

MIMO systems with full and reduced hardware complexity

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Dedicated to Prof. Dr. Ernst Bonek, on the occasion of his retirement.

MIMO (multiple-input-multiple-output) systems propose enormous gains in the capacity of wireless systems without requiring more spectral resources. This paper first gives an overview of the use of MIMO for diversity and spatial multiplexing, and the use of channel state information in MIMO systems. It then explores the use of antenna selection as a means for the reduction of the hardware complexity. It is shown that the performance in a spatial-multiplexing application is almost as good as that of full-complexity systems as long as the number of RF chains is at least as large as the number of data streams.

Keywords: MIMO; diversity; spatial multiplexing; antenna selection

MIMO-Systeme mit voller oder reduzierter Komplexität.

MIMO(multiplier input – multiplier output)-Systeme versprechen enorme Steigerungen der Übertragungskapazität digitaler Mobilfunksysteme, ohne mehr Spektrum zu brauchen. Der vorliegende Artikel gibt zunächst einen Überblick über die Verwendung von MIMO für Diversität oder Raummultiplex und die Verwendung von Information über den Kanalzustand. Der Artikel beschreibt dann die Verwendung von Antennenauswahl zur Reduktion der Hardwarekomplexität. Es wird gezeigt, dass ihre Leistungsfähigkeit für Raummultiplex beinahe so hoch ist wie die von Systemen mit voller Komplexität, solange nur die Zahl der Frequenzumsetzerzüge mindestens so groß ist wie die Zahl der verwendeten Datenströme.

Schlüsselwörter: MIMO; Diversität; Raummultiplex; Antennenauswahl

1. Introduction

Multiple-input – multiple-output (MIMO) systems, also known as multiple-element antenna (MEA) systems, are wireless systems with multiple antenna elements at both link ends. Originally suggested and investigated by Winters in 1987 (Winters, 1987), they attracted great attention through theoretical investigations of (Foschini, Gans, 1998) and (Telatar, 1999). Since that time, interest in those systems has exploded – the majority of the worldwide academic research on the physical-layer of wireless communications is now dedicated to this topic, and the years have also seen the emergence of practical applications, especially definition of wireless communications standards based on MIMO.

The multiple antenna elements of a MIMO system can be used for three different purposes: (i) beamforming (increase of the average SNR), (ii) diversity (reducing the possibility of deep fades), and (iii) spatial multiplexing (transmission of several data streams in parallel). The first two concepts are well known from traditional systems that have multiple antenna elements only at one link end – the concept of antenna diversity, for example, has been known for more than 50 years. Having multiple antennas at both link ends leads to some interesting new technical possibilities, but does not change the fundamental effects of this approach. Spatial multiplexing, on the other hand, is a new concept, and has thus drawn the greatest attention. Diversity and beamforming lead to an indirect increase in the capacity of wireless systems, by improving the SNR; but the capacity of a system increases only logarithmically with the SNR. Spatial multiplexing allows to directly improve the capacity, by transmitting multiple data streams, and the capacity increases linearly with

the number of data streams. The system design and performance also depends on whether the transmitter and receiver know the propagation channel or not.

In the early years of MIMO research, the main emphasis was on the information-theoretic limits. Idealized assumptions about the propagation channels, as well as ideal coding and arbitrary complexity of the transceivers were usually implicit in those analyses. In recent years, the emphasis has shifted more to the question of how the theoretical gains of MIMO can be realized in practice. Several papers in this special issue address aspects of this important question. The current paper also treats one specific aspect, namely the reduction of hardware complexity by antenna selection.

The advances in the practical implementation of MIMO systems have also greatly helped in being adopted in international standards organizations. After two years of deadlock, MIMO seems now poised to be included for the high-speed packet data mode of third-generation cellular systems (3GPP). Also, a recently formed task group of the standardization of wireless LAN, IEEE 802.11n, is currently in the process of specifying a high-throughput mode of the popular WiFi standard with more than 100 Mbit/s; this mode will use MIMO in order to achieve the required spectral efficiency and high data rate. Further systems, especially for fixed wireless access, are following suit.

