are all received coherently by j. In particular, at time t, the amplitude of signal received by j from a node i is $\sqrt{P_i(t)H_{ij}(t)}$. Node j can decode a message transmitted coherently by a subset $\mathcal{B}$ of its neighbors if and only if the total received power, exceeds γ, $\left(\sum_{i\in\mathcal{B}}\sqrt{P_i(t)H_{ij}(t)}\right)^2 \geq \gamma$. Subject to this threshold constraint, the total transmit power consumption is minimized when [21]

$$P_i(t) = \frac{H_{ij}(t)\gamma}{\left(\sum\limits_{k\in\mathcal{B}} H_{kj}(t)\right)^2}.$$

For this scheme to work, each node in $\mathcal{B}$ must know the channel gain and phase of its wireless link to node j, and also the link gain sum $\sum_{k\in\mathcal{B}} H_{kj}(t)$. For more physical layer details of this scheme, we refer the reader to [9] and [25], and references therein.

In this paper, we assume that a node does not store observations from previous times to decode a message. Doing so can further improve performance [26]. Neither do we consider general multi-node to multi-node cooperative transmissions which

[1]We will use frame t and time t interchangeably.

[2]Our analysis can be extended to other ergodic channel fading models such as Rician and Nakagami-m fading [1]. However, this would also involve redoing the analysis in [9] for these channel models for use in Section IV on fast fading in this paper.

[3]Note that, in general, we can consider a very low threshold, which would correspond to fully connected network scenario in which the transmission by a node in the network is heard by all the other nodes. Doing so increases the complexity of finding an optimal route and yields only a marginal performance improvement since the weak links will almost never be used for data transmissions.

require significantly more complicated symbol-level synchronization. We note that preliminary mechanisms for ensuring synchronization among simple distributed nodes for cooperative beamforming were proposed in [27], which showed energy savings even with imperfect synchronization. Unlike the beamforming case, multi-node to multi-node cooperation makes it necessary for nodes to synchronize to multiple relays simultaneously.

The transmission of messages between different source destination pairs can be optimized separately because the bandwidth is assumed to be high enough to mitigate interference. Hence, in the rest of the paper, we will aim to minimize the average energy consumed in the network to transmit a single message from a source, s, to a destination, d, via intermediate cooperative relays, subject to the following constraint: The end-to-end outage probability, i.e., the probability that the destination d is not able decode the packet, should not exceed $P_{\rm out}^{\rm rte}$ after a finite (fixed, but arbitrarily large) number of transmissions. Note that, as we will see, even under the assumption of interference mitigation, the problem of computing an energy optimal cooperative routing scheme is hard.

B. Dynamics

Let $\mathcal{R}(t)$ denote the set of nodes which have decoded the message until time t. The dynamics of the set $\mathcal{R}(t)$ are given by

$$\mathcal{R}(t+1) = \mathcal{R}(t) \cup \left\{ j \in \mathcal{L}(t) : \sum_{i \in \mathcal{N}_1(j)} P_i(t) H_{ij}(t) \geq \gamma \right\}. \tag{1}$$

Here, $\mathcal{R}(0) = \{s\}$, as no relay has decoded the message at time 0. The set $\mathcal{L}(t)$ denotes the set of receivers that can possibly decode at time t. For a broadcast, $\mathcal{L}(t)$ is the set of neighbors of the broadcasting node; and for beamforming, $\mathcal{L}(t)$ is the node its neighbors beamform to coherently. Hence, a node not in $\mathcal{R}(t)$ is included in $\mathcal{R}(t+1)$ if it successfully decodes the message transmitted by one or more of its neighbors.

III. Slow Fading

A. Problem and Computational Complexity

In the case of slow fading, the channel states do not change for a long time. Consequently, the cooperative routing problem, *given the channel states*, is a stochastic network control problem where the underlying dynamics are deterministic. Also, this implies that we must set $P_{\rm out} = 0$, since "outage" in this case implies a service interruption for a long time, which is often unacceptable. In particular, for given channel states and a transmission scheme, the destination d either decodes the message after a finite number of hops with probability one, or else (practically) never decodes the message.

Thus, to compute the optimal cooperative route, we have the following optimization problem:

$$\min_{\{P_i\}} \quad \sum_{i=1}^{|V|} \sum_{t=1}^{N} P_i(t) \quad \text{subject to} \quad d \in \mathcal{R}(N), \text{ and } P \in \mathcal{P} \tag{2}$$

where the dynamics of $\mathcal{R}(t)$ are given by (1). We use $\mathcal{P}$ to denote the class of data transmission schemes such that for each $P \in \mathcal{P}$ and each t, the $P_i(t)$'s correspond to either a broadcast or beamforming transmission. Thus, every $P \in \mathcal{P}$ satisfies one of the following properties at each $t = 0, 1, \ldots$:

i) Only one node broadcasts to one or more nodes, i.e., $|\{i \in V : P_i(t) > 0\}| = 1$, or
ii) Multiple neighbors of only one node, say j, beamform to it, i.e., $\exists\, j \in V$s.t. $\{i \in V : P(i,t) > 0\} \subseteq \mathcal{N}_1(j)$. In this case, we assume only node j attempts to decode the beamforming transmission, as reflected by the dynamics in (1).

In the case of slow fading, the energy consumed in obtaining the desired CSI at any node in the network can be amortized over a large number of message transmissions, and is negligible. Hence, we assume that the energy consumed for obtaining CSI is zero. However, the solution to the problem in (2) requires centralized computation and, hence, coordination between all the nodes in the network. Moreover, the complexity of solving the above optimization problem is prohibitive. In particular, the problem in (2) was shown to be NP-hard in [22]. For a fully connected network of n nodes, an algorithm of complexity $O(n2^n)$ was derived in [21]. The algorithm is equivalent to a shortest path computation on a graph with virtual nodes that correspond to the 2^n different subsets of V. In the algorithm, two virtual nodes, corresponding to subsets $\mathcal{R}_1$ and $\mathcal{R}_2$, are connected by an edge if the message can be forwarded from $\mathcal{R}_1$ to all the (additional) nodes in $\mathcal{R}_2$ using either broadcast from a node in $\mathcal{R}_1$ or beamforming from multiple nodes in $\mathcal{R}_1$ to an additional node in $\mathcal{R}_2$. The link cost for the edge is the energy consumed in forwarding a message from $\mathcal{R}_1$ to $\mathcal{R}_2$ using either a broadcast or a beamforming transmission.

B. Novel Cooperative Routing Policies: Complexity and Performance Trade-Off

In this section, we formulate a sequence of optimization problems that allows us to trade off complexity with performance. Moreover, performance can be traded off with the degree of decentralization as well, with better solutions requiring more CSI knowledge at each node.

The basic idea is the following. We consider a class of routing policies, where each policy is a sequence of sub-policies. Each sub-policy belongs to a policy sub-class which allows cooperation between nodes that are at most h hops from each other on graph G. The optimal sub-policy from this sub-class is chosen to forward a message from node i to node j (where $j \in \mathcal{N}_h(i)$). And, the optimal sequence of sub-policies is then computed for data transfer from the source to the destination.

In other words, we decompose the optimization problem into two smaller optimization problems, where the optimization *within* a local neighborhood of two nodes less than h hops away from each other involves finding "the best collaborative transmission policy", while the optimization over the sequence of sub-policies involves finding "the best sequence of sub-policies" that gets a message from the source to the destination. This approach is computationally much simpler and decentralized, but clearly suboptimal – only in the case where h is the diameter of the network is optimality achieved.

Note that while we consider the neighborhood of cooperation to be defined by the number of hops on graph G, the methods in this paper immediately generalize to the case where the neighborhoods are defined by other metrics, for example, the physical distance between two nodes, the mean channel gain between nodes, or a specific number of closest neighbors (on graph G) of a node.

We now put this qualitative description into a more formal framework. We first state the following definitions that will be required to describe the cooperative routing policies.

We define a directed super-graph G_h that is constructed from the given graph $G = (V, E)$ as follows:

Definition 3.1 (Super-Graph G_h): Two nodes i and j in the directed graph $G_h = (V, E_h)$ are joined by two directed edges in opposite directions, if there exists a path of at most h hops from node i to node j in G, i.e., $j \in \mathcal{N}_h(i)$.

Definition 3.2 (Sub-Graph $G(i,h)$): For each node, i, the undirected sub-graph $G(i,h) = \big(V(i,h), E(i,h)\big)$ of G consists of the h-hop neighbors of i and the edges in G that connect these nodes. Therefore,

$$V(i,h) = \{v \in V : v \in \mathcal{N}_h(i)\},$$
$$E(i,h) = \{(v_1, v_2) \in E : v_1, v_2 \in V(i,h)\}.$$

We now define a class $\mathcal{P}_h(i,j)$ of sub-policies that get a message from node i to node j such that nodes that are not within h hops of i do not transmit, and the message reaches j after a finite number of steps irrespective of which nodes (other than i) decoded the message before.

Definition 3.3 (Policy Sub-Class $\mathcal{P}_h(i,j)$): The class $\mathcal{P}_h(i,j) \subseteq \mathcal{P}$ is the set of sub-policies such that for each sub-policy $P \in \mathcal{P}_h(i,j)$, (i) $P_k(t) = 0,\ \forall\, k \notin V(i,h), \forall t$, and (ii) if $\mathcal{R}(0) = \{i\}$, then $j \in \mathcal{R}(|V(i,h)|)$, where the dynamics of $\mathcal{R}(.)$ are given in (1).

Among all policies in which nodes not in $V(i,h)$ do not transmit, the constraint $j \in \mathcal{R}(|V(i,h)|)$ eliminates unnecessary transmissions that do not successfully deliver the message to any node; hence, this constraint does not compromise energy-efficiency.

Next, we define the energy cost for transmitting a message between two nodes connected by an edge in the super-graph G_h. This definition will be useful when we derive distributed algorithms to compute an optimal sequence of sub-policies.

Definition 3.4 (Edge Cost $C_h(i,j)$): The cost, $C_h(i,j)$ for an edge (i,j) in super-graph G_h is the energy consumed in the network to transmit a message from node i to node j using a minimum energy policy in $\mathcal{P}_h(i,j)$.

Note that to compute $C_h(i,j)$, *we only need the CSI* on the links contained in $E(i,h)$. In particular, it involves solving the optimization problem in (2) associated with the sub-graph $G(i,h)$ with the destination node $d = j$.

Definition 3.5 (Policy Class $\mathcal{T}_h(v_1,\ldots,v_p)$): Consider a path, $(v_1,\ldots,v_p)$, from node v_1 to node v_p on super-graph G_h. The sub-class $\mathcal{T}_h(v_1,\ldots,v_p) \subseteq \mathcal{P}$ is the set of *factored policies* given by the Cartesian product

$$\mathcal{T}_h(v_1,\ldots,v_p) = \mathcal{P}_h(v_1,v_2) \times \cdots \times \mathcal{P}_h(v_{p-1},v_p). \tag{3}$$

Thus, $\mathcal{T}_h(v_1,\ldots,v_p)$ consists of a sequence of transmission schemes in policy sub-classes $P_h(v_i,v_j)$s along the path $(v_1,\ldots,v_p)$ from node v_1 to node v_p on super-graph G_h. Note that even the set of transmission schemes defined by $\mathcal{T}_1$ is more general than the class of the traditional one-hop routing schemes because it allows for cooperation between nodes which are within one hop of each other on graph G.

Thus, every policy in $\mathcal{T}_h(v_1,\ldots,v_p)$ consists of a sequence of sub-policies, where the first sub-policy is in $\mathcal{P}_h(v_1,v_2)$, the second sub-policy is in $\mathcal{P}_h(v_2,v_3)$, and so on. Thus, is it is a sequence of cooperative transmission sub-policies to forward a message from node v_1 to node v_2, node v_2 to node v_3, and so on. We now define a policy class $\mathcal{T}_h$ that is the union of the factored policy classes over all the paths in G_h.

Definition 3.6 (Policy Class $\mathcal{T}_h$): Let Φ_h be the set of all paths on G_h. The policy class $\mathcal{T}_h$ is defined as

$$\mathcal{T}_h = \cup_{(v_1,\ldots,v_p)\in\Phi_h} \mathcal{T}_h(v_1,\ldots,v_p). \tag{4}$$

Thus, the class $\mathcal{T}_h$ consists of all sequences of sub-policies such that each sub-policy belongs to $\mathcal{P}_h(i,j)$ for some $i,j \in V$. Thus, it corresponds to cooperative transmission policies that have as their components smaller cooperative transmission sub-policies, each of which sends data between nodes i and j that are separated by at most h hops. Note that such a sub-policy is in $\mathcal{P}_h(i,j)$ and, hence, can use up to $|V(i,h)| \geq h$ transmissions. *We can interpret h as the degree of cooperation allowed between nodes.* Thus when h is the diameter of the graph G, an optimal policy in $\mathcal{T}_h$ is globally optimal. Also, from the definitions it follows that for $h_1 > h_2$, $\mathcal{T}_{h_2} \subseteq \mathcal{T}_{h_1}$, and hence, the energy consumed by an optimal policy in $\mathcal{T}_{h_1}$ is always less than or equal to the energy consumed by an optimal policy in $\mathcal{T}_{h_2}$.

Remark: Note that the threshold for the mean channel gain used to define the graph G plays a key role in defining the number of hops between two nodes in G. A lower value of threshold can allow nodes to be connected by an edge in G, which otherwise would have been connected by an edge only in G_h for $h > 1$. For example, since in one spectrum use, a message can be exchanged only between nodes that are neighbors in G (see Section II-A), the class of routing policies for graph G_h is smaller than the class of policies for sub-graph $\hat{G}_1$ (corresponding to $h = 1$) obtained from $\hat{G} = G_h$ that is induced by a lower threshold.

C. Example

Consider a wireless network represented by the graph shown in Fig. 1(a), in which nodes that are at most $\sqrt{2}$ distance units apart are connected by an edge. For simplicity, the channel gain equals the path loss, which is taken to be proportional to the inverse fourth power of the distance from the transmitter. We now construct the super-graph G_h and its corresponding edge costs for $h = 1, 2, \ldots$.

The first super-graph G_1 equals G, by definition. We now describe the computation of the link costs for links emanating from node a, i.e., $C_1(a,v),\ v \in \{b, g, h\}$ using the cost calculations of Section II-A. Other link costs (shown adjacent to the corresponding links in Fig. 1(a)) can be computed in a similar manner. Note that a unit of energy refers to the energy required to transmit one packet over a distance of 1 m.

Consider the sub-graph $G(a,1)$, which consists of nodes a, b, g, and h, and edges (a,b), (a,g), (a,h), and (g,b). Then the sub-class of policies $\mathcal{P}_1(a,v)$ is the set of policies that get a

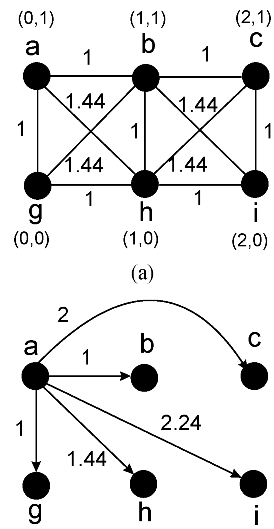

Fig. 1. Construction of graph G_h. (a) Graph $G = G_1$ with link costs. (b) Graph $G(a, 2)$ with link costs.

message from node a to node v using only nodes a, b, g, and h, for $v \in \{b, g, h\}$. The optimal policies in these classes and the corresponding energy consumption can be computed using the dynamic programming algorithm in [21]. For data transmission to b, it turns out that the optimal policy is for a to transmit directly to b, which costs unit energy. Similarly, direct transmission is optimal to get a message to g. For data transmission to h, it turns out that the optimal policy in $\mathcal{P}_1(a, h)$ is one where a first broadcasts to nodes b and g using unit energy, and then a, b, and g beamform to h; this consumes a total energy of 1.44 units.

The second super-graph G_2 is obtained from G by connecting nodes that are within two hops of each other. Again, we describe the construction of links costs for links emanating out of node a. The subgraph $G(a, 2)$ of G is shown in Fig. 1(b). The class of policies $\mathcal{P}_2(a, v)$, for $v \in \{b, c, g, h, i\}$, is the set of all policies in $\mathcal{P}$ that get the data from node a to node v. It turns out that energy optimal policies for node a to send a message to nodes b, g, and h are the same as those in $G(a, 1)$. Node a communicates with node c by first transmitting the message to node b, which then forwards it to node c. For communication with node i, node a first broadcasts the message to nodes b and g, after which a, b, and g beamform to node h. Then, nodes b and h beamform to forward the message to i.

An optimal point-to-point conventional route on graph G from node a to node i consumes an energy of 3 units. The optimal policy in $\mathcal{T}_1$, in which data is forwarded from one node to another node within 1 hop, is to send data from a to h (which involves a broadcast to b and g, and cooperative beamforming from a, b, and g to h), followed by a direct transmission from h to i. The energy consumption for this policy is 2.44. The optimal policy in $\mathcal{T}_2$, for the same purpose, consumes an even lower energy of 2.24 units. For this graph, an optimal policy in $\mathcal{T}_2$ is also globally optimal as all nodes are at most two hops apart from each other.

D. Distributed Optimization and Complexity

Let $\Delta(h)$ be the degree of the super-graph G_h consisting of n nodes. The following result about complexity follows from the dynamic programming approach of [21].

Lemma 3.7: All the costs, $\{C_h(i,j) : (i,j) \in V_h\}$, for G_h can be computed in $O\big(n\Delta(h)2^{\Delta(h)}\big)$ time.

Note that this can be significantly lower than the complexity of $O\big(n2^n\big)$ of computing a globally optimal policy using the dynamic programming algorithm in [21]. Furthermore, only the CSI over the edges in the sub-graph $G(i, h)$ is needed to compute $C_h(i, j)$.

The following result shows that the computation of an optimal policy, π in $\mathcal{T}_h$, can be done in a distributed manner. Specifically, we show that given $C_h(i, j)$'s, the problem is equivalent to solving a shortest path problem on the super-graph G_h, which can be done using the distributed Bellman-Ford algorithm [20]. Let the optimal route from source s to destination d on the super-graph G_h be $s = v_1^*, v_2^*, \ldots, v_{p^*}^* = d$. Let $\pi^*(v_j, v_{j+1})$ denote a minimum energy sub-policy in $\mathcal{P}_h(v_j^*, v_{j+1}^*)$, for $j = 1, \ldots, p^* - 1$.