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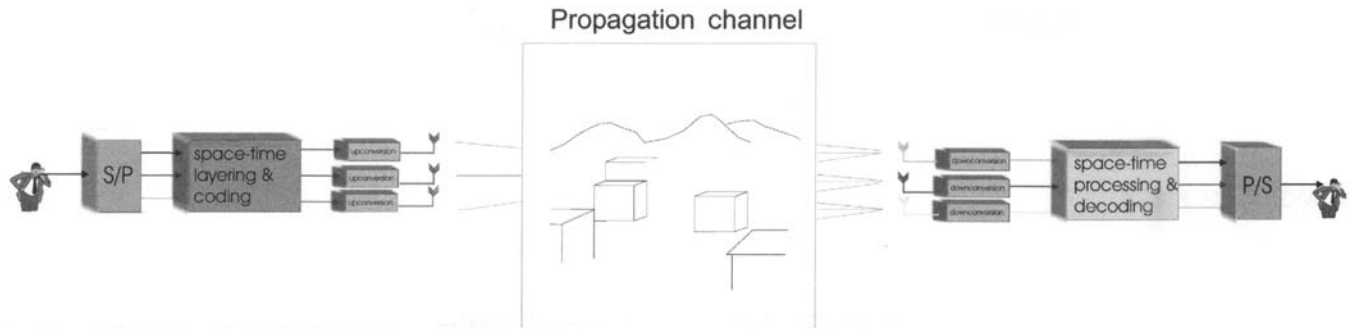


Fig. 1. Principle of spatial multiplexing (Molisch, Win, 2004)

The remainder of the paper is organized the following way: Section 2 gives an introduction to the operating principle of MIMO, with a special emphasis on spatial multiplexing. Next, we analyze the capacity and algorithms of MIMO systems with different amounts of channel state information. Section 4 is dedicated to antenna selection and the performance results and hardware savings that can be achieved with this method. A summary and conclusions wrap up this paper. In the interest of general readability and space limitations, this paper gives very few references, and almost no equations. The interested reader can find more of those in (Gesbert *et al.*, 2003) for MIMO in general, (Molisch, Tufvesson, 2005) for capacity and channels, and (Molisch, Win, 2004) for antenna selection.

2. Principle of MIMO

As mentioned in the introduction, the multiple antenna elements of MIMO can be used for beamforming, diversity, or spatial multiplexing. The concept of beamforming is well known from phased-array antennas. Linearly weighted replicas of the same signal are sent from the different antenna elements; this leads to the forming of an antenna pattern of the whole array that can be adapted to the different propagation conditions. The pattern focuses the energy into the direction of the receiver, and thus increases the average SNR. In a MIMO system, beams can be formed at both link ends. For the case of a LOS connection, the gains at the two link ends (measured in dB) add up.

While beamforming increases the average SNR, antenna diversity is used to combat fading, i.e., the fluctuations of the SNR with time. Multiple independent copies of the same signal can be combined into a total signal with small fluctuations. For receive antenna diversity, this concept has been used for many years: the signals at the receive antenna elements are weighted (optimum weights follow from the principle of matched filtering, and are called maximum ratio combining MRC) and added up. The effectiveness of such a diversity scheme is described by the diversity order, which is related to the slope of the SNR distribution of the sum signal. With N antenna elements, a diversity order of N can be achieved. Receive diversity by combining also results in a gain of the average SNR (a factor of N for maximum ratio combining). In case the transmitter knows the channel, antenna diversity with linear weighting can also be used at the transmitter. The optimum weights can again be obtained from the principle of matched filtering (maximum ratio transmission MRT); MRT gives the same diversity order and same average SNR gain as MRC. In case the channel is unknown at the transmitter, the same diversity order can be achieved, the available average SNR is reduced compared to MRT. For a MIMO system with transmit and receive antenna diversity, the diversity order is the product of the number of antenna elements at TX and RX.

An alternative way of using the multiple antenna elements is the transmission of parallel data streams; this is known as “spa-

tial multiplexing”, see Fig. 1. An original (encoded) high-rate data stream is multiplexed into several parallel streams, each of which is sent from one transmit antenna element. The channel “mixes up” those data streams, so that each of the receive antenna elements “sees” a combination of the TX data streams. If the channel is well-behaved, the received signals represent linearly independent combinations. In that case, appropriate signal processing can separate the data streams as long as the number of receive antenna elements is at least as large as the number of transmit data streams. It is clear that this approach allows to drastically increase the data rate – namely by a factor that is equal to the smaller of the number of transmit and receive antenna elements.

The three main benefits of MIMO systems cannot be realized simultaneously to their full extent. For a LOS environment, full beamforming gain is available, but there is little diversity. For a rich scattering environment, on the other hand, we can achieve full diversity, while the beamforming gain is limited (i.e., it is not possible any more to “point beams at each other”). Even more importantly, exploiting the spatial multiplexing gain limits the diversity order to something less than the product of transmit and receive antenna elements, and vice versa. It is thus a task of a MIMO system designer to find the optimum trade-offs for a specific application.

3. Theoretical performance limits of MIMO

In this section we describe some basic MIMO architectures as well as basic (information-theoretic) performance limits. Necessarily, this section is somewhat more mathematical. We will also show that the performance strongly depends on the channel state information (CSI) that is available at the transmitter and receiver. For this reason, we first need to discuss different ways of obtaining CSI.

At the receiver, CSI is normally obtained from “training” or “pilot” bits. At regular intervals, known bits are transmitted over the channel, from which the receiver can conclude the current channel state. This approach, which is used in all current wireless systems, can easily be generalized to MIMO systems. The major difference is that now the channel state information (impulse response, transfer function) from each transmit antenna element to each receive antenna element has to be determined. This means in practice that the number of training bits, and thus the overhead, is increased. While it is possible to eschew training bits and use modulation formats that allow a blind demodulation, such approaches are computationally very expensive, and thus not used in practice.

On the transmitter side, CSI can be either obtained from feedback, or from the principle of reciprocity. Feedback is the more straightforward solution: the receiver sends back information to the TX about the CSI it sees. The drawback is the requirement for a separate feedback channel, which increases

the overhead. Again, this problem is compounded in MIMO system by the requirement to feed back the CSI from each TX to each RX antenna element. Alternatively, we can use reciprocity, so that the transmitter uses the CSI from its reception phase for the transmit phase. This is possible in time-domain duplexing, but not for frequency-domain duplexing systems, as the small-scale fading is different at the transmit and receive frequencies. In that case, the use of average CSI, which is almost independent of frequency, is preferable.

In the following, we will consider the performance of MIMO systems that have a small bandwidth (i.e., no variations of the channel over the system bandwidth). The transfer function from the TX to the RX can then be described by a $N_r \times N_t$ matrix of the channel as

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1N_t} \\ h_{21} & h_{22} & \dots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r1} & \dots & h_{N_rN_t} \end{pmatrix}, \quad (1)$$

whose entries h_{ij} are the attenuations (transfer functions) from the j -th transmit to the i -th receive antenna. The received signal vector

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{x} + \mathbf{n} \quad (2)$$

is received by N_r antenna elements, where $\mathbf{s}$ is the transmit signal vector, and $\mathbf{n}$ is the noise vector.

Let us first analyze systems that use spatial multiplexing. The key quantity is the capacity, i.e., the data rate that can be transmitted without errors, for the case that the transceiver uses optimum transceiver schemes. The capacity equations, as well as many space-time structures, can be best understood when considering the singular value decomposition of the channel $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{W}^H$, where $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values, and $\mathbf{U}$ and $\mathbf{W}^H$ are unitary matrices composed of the left and right singular vectors, respectively. Then, multiplication of the transmit data vector with a matrix $\mathbf{W}$ at the TX and $\mathbf{U}^H$ at the receiver diagonalizes the channel, resulting in R independent channels with SNR σ_k^2 , where σ_k is the k -th singular value of $\mathbf{H}$, and R is the rank of $\mathbf{H}$ (number of nonzero singular values).