Lemma 3.8: The policy defined by the ordered sequence of sub-policies $\pi^* = \Big(\pi^*(v_1, v_2), \ldots, \pi^*(v_{p-1}, v_p)\Big)$ is a minimum energy policy in $\mathcal{T}_h$ for data transfer from v_1^* to v_p^*. Also, the energy consumption to forward a message from s to d using this policy is given by $\sum_{j=1}^{p^*-1} C_h(v_j^*, v_{j+1}^*)$.

Proof: Consider a path $s = v_1, v_2, \ldots, v_p = d$ from s to d on the super-graph G_h from s to d, and the corresponding class of policies $\mathcal{T}_h(v_1, v_2, \ldots, v_p)$. Any policy, π, in this class consists of an ordered sequence of sub-policies $\pi = \Big(\pi(v_1, v_2), \ldots, \pi(v_{p-1}, v_p)\Big)$, where $\pi(v_j, v_{j+1}) \in \mathcal{P}_h(v_j, v_{j+1})$. The sub-policy $\pi(v_j, v_{j+1})$ consumes energy, say $C_\pi(v_j, v_{j+1})$, to forward a message from v_j to v_{j+1}. Then the total energy consumption to forward a message from s to d using policy $\pi \in \mathcal{T}_h(v_1, v_2, \ldots, v_p)$ is given $\sum_{j=1}^{p-1} C_\pi(v_j, v_{j+1})$. Using the definitions of $\mathcal{P}_h(v_j, v_{j+1})$ and $\mathcal{T}_h(v_1, v_2, \ldots, v_p)$, we can compute an optimal policy $\hat{\pi} \in \mathcal{T}_h(v_1, v_2, \ldots, v_p)$ as follows:

$$\hat{\pi} = \underset{\{\pi:\pi(v_j,v_{j+1})\in\mathcal{P}_h(v_j,v_{j+1}),j=1,\ldots,p-1\}}{\operatorname{argmin}} \sum_{j=1}^{p-1} C_\pi(v_j, v_{j+1})$$
$$= \Bigg(\underset{\pi(v_1,v_2)\in\mathcal{P}_h(v_1,v_2)}{\operatorname{argmin}} C_\pi(v_1, v_2),$$
$$\ldots, \underset{\pi(v_{p-1},v_p)\in\mathcal{P}_h(v_{p-1},v_p)}{\operatorname{argmin}} C_\pi(v_{p-1}, v_p)\Bigg)$$

since $C_\pi(v_j, v_{j+1})$ only depends on $\pi(v_j, v_{j+1})$, which forwards the message from v_j to v_{j+1} without making use of previous transmissions in $\pi(v_1, v_2), \ldots, \pi(v_{j-1}, v_j)$ (see Def. 3.3). The total energy consumption corresponding to the policy $\hat{\pi}$ is given by $\sum_{j=1}^{p-1} C_{\hat{\pi}}(v_j, v_{j+1}) = \sum_{j=1}^{p-1} C_h(v_j, v_{j+1})$, which follows from the definition of $C_h(v_j, v_{j+1})$. Thus, the energy cost of using $\hat{\pi}$ to forward a message from s to d using an optimal policy in $\mathcal{T}_h(v_1, v_2, \ldots, v_p)$ is equal to the sum of the costs on the edges in the path $(v_1, v_2, \ldots, v_p)$ in graph G_h. Since, the set $\mathcal{T}_h$ is the union of $\mathcal{T}_h(v_1, v_2, \ldots, v_p)$'s over all possible paths from $s = v_1$ to $d = v_p$ on the graph G_h, the optimal policy in

$\mathcal{T}_h$ is the minimum energy policy in $\mathcal{T}_h(v_1^*, v_2^*, \ldots, v_{p^*}^*)$, where $s = v_1^*, v_2^*, \ldots, v_{p^*}^* = d$ is the shortest path in graph G_h. The result then follows. ■

The following result then follows from Lemmas 3.7 and 3.8.

Theorem 3.9: The complexity of computing an optimal policy in the class $\mathcal{T}_h$, using dynamic programming for computing costs and the distributed Bellman-Ford algorithm for solving the shortest path problem on G_h, scales as $O\big(n\Delta(i)2^{\Delta(i)} + n^2\log(2n)\big)$.

We now summarize the main steps for computing an optimal policy in $\mathcal{T}_h$ for data communication between nodes s and d.

1) Construct the super graph G_h, in which two nodes are connected by an edge if they are separated by at most h hops in the original graph G.
2) For each node i in the node set V,
   a) Construct the sub-graph $G(i,h) = \big(V(i,h), E(i,h)\big)$, which consists of the node i and the nodes which are within h hops of i, and the edges between these nodes in the graph G.
   b) For all nodes $j \in V(i,h)$, compute the optimal policy in the class $\mathcal{P}_h(i,j)$ using dynamic programming, and the corresponding minimum cost, $C_h(i,j)$ to forward a message from i to j.
3) Compute a shortest path on graph G_h between s and d, where cost of edge (i,j) is given by $C_h(i,j)$, using the distributed Bellman-Ford algorithm. The sequence of sub-policies corresponding to this path gives an optimal policy in $\mathcal{T}_h$.

Remark 1: The computational complexity of computing an optimal policy in $\mathcal{T}_h$ grows quickly with h (from Lemma 3.7). To reduce it, one option is to reduce h. Alternately, for a given h, we can consider a smaller policy sub-class $\hat{\mathcal{P}}_h(i,j) \subseteq \mathcal{P}_h(i,j)$. The optimal sequence sequence of these sub-policies can then be computed using the above approach.

Remark 2: For neighbor inclusion in the super-graph based on the distance d or the closest η neighbors, we can define classes of sub-policies, $\mathcal{P}_d(i,j)$ and $\mathcal{P}_\eta(i,j)$, respectively, in a manner identical to $\mathcal{P}_h(i,j)$. These policy subclasses, in turn, define the corresponding classes of policies (analogous to $\mathcal{T}_h$) over which we can optimize (for energy-efficiency) by computing a shortest path for the appropriate induced super-graphs and their corresponding edge costs.

Remark 3: Our approach provides a way to trade-off computational complexity with performance. Thus, for a given amount of computational budget, we can solve the local cooperation problem for the highest possible h, and then route over the induced super-graph.

E. Sequence of Solutions

Consider the graph G shown in Fig. 2. The channel gain on each link is assumed to be constant, and is determined by the distance between the nodes and the path loss model described in Section II-A. Since any two nodes in G are at most 4 hops apart, we compute the optimal policy in classes $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$, and the globally optimal $\mathcal{T}_4$ for forwarding a message between from any node to any other node. The energy consumed per message by optimal policies in different classes, when averaged over all the 72 possible node-destination pairs in G, were as follows: An optimal policy in $\mathcal{T}_1$ consumed an average of 2 units of energy,

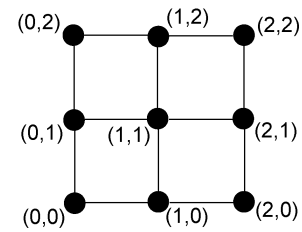

Fig. 2. Example: Graph G.

while optimal policies in $\mathcal{T}_2$, $\mathcal{T}_3$, and $\mathcal{T}_4$ were identical and consumed an average of 1.722 units of energy. (Recall that a unit of energy is the energy required to forward a message from one node to another node at a distance of 1 m.) Thus, increasing the possibilities for cooperation is quite beneficial, though the gains saturate eventually.

F. Computational Results for a Random Network

The above examples assumed a specific graph. We now consider optimal policies in different classes for a random network that consists of 20 nodes and is located within a square of length 5 units. Each node's location is random, and is uniformly distributed over the square's area. For each instance, nodes within a distance of 1.7 units were taken to be connected by an edge in G.

We limit our attention to $h = 2$. The computation of an optimal sub-policy in some sub-classes $\mathcal{P}_2(i,j)$'s becomes prohibitive, and thus makes the computation of an optimal policy in $\mathcal{T}_2$ difficult. Hence, as discussed in Section III-D, we consider the following subclasses of $\hat{\mathcal{P}}_2(i,j)$ of $\mathcal{P}_2(i,j)$ for transmitting a message from i to j, where $(i,j) \in E_2$.

1) No Cooperation (NC) Policy: This corresponds to traditional point-to-point routing policies, with no cooperation between nodes. The optimal policy in this class is given by a shortest path computation on graph G, which can be done in a distributed manner using the Bellman-Ford algorithm.

2) $\mathcal{T}_1$: This class, corresponding to $h = 1$ was defined in Section III-B.

3) Broadcast-Cooperate Policies (BC): This class consists of the following type of transmissions from node i to node j for $(i,j) \in E_2$. Node i broadcasts a message to some or all of its neighbors. The nodes in the set of neighbors common to i and j, given by $\mathcal{M}_2 = \mathcal{N}_1(i) \cup \mathcal{N}_1(j)$, that successfully decode the message then cooperatively beamform to forward the message to node j. Policies in this class route data from one relay subset to another via single nodes. The complexity of computing an optimal policy in this class can be shown to scale as $n^2\log(2n)$ for any graph G with n nodes.

4) $\mathcal{T}_1 \cup \mathrm{BC}$: This is the class of policies given by the union of the policies in $\mathcal{T}_1$ and the class BC.

The energy cost of sending a message from a node a to a node b need not be the same as that from b to a even though the links are reciprocal. This is because of the inherent asymmetry in the energy cost of broadcast and beamforming transmissions. The network thus consists of 380 ordered source-destination pairs. To get a complete picture of the energy savings possible anywhere in the network, we compare the statistics of the average energy consumed to send a message between different source-destination pairs. The cumulative distribution function

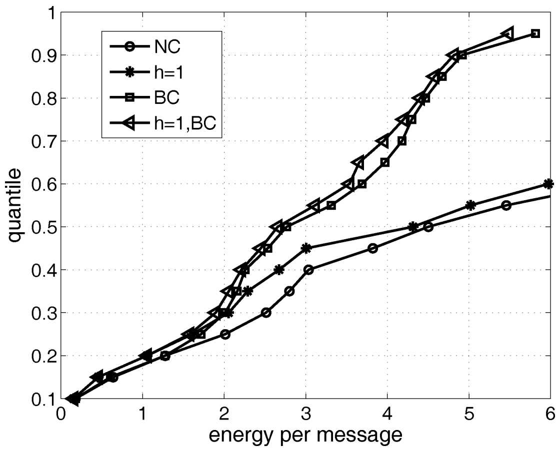

Fig. 3. CDF of energy per message for optimal policies in NC, $\mathcal{T}_1$, BC, ($\mathcal{T}_1 \cup$ BC) for a 20-node random network.

TABLE I
AVERAGE ENERGY PER MESSAGE FOR OPTIMAL POLICIES IN NC, $\mathcal{T}_1$, BC, AND ($\mathcal{T}_1 \cup$ BC) FOR A 20-NODE NETWORK

Class	Avgerage energy for optimal policies
NC	5.13
$\mathcal{T}_1$	4.54
BC	3.05
$\mathcal{T}_1 \cup$ BC	2.93

(CDF), which gives an idea of the entire probability distribution, of the energy consumed for different source-destination pairs is plotted in Fig. 3 for the four different classes. And, the energy consumption (averaged over all source-destination pairs) is shown in Table I.

We see that the optimal point-to-point routing policies perform poorly compared to even simple cooperative policies, namely, optimal policies in classes $\mathcal{T}_1$ and BC. An interesting observation is that optimal BC policies, which only consist of two transmissions to forward a message from i to j that are within 2 hops of each other, perform almost as well as the more complex policies in $\mathcal{T}_1 \cup \mathrm{BC}$, which, in addition, allows arbitrary cooperation in forwarding a message from i to j on sub-graph $G(i, 1)$. This observation will motivate the class of policies that we will consider for fast fading channels in the next section.

The heuristics proposed in [21], [22] take advantage of cooperative diversity to forward a message from a source to destination. Unlike our approach, these heuristics do not take advantage of broadcasts after the first transmission. While finding the performance bounds for an optimal policy in $\mathcal{T}_h$ for any h less than the diameter of the graph G remains an open problem, the following simple example illustrates how exploiting broadcasts in intermediate steps can help improve the energy-efficiency of the overall route.

Consider five nodes whose coordinates are shown in Fig. 4(a), and a graph G induced as shown in Fig. 4(b). Then the two heuristics in [21] and the heuristic in [22] all yield the following policy: (1) a broadcasts to b, (2) a and b beamform to c (or g), (3) a, b and c (g) beamform to e. Instead, the optimal policy in $\mathcal{T}_1$ for the graph G consists of the following steps: (1) a broadcasts to b, (2) b broadcasts to c and g, and (3) c and g beamform to e. Thus, the optimal policy utilizes a broadcast as an intermediate step, which is not allowed by the heuristics in [21], [22]. The first policy consumes 7.8% more energy than the optimal policy in $\mathcal{T}_1$.

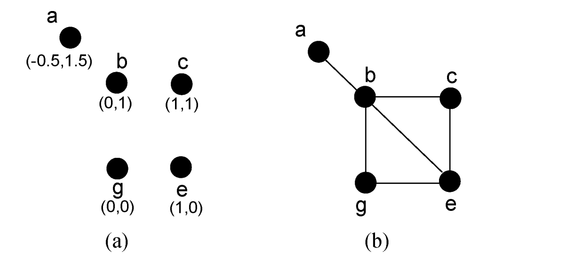

Fig. 4. Example: Broadcast advantage. (a) Node locations. (b) Induced graph G.

G. Improving Energy-Efficiency Further Using Node Reuse

We now describe a refinement of the optimal policy in class $\mathcal{T}_h$ that can only reduce the total energy consumed to send a message from the source to the destination. Specifically, for a policy composed of a sequence of sub-policies, we allow a sub-policy to exploit the nodes that decoded the message during the execution of earlier sub-policies in the sequence.

The following theorem provides the motivation for the approach. It shows that if node v_l precedes node v_m in the minimum energy path in the super-graph G_h, then node v_m successfully decodes a message only after v_l has decoded it. The result is obvious for point-to-point routing policies.

Theorem 3.10: Let the minimum cost path on the graph, G_h, between the source, s, and the destination d, consist of the ordered sequence of nodes, say $s, v_1, \ldots, v_p, d$. If $l < m$, then v_m successfully decodes the message only after v_l:

$$\min\{t : v_m \in \mathcal{R}(t)\} > \min\{t : v_l \in \mathcal{R}(t)\}. \quad (5)$$

Proof: See Appendix A. ■

Consider a transmission policy that forwards a message along a path $s, v_1, \ldots, v_p, d$ in G_h. Recall that the message is forwarded from v_{l-1} to v_l using a minimum energy sub-policy in $\mathcal{P}_h(v_{l-1}, v_l)$. In general, such a sub-policy can forward the message to intermediate nodes, say in the set $\mathcal{S}(v_{l-1}, v_l)$, before the message is decoded by v_l. Therefore, the nodes that already have successfully decoded the data can be additionally used in cooperatively forwarding the message from v_l to v_{l+1} and save energy. Formally, these nodes belong to the set $\Big(\mathcal{S}(s, v_1) \cup \cdots \cup \mathcal{S}(v_{l-1}, v_l)\Big) \cap \mathcal{M}_h(v_l, v_{l+1})$, where $\mathcal{M}_h(v_l, v_{l+1}) = \mathcal{N}_h(v_l) \cap \mathcal{N}_h(v_{l+1})$ is the set of common neighbors of v_l and v_{l+1} in the super-graph G_h.

We now illustrate the above refinement by revisiting the example of Section III-C. As we saw, the optimal policy in the class $\mathcal{T}_1$ for sending data from node a to node i consists of three steps. First node a broadcasts a message to nodes b and g. Then nodes a, b, and g beamform to transmit the message to node h. Finally, node h forwards the message to node i. This cooperative policy is illustrated in Fig. 5. Using the node reuse refinement, node b, which is a neighbor of node i and has successfully decoded the message when it was transmitted from a to h, can simultaneously beamform with node h to forward the message to node i, as shown in Fig. 5. Doing so reduces the total energy

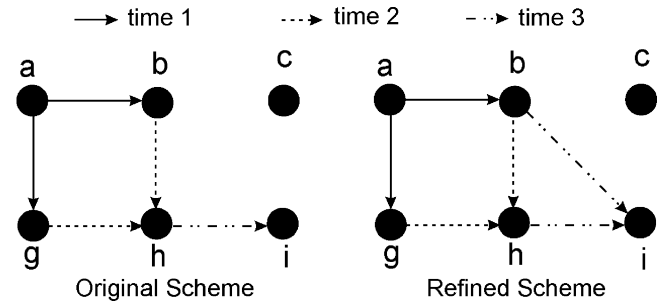

Fig. 5. Refinement of optimal solution in $\mathcal{T}_1$.

consumption on the second hop from 1 to 0.8. In fact, the refined policy in $\mathcal{T}_1$ is actually the optimal policy in $\mathcal{T}_2$.

IV. Fast Fading

We now consider the case where the channel states vary with time. In particular, we assume a block fading model where the channel state on a given link during frame t is independent of the channel state on any link at other times $t' \neq t$. Given that the channel states change from one frame to another, outage no longer occurs in long bursts with a high probability. Hence, one can allow for an average end-to-end outage of $P_{\text{out}}^{\text{rte}}$. This is a reasonable system model for best effort service, or for the case where the higher layers of the protocol stack can tolerate a certain packet loss probability.

Moreover, as we argue in Section I, at any time t, nodes should only obtain CSI of local links to decide the transmission policy at time t. In particular, in this section, we focus on CSI acquisition schemes that enable a node to obtain CSI that is a function of the channel gains on the links in $G = (V, E)$ that are incident either on the node on its neighbors of the node. This corresponds to the links contained in $E(i, 2)$. The allowance to obtain CSI on a link in $E(i, 2)$, versus just $E(i, 1)$, is motivated by beamforming, in which a transmitting node needs to know the sum of the channel gains on all the links to the destination. Generalizing to $E(i, k),\ k \geq 2$, while possible, is less practical because of the limited time available to obtain the CSI over multiple hops in a fast fading channel.