The capacity of the channel $\mathbf{H}$ is thus given by the sum of the capacities of the subchannels (modes of the channel):

$$C = \sum_{k=1}^R \log_2 \left[1 + \frac{P_k}{\sigma_n^2} \sigma_k^2 \right] \quad (3)$$

where σ_n^2 is the noise variance, and P_k is the power allocated to the k -th mode; we assume that $\mathbf{\Sigma} \mathbf{P}_k = P$ is independent of the number of antennas. Eq. (3) shows that to a first approximation, the capacity increases linearly with the number of antenna elements. Expression can be shown to be equivalent to

$$C = \log_2 \left[\det \left(\mathbf{I}_{N_r} + \frac{\bar{\Gamma}}{N_t} \mathbf{H} \mathbf{R}_{xx} \mathbf{H}^H \right) \right], \quad (4)$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix, $\bar{\Gamma}$ is the mean signal-to-noise ratio (SNR) per receiver branch, and $\mathbf{R}_{xx}$ is the correlation matrix of the transmit data (for uncorrelated data, it is a diagonal matrix with entries that describe the power distribution among the antennas).

The capacity given in Eq. (3) and (4) is for a deterministic, time-invariant channel matrix $\mathbf{H}$. In case the channel changes, the following definitions are in use:

- ▶ the ergodic (Shannon) capacity, i.e., the expected value of the capacity, taken over all realizations of the channel;
- ▶ the outage capacity, i.e., the minimum capacity that is achieved a certain fraction of the time, e.g., 90 %. This definition assumes that the data are encoded with a near Shannon limit achieving code that extends over a period that is

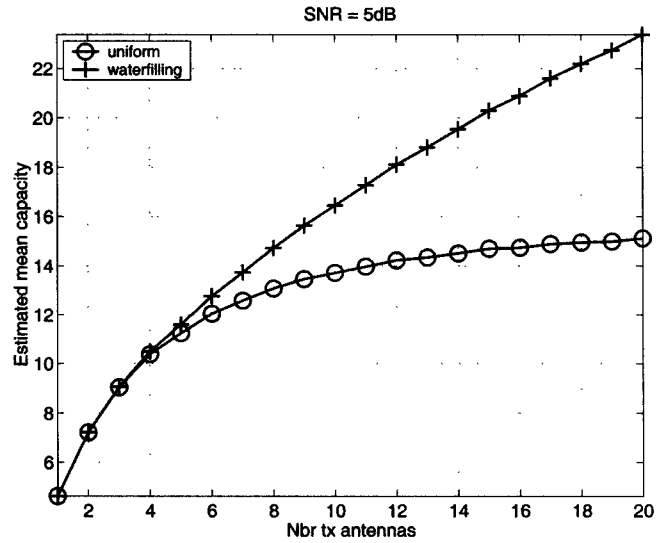


Fig. 2. Capacity with and without CSI at the TX with $N_t = 8$ antennas and SNR = 5 dB (Molisch, Tufvesson, 2005)

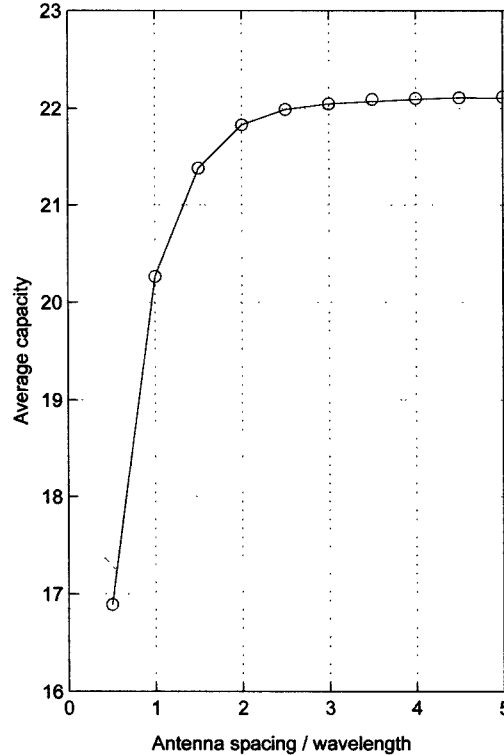


Fig. 3. Average capacity of a 4×4 MIMO system with 10 degree rms angular spread as seen from the BS as a function of the antenna spacing at the BS (Molisch, Tufvesson, 2005)