We first design the CSI acquisition processes to enable broadcasts and beamforming transmissions. We then model the optimal cooperative routing problem for fast fading channels as a *stochastic network control* problem. We show that the optimization problem can be written as a dynamic programming problem that is hard to solve even offline. We therefore consider a sub-class of the general class of transmission schemes, derive an optimal scheme in this sub-class, and demonstrate that we can still get large energy savings. What makes this approach attractive is that the computation associated with finding this policy scales well with the network size and can be carried out in a decentralized manner with fast convergence. The main ideas used in this section are similar to those in the previous section for optimization in $\mathcal{T}_1 \cup \text{BC}$, but more involved due to the fast fading on each link.

A. CSI Acquisition Processes

We first introduce additional notation. We use 2-neighborhood of a node i to refer to the nodes and links in $G(i, 2)$. We denote by $f_i : 2^{|V(i,2)|} \times 2^{|E(i,2)|} \mapsto \mathbb{R}^\kappa$ the function which maps the set of nodes within the 2-neighborhood of i that have decoded the message at time t (this is a random process for fast fading channels) and the instantaneous channel gains on links in $E(i, 2)$ to the CSI available at node i. Thus, the CSI available at node i at time t is $f_i\big(\mathcal{R}(t), \{H_{ij}(t) : (i,j) \in E(i,2)\}\big)$, Here, κ is the maximum node degree of the graph $G(V, E)$. We consider the range to be $\mathbb{R}^\kappa$ because for broadcast, the broadcasting node needs to obtain an estimate of the channel gains on the links to the receivers. For beamforming, each node needs to know its own channel gain and phase to the receiver, and the sum of the channel gains over all the links which beamform to the receiver. The transmission power at a node i at time t is thus a function of the CSI available at that time, in particular it is given by $P_i\big(f_i\big(\mathcal{R}(t), \{H_{ij}(t) : (i,j) \in E(i,2)\}\big)\big)$, where P_i is now a function that maps $\mathbb{R}^\kappa$ to $\mathbb{R}_+$.[4] We will simplify notation and denote $f_i\big(\mathcal{R}(t), \{H_{ij}(t) : (i,j) \in E(i,2)\}\big)$ by $f_i(t)$ and $P_i\big(f_i\big(\mathcal{R}(t), \{H_{ij}(t) : (i,j) \in E(i,2)\}\big)\big)$ by $P_i(t)$ when the underlying mappings are clear from the context. Also, note that the nodes are assumed to know the mean channel gains on the links in their 2-neighborhood – these can be updated at the timescale over which shadowing changes. We neglect the energy cost of obtaining these mean values because they get amortized over a large number of transmissions. Finally, we denote by $\mathcal{E}_{\text{FB}}(f_1, \ldots, f_{|V|}, \mathcal{R}(t), H(t))$ the energy consumption to obtain the CSI $f_i\big(\mathcal{R}(t), \{H_{ij}(t) : (i,j) \in xE(i,2)\}\big)$ at node i at time t.[5]

We now describe the different CSI functions f_i that we consider, and the underlying physical mechanisms to obtain this CSI. While we only describe one particular choice of a CSI acquisition process, different such processes are possible, and would lead to different amounts of energy consumption to obtain the same CSI.

1) CSI for Broadcast: For a node i to obtain the CSI on channels from itself to a subset of its neighbors $\mathcal{M} \subseteq \mathcal{N}_1(i)$, the neighbors in this subset transmit training symbols of 1 symbol duration time each at a fixed power P_t. P_t is chosen such that it is sufficiently high for i to estimate the channel gain for the channels to nodes which are intended to receive the broadcast. Then, if $\mathcal{M} = \{n_1, \ldots, n_i\}$, the CSI at node i and time t and the corresponding energy consumption to obtain it are

$$f_i(t) = [H_{in_1}(t), H_{in_2}(t), \ldots, H_{in_i}(t), -1, \ldots, -1],$$
$$\mathcal{E}_{\text{FB}}(t) = 1 \times |\mathcal{M}| P_t$$

and no CSI is acquired by nodes other than i. We use -1 to denote the fact that no CSI is received on a particular link.

2) Beamforming CSI: Here, a set of nodes $\mathcal{D} \in \mathcal{N}_1(j)$ beamform to send a message to node j. We consider a class of CSI feedback processes, in which partial CSI of the channels from nodes in $\mathcal{D}$ to j is obtained by each node in a subset $\mathcal{K}(\mathcal{D})$. In general, $\mathcal{K}(\mathcal{D})$ may not be the same as $\mathcal{D}$ because feeding back the CSI to nodes which have low instantaneous channel gains to j costs a lot of energy. Node j then feeds back the following CSI to every node ν in $\mathcal{K}(\mathcal{D})$: (i) to a node, ν, the gain $H_{\nu j}(t)$ and the corresponding phase, $\phi_{\nu j}(t)$, and (ii) to all nodes, sum

[4]This corresponds to the term *information structure* in the decentralized control literature [28].

[5]We need appropriate measurability restrictions on f_is, P_is, and $\mathcal{E}$ – the functions we consider in this paper satisfy such technical conditions.

of channel gains $\sum_{\nu \in \mathcal{K}(\mathcal{D})}(H_{\nu j}(t))$. Each feedback is assumed to take c symbols.

To obtain this CSI, each node in the set $\mathcal{D}$ transmits a training signal with power P_t to node j, for one symbol time. Node j then estimates the channels gains and phases, and feeds them back to the nodes in $\mathcal{K}(\mathcal{D})$. The sum of the channel gains are fed back to all the nodes using a single broadcast, i.e., by inverting the weakest channel with gain $\min_{m \in \mathcal{K}(\mathcal{D})} H_{mj}(t)$. For $k \notin \mathcal{K}(\mathcal{D})$, no CSI is available. For $k \in \mathcal{K}(\mathcal{D})$, the available CSI function is given by

$$f_k(t) = \left[H_{kj}(t), \phi_{kj}(t), \sum_{m \in \mathcal{K}(\mathcal{D})} H_{mj}(t), -1, \ldots, -1 \right].$$

The energy expended during time t in the training and feedback process during time t described above is given by

$$\mathcal{E}_{\mathrm{FB}}(t) = |\mathcal{D}| P_t + c\gamma \left(\sum_{m \in \mathcal{K}(\mathcal{D})} \frac{1}{H_{mj}(t)} + \max_{m \in \mathcal{K}(\mathcal{D})} \frac{1}{H_{mj}(t)} \right).$$

Here, $|\mathcal{D}|P_t$ is the energy consumed to send training sequences to j, and $c\sum_{m \in \mathcal{K}(\mathcal{D})} N_0 B(2^r - 1)/H_{mj}(t)$ is the energy to feedback the individual channel gains and $c \max_{m \in \mathcal{K}(\mathcal{D})} N_0 B(2^r - 1)/H_{mj}(t)$ is the energy required to feedback the sum of channel gains using a single broadcast.

B. Stochastic Networked Control

We now formulate the problem of computing an optimal transmission scheme for fast fading channels formally in a stochastic networked control setting.

We consider the optimization of both the available CSI given by the functions f_is, and the control policy which defines the functions P_is. The goal is again to minimize the average energy consumption for both CSI acquisition and data transmission to send a message from the source to the destination, subject to an end-to-end outage constraint. Thus, this optimization problem is a *stochastic control problem*, where we design the *information structure* (available CSI) as well as the *control policy*.

The optimization problem can be stated as

$$\begin{aligned} \min_{\{P_i, f_i\}} \mathbb{E} \sum_{i=1}^{|V|} \sum_{t=1}^{N} \Big(T_d P_i \big(f_i(\mathcal{R}(t), \{H_{jk}(t), (j,k) \in E(i,2)\}) \big) \\ + \mathcal{E}_{\mathrm{FB}}(f_1, \ldots, f_{|V|}, H(t), \mathcal{R}(t)) \Big) \\ \text{s.t. } \Pr\big(d \in \mathcal{R}(N)\big) \geq (1 - P_{\mathrm{out}}^{\mathrm{rte}}), \text{ and } P \in \mathcal{P}, f \in \mathcal{F} \end{aligned} \tag{6}$$

where the dynamics of the set $\mathcal{R}(t)$ are given by (1). The sets $\mathcal{P}$ and $\mathcal{F}$ are similar to the definition of $\mathcal{P}$ in Section III, i.e., they correspond to a sequence of broadcasts and beamforming schemes, where during each frame t the CSI for broadcast (or beamforming) is first acquired, and then data is transmitted using a broadcast (or beamforming transmission). We would like to optimize over the functions f_is and P_is.

We can write down the above system as a Markov decision process (MDP) with state $(\mathcal{R}(t), H(t))$ and solve the stochastic optimization problem using dynamic programming. However, the cost is again prohibitive, as $\mathcal{R}(t)$ itself belongs to a set of cardinality $2^{|V|}$. Even when we fix f_is, the problem is at least as complex as that in the slow fading case. We now describe a set of local cooperation schemes similar in spirit to the schemes in the sub-policy class $\mathcal{P}_2(v_1, v_2)$, but more restrictive.

C. Local Cooperative Communication Schemes

Specifically, we consider direct transmission between two nodes, and a scheme where a broadcast from a node v_1 to multiple relays is followed by a beamforming from the relays to a node v_2. Again, we optimize over each such class of local cooperation schemes. This is a smaller set of schemes than those in $\mathcal{P}_2(v_1, v_2)$. For example we do not allow a sequence of two beamforming transmissions, even if the beamforming during each transmission is between nodes which are neighbors of the destination of that beamforming. Still, the analysis of the local cooperation schemes for fast fading is more involved than that for $\mathcal{P}_2(v_1, v_2)$ in slow fading. However, we note that the techniques in this section of the paper can be used to design more complex schemes for fast fading.

We describe direct transmission and relay cooperation schemes (broadcast followed by beamforming) and show how to compute the optimal strategy for each of these kinds of schemes. Each local transmission is subject to an outage probability constraint which mandates that the data should reach the intended node with a probability of at least P_{out}. In particular, we focus on the transmission of a message from node i to node j, over one frame t (direct transmission) or over two frames t and $t+1$ when common neighbors are used as relays.

1) Direct Transmission With CSI: If $(i,j) \in E$, for direct transmission, node i first obtains the CSI, $H_{ij}(t)$, using the CSI acquisition process for broadcasts. Node i then forwards the data message with a transmit power that depends on $H_{ij}(t)$ so that the received power at node j exactly equals the power threshold γ with a probability of P_{out}. Mathematically, the transmit power of node i is $P_i(t) = \mathbf{1}_{[H_{ij}(t) \geq \delta_{ij}]} \gamma/(H_{ij}(t))$, where $\mathbf{1}_{[.]}$ denotes the indicator function. Given the outage constraint, δ_{ij} must be set such that $H_{ij}(t)$ exceeds it with probability $(1 - P_{\mathrm{out}})$. Therefore, using the fact that $H_{ij}(t)$ is an exponential random variable, $\delta_{ij} = -\bar{H}_{ij} \log_e(1 - P_{\mathrm{out}})$. The total average energy consumed by this scheme (including the cost of acquiring CSI) to forward a message from i to j is given by

$$C_1(i,j) = P_t + T_d \frac{\gamma}{\bar{H}_{ij}} \mathrm{Ei}\left(\frac{-\delta_{ij}}{\bar{H}_{ij}}\right), \tag{7}$$

where $\mathrm{Ei}(x) = \int_{-\infty}^{x} (\exp(u)/u) du$ is the standard exponential integral [29].

Note that as long as we can track the channel well by sending training sequences for a small duration compared to the frame duration, acquiring CSI is always beneficial. In this case, even though the training sequence is sent with a fixed power, P_t, which needs to account for bad channel states over which data may be sent, it saves energy because the data transmission power (over a longer time T_d) can be adapted to the channel state.

2) Optimized Broadcast With CSI & Cooperation: We first describe a parameterized set of schemes, in which node i uses a two-hop transmission to forward a message to node j, using a combination of broadcast from i to intermediate relays at time t and beamforming by the relays to j at time $t+1$. Clearly, $(i,j) \in E_2$, and the relays must be common neighbors of i and j, i.e., they belong to the set $\mathcal{M}_2(i,j) = \mathcal{N}_1(i) \cap \mathcal{N}_1(j)$. The optimized broadcast with CSI and cooperate scheme can then be computed easily, as describe below.

Node i obtains the CSI about the links to its neighbors (in the set $\mathcal{M}_2(i,j)$), and then broadcasts data, which its neighboring nodes (including j if $(i,j) \in E$) try to decode, incurring an energy cost of $|\mathcal{M}_2(i,j)|P_t$. Node i then broadcasts the message to a subset of M relays with the highest instantaneous channel gains to i with probability P_{out}, i.e., $P_i(t) = \mathbf{1}_{[H_{i[M]} \geq \delta]}\gamma/(H_{i[M]}(t))$, where $[M]$ is the index of the node in $\mathcal{M}_2(i,j)$ with the Mth highest instantaneous channel gain. As before, δ is chosen such that $\Pr([H_{i[M]} \leq \delta) = P_{\text{out}}$. (Also, $P_k(t) = 0$, for all $k \neq i$.)

When node i does not declare outage, let the set of M relays which successfully decoded the data broadcast by i be denoted as $\mathcal{D}(i,j) \subseteq \mathcal{M}_2(i,j)$. (For all $k \in \mathcal{D}(i,j)$, $P_i(t)H_{ik}(t) \geq \gamma$.) To beamform to j, when node j is unable to decode, these relays need to first acquire the required CSI using the CSI acquisition process for beamforming described earlier. In particular, the node j selects a subset, $\mathcal{K}(\mathcal{D}(i,j))$, of relays with the highest instantaneous channel gains to j, and it feeds back to every selected node, k, the gain and phase of the channel on the link (k,j) along with the sum of the channel gains. Each selected node, $k \in \mathcal{K}(\mathcal{D}(i,j))$, then cooperatively beamforms to forward the data to j with its *optimal* transmit power given by

$$P_k(t+1) = \frac{\gamma H_{kj}(t+1)}{\left(\sum_{m \in \mathcal{K}(\mathcal{D}(i,j))} H_{mj}(t+1)\right)^2}.$$

(All nodes not in $\mathcal{K}(\mathcal{D}(i,j))$ do not transmit.) This ensures that the data message arrives at node j with a total received power that exactly meets the decodability threshold γ. The above relay selection is similar to *hybrid selection* and *maximum ratio combining* in systems with multiple transmit and/or receive antennas [1], and exploits fast fading of the links between the relays and i and j.

Any such scheme described above is parameterized by (i) M, the number of relays to which node i broadcasts, and (ii) $|\mathcal{K}(\mathcal{D}(i,j))|$, which determines the actual set $\mathcal{K}(\mathcal{D}(i,j))$ of relays with highest instantaneous channel gains to j that beamform to j.[6] The optimal values of these parameters and the *average* total energy consumed $C_2(i,j)$, (including the cost of acquiring CSI) can be obtained using techniques very similar to those in [9]. The computations involved are very simple and can be done in real time using Ei function tables [29]. We provide a brief overview of the computations in Appendix B. The Appendix also describes the suitable modifications that are needed when $M = 1$, in which only one relay forwards a message to j. This is a special case because zero outage costs infinite energy for Rayleigh fading channels.

D. Distributed Cooperative Routing

We now show how to compute, using a tight approximation bound, an approximately optimal cooperative route that consists of a sequence of the local cooperative schemes described above. Limiting the set of transmission schemes to the above set in which messages are always (in one or two hops) destined for one node with a local maximum outage, enables the decomposition of the general intractable cooperative routing problem into a local physical layer optimization problem, and a global optimization of the cooperative route. Notably, both these optimizations problems need to be re-solved at the time scale of slow fading or shadowing, and the total optimization complexity for a wireless network with n nodes turns out to be $O(n^2 \log n)$. At the same time, both the CSI acquisition and data transmission still adapt locally to the instantaneous channel states and exploit the advantages of cooperative communication; the rules for this adaptation are very simple, and allow a real-time implementation that can cope with fast fading and benefit from it.

The transmission of messages in each of these schemes can be optimized separately, as done above, because the interference power is not assumed to change. Specifically, our aim is to minimize the average energy consumed in the network to transmit a single message from a source, s, to a destination, d, via intermediate cooperative relays subject to a per-hop outage constraint. For an end-to-end maximum outage probability, $P_{\text{out}}^{\text{rte}}$, we set P_{out} such that $P_{\text{out}}^{\text{rte}} = (1 - P_{\text{out}})^{|V|}$. Joint optimization of P_{out} and the cooperative route is beyond the scope of the paper.

All local transmissions, as defined in the previous section, can be associated with an edge on the supergraph G_2. (Recall that G_2 connects any two nodes in G that are at most two hops away from each other.) For each edge in E_2, we can compute an optimized broadcast with CSI and cooperation scheme (along with $C_2(i,j)$) as described in Section IV-C, and the average energy cost for direct transmission $C_1(i,j)$. Hence, we can compute an optimal local scheme that consumes the minimum energy including that for CSI acquisition. Note that the computations involved are simple [9] and the computational complexity depends only on the degree of G, which is independent of n for constant density networks.

The total energy consumption to forward a message from i to j, conditioned on i receiving the message is then given by $C_*(i,j) = \min_{k=1,2} C_k(i,j)$. Then the total energy consumed, $C_{\text{tot}}^P(v_0, v_n)$, to forward a message from the source, v_0, to the destination, v_n, using optimized local cooperative data transmission schemes corresponding to the n-hop path $P = (v_0, v_1, \ldots, v_n)$, where $(v_k, v_{k+1}) \in E_2$, for all $k = 0, \ldots, n-1$, is given by

$$C_{\text{tot}}^P(v_0, v_n) = \sum_{k=0}^{n-1} (1 - P_{\text{out}})^k C_*(v_k, v_{k+1}) \tag{8}$$

where $(1 - P_{\text{out}})^k C_*(v_k, v_{k+1})$ is the energy per message over hop (v_k, v_{k+1}). The factor $(1 - P_{\text{out}})^k$ occurs because node v_k transmits only if it successfully receives and decodes the message, which happens with probability $(1 - P_{\text{out}})^k$.

Computing an optimal path which minimizes the cost in (8) is a combinatorial problem since the energy consumption on a given hop depends on the *number of hops that precede it*. However, using $(1 - P_{\text{out}})^k \leq (1 - P_{\text{out}}^{\text{rte}})$, we can derive and minimize the following tight and tractable upper bound on $C_{\text{tot}}^P(v_0, v_n)$

$$C_{\text{tot}}^P(v_0, v_n) \leq (1 - P_{\text{out}}^{\text{rte}}) \sum_{k=0}^{n} C_*(v_k, v_{k+1}). \tag{9}$$

The energy consumption on a path that minimizes (9) is guaranteed to be within $1/(1 - P_{\text{out}}^{\text{rte}})$ of that of an optimal path that

[6]Note that the number of relays selected for beamforming in [9] is based on the set of relays which decode the previous transmission; such a simple rule enables real-time implementation even in very fast fading channels as long as we can track the channel.

minimizes (8). For example, when $P_{\text{out}}^{\text{rte}} = 0.05$, the above approximation factor is at most 1.05.