much shorter than the channel coherence time. In that case, each channel realization can be associated with a (Shannon) capacity value. The capacity thus becomes a random variable (r.v.), with an associated cumulative distribution function (cdf).

The correlation matrix of the transmit signal depends strongly on the available CSI. In case no CSI is available, it is simply the identity matrix – physically speaking, each transmit antenna element transmits the same power, and into all directions. When not knowing the channel, this is obviously the best that can be done. When the transmitter has full CSI, then the

optimum way to allocate power to the different eigenmodes follows from Shannon's principle of "waterfilling", where more power is assigned to the eigenmodes that have a good SNR, while modes with very poor SNR might not carry any energy at all. The effect on the capacity is shown in Fig. 2, which shows the capacity as a function of the number of TX antennas for a fixed number of receive antennas.

The capacity is considerably decreased by correlation of the signals at the antenna elements. Such a correlation makes it much more difficult to separate the data streams. Fading correlation is influenced by the angular spectrum of the channel, as well as the arrangement and spacing of the antenna elements. For antennas that are spaced half a wavelength apart, a uniform angular power spectrum leads approximately to a decorrelation of the incident signals. For a smaller angular spread, the correlation increases; Fig. 3 shows the capacity of a 4×4 MIMO system with uniform linear arrays at TX and RX as a function of the angular spread at one link end.

For a pure diversity system, i.e., a system that transmits a single data stream, the capacity can be derived from the SNR statistics

$$C = \log_2 \left[1 + \frac{P}{\sigma_n^2} \max(\tilde{\lambda}_i^2) \right]. \quad (5)$$

The considerations about the difference between ergodic capacity and outage capacity remain valid for this case.

4. Antenna selection

4.1 Hardware complexity for MIMO systems

One of the major drawbacks of MIMO systems is the increased complexity and cost associated with the use of several antenna elements:

- (1) Obviously, MIMO systems require multiple antenna elements. The production of such antenna elements is inexpensive, especially if they can be integrated into the handset or other communication device. The number of antenna elements is limited for mobile handsets, (typically to four elements, with a combination of polarization diversity and spatial diversity), while much larger numbers are possible for antennas on laptops and other devices for data communication. Also, at the base station (access point), larger numbers of antenna elements are feasible.
- (2) Each antenna element requires a complete RF chain (low-noise amplifier, frequency downconverter, filters, analogue-to-digital conversion, etc.). These components increase both cost and power consumption. As the general progress for analogue circuit elements is slower than for digital devices, the relative impact of these devices on total system costs will increase with time.
- (3) Digital signal processing: Obviously, sophisticated signal processing is required to separate the data streams from each other in the receiver. At the moment, suboptimum, but simple approaches (e.g., minimum mean-square error detection for each antenna) are favoured by the industry, especially for systems with large average SNR and thus high-order modulation formats. However, more complicated (and thus better performing) ones are anticipated to be used in the future, when progress in integrated-circuit density and processing power has made this feasible.

From the above enumeration, it becomes clear that the RF chains constitute a major factor in the costs of a multi-antenna system. There is thus an increased interest in antenna selection.

4.2 Principle of antenna selection

Conventional antenna selection at the receiver is an old concept. Multiple antenna elements are available, but only one RF chain. The receiver determines the channel quality (usually the SNR) for each of the antenna elements, and then connects the antenna with the best signal to the RF chain for the downconversion and reception of the desired signal. It has been shown that this method gives a considerable performance loss compared to a full complexity system.