Thus, computing a global route P^* for which $C_{\text{tot}}^{P^*}(v_0, v_n)$ is within $1/(1 - P_{\text{out}}^{\text{rte}})$ of that of an optimal route consists of the following main steps: (a) determining the super-graph G_2, (b) optimizing the local cooperative transmission schemes to determine the edge costs $C_*(i,j)$ for each $(i,j) \in E_2$ which can be done in $O(n^2)$ time if degree of G does not grow with n, (c) computing in a distributed manner the shortest path on G_2 with edge costs $C_*(i,j)$ using the Bellman-Ford algorithm, which can be done in $O(n^2 \log(n))$ time.

Note that only the relay selection and the transmit powers and phases of the transmitting nodes have to be adjusted with the instantaneous local channel states. The costs, $C_*(i,j)$, for the graph G_2, and, therefore, the optimum route, depend only on the mean channel gains on the local links. Hence, the optimization of the local relay cooperation scheme and the Bellman-Ford algorithm needs to be executed only once for a given set of mean channel gains. As the mean channel gains change with time due to shadowing, the algorithm will have to be re-executed. However, given that the optimal cooperative route obtained in the previous iteration would be close to optimal when the underlying network changes gradually, the number of iterations required by the Bellman-Ford algorithm to re-converge to the optimal solution would typically be small (see .Fig. 6).

E. Example

We now illustrate the above steps by an example. Consider the wireless network represented by the graph G in Fig. 7. The channels on all the links are assumed to undergo Rayleigh fading with an average channel power gain of 1. Nodes that are not connected by an edge have very weak channels between them. The threshold, γ, for successful reception was set such that the instantaneous channel gains exceeded the threshold with probability of 0.95. To compute an optimal scheme as described above, we first form the super-graph G_2, in which an edge exists between two nodes that are within two hops of each other in G. This is illustrated in Fig. 7. For example, nodes 1 and 6, which are not connected by an edge in G get connected by an edge in G_2 because their common neighbors, nodes 2, 3, and 4, can possibly act as relays.

The next step consists of the computation of the costs associated with each of these edges. To do this, each node obtains the *mean* channel gains on the links which connect it to the other nodes in G_2. For example, for the network in Fig. 7, node 1 obtains mean channel gains of links (1, 2), (1, 3), (1, 4), and also of links (2, 3), (2, 5), (3, 4), (2, 6), (3, 6), (4, 6). This enables it to compute the minimum energy schemes to forward a message to nodes 2, 3, 4, and 6. The cost associated with the using the optimal local transmission scheme is shown for each of the edges in G_2 in Fig. 7. Then, using Bellman-Ford algorithm, the minimum cost route from source node 1 to destination node 7 can be computed in a distributed manner. It turns out to consist of the following two hops: (1, 6), in which optimized broadcast with CSI and cooperation scheme (with nodes 2, 3, and 4 as relays) is used, and (6, 7), in which direct transmission with CSI scheme is used. All the steps of the resulting optimal cooperative transmission scheme – including CSI acquisition – to transmit a message from node 1 to node 7 are described below.

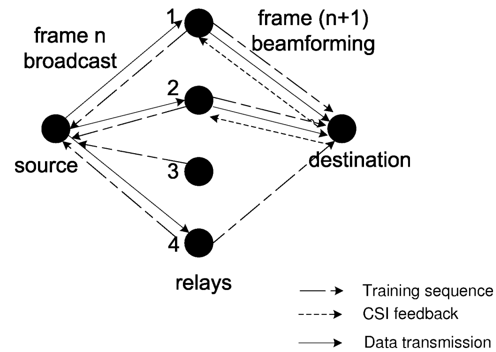

Fig. 6. Broadcast and cooperative beamforming with CSI acquisition process. In frame n, relays 1, 2,3, and 4 transmit training sequences to the source, which, in turn, broadcasts the data to relays 1, 2, and 4. These relays then transmit training sequences to the destination in frame $(n+1)$, which feeds back the CSI to relays 1 and 2; these relays beamform to transmit the data to the destination.

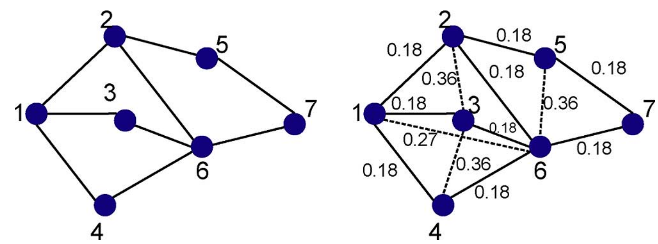

Fig. 7. *Left.* Wireless network graph, G. *Right.* Computation of optimal scheme using super-graph G_2. The solid edges are those in G, while the dashed edges are those in G_2 but not in G. Optimal edge costs are shown for each edge.

1) Broadcast With CSI: Nodes 2, 3, and 4 transmit training sequences to node 1, which then broadcasts the message (with power control) to the two nodes in $\{2, 3, 4\}$ to which it has the best channel gains. It does this with outage P_{out}. This selection of 2 best nodes out of three nodes provides a diversity order of 3 [1]. Let us denote these nodes as X and Y, which are a function of the channel realizations.

2) Relay Selection: Nodes X and Y transmit training sequences to node 6, which then feeds back the CSI to the node, say Z, with the best channel to node 6. Z then forwards the message to node 6. This selection of the best one node out of two nodes provides a diversity order of 2.

3) Direct Transmission With CSI: Node 6 then inverts the channel to node 7 with outage P_{out} after obtaining the CSI as described in the previous section.

For direct transmission, since the outage on each hop is at most 0.05, the total average energy consumption to forward a message from node 1 to node 7 over the optimal path (1, 3), (3, 6), (6, 7) is given by $0.18(1+0.95+0.95^2) = 0.513$, while that for the cooperative scheme computed above is $0.27 + 0.95 \times 0.18 = 0.441$. Even for such a small network, for transmitting a message from node 1 to node 7, allowing for local cooperation along with direct transmissions leads to energy savings of 14% over just direct transmissions.

F. Computational Results: Performance Improvement

To study the benefits of local cooperation in larger networks, we simulate a wireless network with 20 nodes. The mean channel gain on each link between a pair of nodes was chosen randomly and independently of the other mean gains. Each

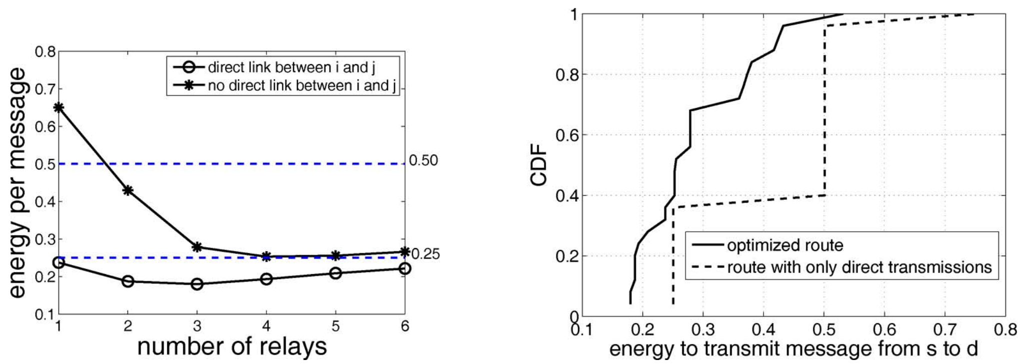

Fig. 8. *Left.* Energy per message for an edge (i,j) in E_2 when $(i,j) \in E$ and when $(i,j) \notin E$. *Right.* CDFs for energy required to transmit a message from source to destination when optimized local schemes are used and when only direct transmissions are used.

link had a mean gain of 1 with probability 1/3, and a mean gain of 0.1 with a probability 2/3. Two nodes were connected by an edge in the graph G if the mean gain of the channel between them was 0.8. While this is a simplified abstraction of a wireless network, it will prove sufficient to compare the various transmission schemes. It models a scenario in which the channel between two nodes can either be good (mean gain of 1) or bad (mean gain of 0.1). (We emphasize that the theory developed in Sections III and IV can be applied to more detailed channel models with pathloss and shadowing as well.) The channel fading and the threshold γ are as in Section IV-E. Unlike the example in Section IV-E, the maximum end-to-end outage probability is set as $P_{\rm out}^{\rm rte} = 0.95$, which implies that the link outage probability is now $P_{\rm out} = (1 - 0.95^{1/20}) = 0.002$. We set the time to feedback CSI from one node to another as $c = 4$ symbol times, and the time for transmission of one data message as $d = 100$ symbol times.

We first study the energy savings obtained by local cooperation on a per hop basis in G_2. We then consider an end-to-end optimized cooperative route.