As a compromise, "hybrid selection/maximum-ratio-combining" (H-S/MRC), also known as "generalized selection combining", has been investigated for about 10 years. In this approach, the best L out of N antenna signals are chosen. This reduces the number of required RF chains from N to L . The approach has been adopted for MIMO systems, both for diversity and for spatial multiplexing. For diversity, the H-S/MRC can be applied at only one link end (transmitter or receiver), or at both link ends. In either case, the concept is identical to the one described above. For MIMO systems with spatial multiplexing, the concept of downconverting and processing only L out of the N signals from the available antenna elements (hybrid-selection – MIMO) can also be used at the receiver. The main difference is that in this case, the processed L signals are not linearly combined, but rather used to detect the multiple data streams by a conventional spatial-multiplexing receiver that is designed for L signals. Note that the antenna selection can also be used at the transmitter. In order to encompass both the diversity and the spatial multiplexing, we will henceforth speak of "hybrid antenna selection" when referring to the concept of selecting and processing L out of N antennas. To simplify the discussion, we will also henceforth assume that antenna selection occurs only at the receiver, while the transmitter has full complexity.

4.3 Performance of antenna selection

When identifying the number of RF chains that are required for a certain performance, the following facts are useful:

- the diversity order is determined by the number of available antenna elements N ,
- the possible beamforming (mean SNR) gain is determined by the number of RF chains L (see, however, the subsection on RF preprocessing for methods that circumvent this restriction),
- the maximum number of data streams (spatial multiplexing gain) is given by $\min(L_r, L_t)$, where L_r (L_t) is the number of RF chains at the receiver (transmitter). Note that for a full-complexity system, $L = N$. Obviously, the number of selected signals should never be smaller than the number of data streams in a spatial multiplexing system.

The capacity of a diversity system with hybrid antenna selection at the receiver immediately follows from the fact that we select the best antennas. In mathematical terms, that can be formulated the following way: define a set of matrices $\tilde{H}$, where $\tilde{H}$ is created by striking $N_r - L_r$ rows from H , and $S(\tilde{H})$ denotes the set of all possible $\tilde{H}$, whose cardinality is $\binom{N_r}{L_r}$. The achievable SNR γ of the reduced-complexity system (for a specific channel realization) is now

$$\gamma = \max_{S(\tilde{H})} \left(\max_i (\tilde{\lambda}_i^2) \right), \quad (6)$$

where the $\tilde{\lambda}_i$ are the singular values of $\tilde{H}$. For a spatial-multiplexing system, the capacity is given as

$$C_{H-S/MIMO} = \max_{S(\tilde{H})} \left(\log_2 \left[\det \left(I_{N_r} + \frac{\bar{P}}{N_t} \tilde{H} R_{xx} \tilde{H}^H \right) \right] \right). \quad (7)$$

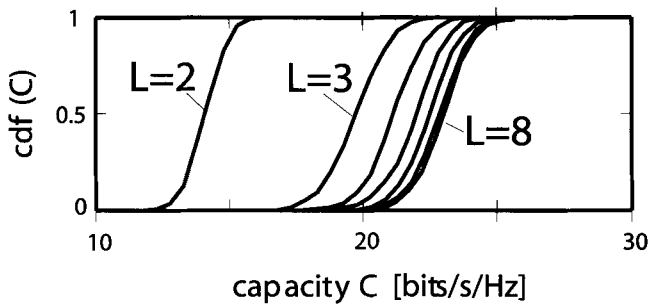


Fig. 4. Capacity of HS-MIMO for $N_r=8$, $N_t=3$, SNR = 20 dB.

The evaluation of this equation requires the evaluation of $\binom{N_r}{L_r}$ determinants (one for each possible set of selected antennas); however, approximations have been published.

Figure 4 shows the cdf of the capacity obtained by Monte Carlo simulations for $N_r=8$, $N_t=3$, and various L_r . With full exploitation of all available elements, a mean capacity of 23 bit/s/Hz can be transmitted over the channel. This number decreases gradually as the number of selected elements L_r decreases, reaching 19 bit/s/Hz at $L_r=3$. For $L_r < N_t$, the capacity decreases drastically. As we have discussed above, we require a sufficient number of antennas to spatially multiplex N_t independent transmission channels in order to achieve high capacity.