1) Energy Savings on One Hop in G_2: Consider an edge $(i,j) \in E_2$, i.e., two nodes i and j at most two hops away from each other in G. When $(i,j) \in E$, i.e., there exists a direct wireless link from i to j, the average energy consumption to forward a message from node i to node j using a direct transmission with CSI is 0.25, while when $(i,j) \notin E$, the value corresponding to two such direct transmissions (from i to a neighbor, and from the neighbor to j) is 0.5. These and the corresponding values when a varying number of common neighbors are present and used as relays are shown in Fig. 8. When $(i,j) \notin E$, the energy consumption of local cooperation scheme, which consists of two hops and only one common relay node, can exceed 0.5. This is because in the broadcast with CSI and cooperate scheme, the outage requirements for sending a message from node i to the single common relay and then from the relay to node j are tighter so as to ensure that the total outage probability of the scheme is 0.002. In this case, the routing algorithm will choose the direct transmission scheme instead of cooperation using one relay for that edge.

As the number of relays used for local cooperation increases, the average energy per message first decreases and then increases. The reason for the increase is that, with a large number of relays, the marginal diversity gain for an additional relay is too small to offset the additional energy required to acquire the CSI for it. While the above local optimization is for the case in which all source to relay links and relay to destination links are independent and identically distributed (i.i.d.), the general case of non-i.i.d. links can also be handled [9].

2) End-to-End Energy Savings: We now compute the average end-to-end energy consumed to forward a message from a source to destination for all the 380 source-destination pairs that can occur in a network of 20 nodes, when (i) only direct transmissions are used between neighbors in G, and when (ii) an optimized sequence of local transmission schemes computed based on the methods in the previous two sections, is used. We then plot the cumulative distribution function (CDF) of the energy consumed for all the different source-destination pairs in Fig. 8. The average energy consumption (over all source destination pairs) when the route uses only direct transmissions is 42.0% to 44.4% higher than when the route uses the optimized sequence of local transmission schemes.[7] Thus, the right amount of local cooperation over each hop $(i,j) \in E_2$, where the number of relays a node i broadcasts to and the number of relays that j selects for beamforming is optimized, leads to tremendous energy savings in spite of the additional CSI acquisition that such cooperation necessitates.

V. Conclusion

The problem of decentralized routing in cooperative diversity wireless networks was cast as a stochastic network control problem. This framework allowed us to consider two significantly different variants of the problem that differ in their underlying channel models and the channel knowledge available within the network – slow fading and fast fading. For both these models, we considered a class of transmission policies which allowed local cooperative communication via broadcasts and beamforming. The cooperative scheme was computed by a graph induced by the local cooperative communication schemes. For slow fading, we proposed a class of cooperative policies which trade-off complexity and the degree of decentralization with energy-efficiency. For fast fading, we showed how to compute the optimal scheme in a class of schemes, which allowed cooperation within a local relay network, in a decentralized and efficient manner. The key to reducing the computational complexity was to decouple the

[7]The range 42.0% to 44.4% occurs because the upper bound in (8) on the total energy consumed exceeds the actual value by a factor that is at most 1.05.

design of the local cooperation scheme from routing over the induced super-graph. This approach works well because, with high probability, a node cannot hear broadcasts from a distant transmitter, nor is it very useful as a relay to beamform to a distant receiver. Also, the route designs we arrived at were scalable because the nodes used only locally available CSI to determine their transmission powers.

We believe that the general framework for formulating the cooperative communication routing problem as a decentralized stochastic control problem, as well as our general approach of computing local cooperative communication schemes to induce a graph for routing, open up interesting avenues for future work. One obvious question is that of deriving sub-optimal policies with approximation bounds in the slow fading case. For fast fading, the available CSI and its overhead depends on the specifics of the physical mechanism used, for which many different variants are possible. In addition to the ones considered here, another extension is space-time codes.

Appendix A

Proof of Theorem 3.10: We will prove the result by contradiction. Note that the costs assigned to the edges of the graph G_h are strictly positive. Let us assume that there is a minimum cost path, $\mathcal{W}$, given by $s, v_1, \ldots, v_p, d$, on the graph G_h such that for some $l < m \le p$, $t_m = \min\{t : v_m \in \mathcal{R}(t)\} < \min\{t : v_l \in \mathcal{R}(t)\}$. Then at time t_m, node v_m has successfully decoded the message. This time corresponds to a transmission used to forward the message over some hop, from some node k_1 to node k_2, in the minimum cost path. Moreover, $(k_1, v_m) \in E_h$, and $C(k_1, v_m) \le C(k_1, k_2)$. Now consider another path on graph G_i, where the sub-path $k_1, k_2, \ldots, v_m$ is replaced by k_1, v_m. This new path has a lower total cost than the shortest path, $\mathcal{W}$. Thus we have a contradiction. This completes the proof.

Appendix B

We now show how to compute the energy consumption for a relay cooperation scheme (described in Section IV-C.II) parameterized by M and $|\mathcal{K}(\mathcal{D}_{ij})|$ for transmission of a message from i to j using common neighbors as relays. Specifically, we focus on the case where all the links undergo i.i.d. fading; the general case is similar, and we refer the reader to [9] for the details. For the case of i.i.d. channel gains we can parameterize the scheme by M (number of relays to which node i broadcasts the message) and K (number of relays which beamform to forward the message to node j). We denote the common mean channel gain by $\bar{h}$. The average energy consumption to forward a message from i to j using a relay cooperation scheme (M, K) is given by

$$|\mathcal{M}_2(i,j)|P_t + \mathbb{E}\mathbf{1}_{[H_{i[M]}\ge\delta]}\frac{\gamma}{(H_{i[M]}(t))} + MP_t + c\gamma\mathbb{E}\left(\sum_{\nu=1}^{K}\frac{1}{H_{[\nu]j}(t)} + \frac{1}{H_{[K]j}(t)}\right) + \frac{T_d}{\sum\limits_{\nu=1}^{K} H_{[\nu]j}(t)} \quad (10)$$

where we recall that $\mathcal{M}_2(i,j)$ are the common neighbors of i and j, and $H_{i[M]}$ ($H_{[K]j}$) denotes the Mth (Kth) ordered channel gain from (to) i (j) to (from) the relays. Using the analysis in [9], we now provide closed form expressions which can be evaluated in terms of Ei tables with an arbitrary accuracy for each of the above terms. The derivations use a change of variables to the differences between the ordered channel gains, which turn out to be independent and exponentially distributed [30]. We have, for $M > 1$,

$$\mathbb{E}\mathbf{1}_{[H_{i[M]}\ge\delta]}\frac{\gamma}{(H_{i[M]}(t))} = \frac{N!}{\bar{h}(M-1)!}\sum_{p=n}^{M}\frac{\mathrm{Ei}\left(\frac{-p\delta}{\bar{h}}\right)}{\prod_{l=n,l\neq p}^{M}(l-p)}$$

where δ is chosen such that

$$\frac{N!}{(M-1)!}\sum_{p=M}^{N}\frac{1-e^{-p\delta/\bar{h}}}{p\prod_{l=M,l\neq p}^{N}(l-p)} = P_{\mathrm{out}}$$

where $N = |\mathcal{M}_2(i,j)|$ is the number of relays. Such a $0 < \delta < 1$ can be found using a bisection search since the function on the left is an increasing function of δ. Also,

$$\mathbb{E}\frac{1}{h_{[\nu]j}} = \frac{M!}{\bar{h}(n-1)!}\sum_{p=n}^{M}\frac{\left(\sum\limits_{l=1}^{k}(-1)^l\frac{(p\eta)^l}{(l-1)!\bar{h}^l} + \mathrm{Ei}\left(\frac{-p\eta}{\bar{h}}\right)\right)}{\prod_{l=n,l\neq p}^{M}(l-p)} + o(\eta^{k+1}),$$

$$\begin{aligned}\mathbb{E}\frac{1}{\sum\limits_{\nu=1}^{K} H_{[\nu]j}(t)} &= c_1\left(\sum_{l=1}^{k}(-1)^l\frac{(\eta)^l}{(l-1)!\bar{h}} + \mathrm{Ei}\left(\frac{-\eta}{\bar{h}}\right)\right)\\ &\quad + \sum_{p=2}^{K}c_p(\bar{h})^{p-1}(p-2)! + \sum_{p=K+1}^{M}c_p\mathrm{Ei}\left(\frac{-p\eta}{\bar{h}K}\right)\\ &\quad + \sum_{p=K+1}^{M}c_p\left(\sum_{l=1}^{k}(-1)^l\frac{(p\eta)^l}{(l-1)!(\bar{h}K)^l}\right) + o(\eta^{k+1})\end{aligned}$$

where, for $1 \le p \le K$,

$$c_p = \frac{\frac{M!}{K!}}{(K-p)!\bar{g}^K}\frac{d^{K-p}}{ds^{K-p}}\prod_{n=K+1}^{M}\frac{1}{Ks+n}\Bigg|_{s=-1}$$

and, for $K+1 \le p \le M$,

$$c_p = \frac{\frac{M!}{K!}}{\bar{h}K\left(1-\frac{p}{K}\right)^K\prod_{n=K+1,n\neq p}^{M}(n-p)}.$$

For $M = 1$, as discussed earlier, we allow for a finite outage from the relay to j; this changes the analysis only slightly [9]. Computing an optimal scheme involves computing the above expressions for each value of K and M, and comparing them to obtain the optimal values of K and M.

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