4.4 RF preprocessing

Beamforming gain is especially relevant in highly correlated channels. In the extreme case of a LOS channel, there is no diversity available (since there is no fading), and all gains from the antenna array are due to beamforming. The limitation of the possible beamforming gain in hybrid selection schemes have motivated us to investigate schemes that can overcome this limitation. The main idea is to perform a preprocessing (transformation) in the RF domain, i.e., before the selection occurs. This preprocessing can be viewed as a matrix M that multiplies the received signal vector. It can be most easily implemented if the entries of the matrix are phase-shift-only operations, since phase shifters can be easily realized in the RF domain, while adjustable attenuators are more difficult to manufacture, and lead to a reduction of the SNR.

The most simple transformation matrix is a FFT matrix, which can be realized as a Butler matrix. In essence, such a transformation changes the antenna selection to a beam selection. It is obvious that for a channel with a small angular spread, such a beamselection leads to an improvement of the average SNR. In the limit of a pure LOS channel, the beam selection gives optimum performance. However, this good performance is only true if the mean angle of incidence is identical to the center directions of one of the beams of the FFT matrix (notice that those beams are fixed in space); otherwise the performance is suboptimum even for strongly correlated channels.

Better performance can be achieved by using a matrix with variable phase shifters, where the phases are adapted to the average channel state information (i.e., the angular spectrum averaged over the small-scale fading). This angular spectrum is (almost) independent of frequency, and changes only on a time scale that is very long compared to the inverse Doppler frequency. Average CSI can thus be obtained more easily at the transmitter.

An even better performance can be achieved by the use of phase shifts based on instantaneous CSI. It can be shown that as long as L is at least 2, the performance of full-complexity receivers can be reached exactly. Further investigations have also

demonstrated that phase errors less than about 30 degrees hardly influence the capacity of the system.

4.5 Practical considerations

There are several practical aspects that make hybrid antenna selection challenging. While they are not prohibitive for the deployment of antenna selection, they have to be kept in mind.

The first question is how the channel can be estimated. By definition, only L RF chains are available to estimate the channels (from each TX antenna element) to N receive antenna elements. In order to estimate all relevant channel coefficients, the training sequence, from which the channel estimates are derived, has to be repeated at least $\lceil N/L \rceil$, where $\lceil x \rceil$ denotes the smallest integer larger than x . This implies an increase in the total overhead. The relative importance of that overhead decreases with the data rate, as the channel has to be retrained only after a time that is shorter than the inverse of the Doppler frequency (and thus independent of the data rate). Also, noise in the channel estimates can lead to erroneous selection of antennas. However, more detailed investigations have shown that as long as the SNR during the channel estimation phase is at least as large as the SNR during data reception, the performance does not suffer.

Once the CSI for the full channel matrix H has been identified, the next task is to find the antenna subset $S(\vec{H})$ that maximizes the capacity. In order to achieve truly optimum performance, an exhaustive search has to be performed. However, this is computationally intensive. For this reason, a wealth of different techniques has been developed for suboptimum antenna selections. For diversity, a simple selection algorithm, based on the power of the received signals, has proven to be effective. For spatial multiplexing, however, the received power is not the main criterion; rather the correlation between the signals (which determines the separability of the data streams) is most important. For this case, it proves to be advantageous step-by-step select the best, second best (given the first antenna), ... The performance loss for these algorithms is usually less than 1 bit/s/Hz in typical system configurations.

Finally, we have to keep in mind that the small-scale (Rayleigh) fading changes with frequency. Thus, in an OFDM wideband system, different antennas tend to be optimum for different considered frequencies. However, an antenna selection is always effective for all frequencies. For this reason, selection based on the instantaneous channel state information is less effective than in a narrowband system.

5. Summary

In this paper, we have reviewed some basic characteristics of MIMO systems, and compared full complexity with reduced-complexity systems. MIMO systems can be used both for providing diversity, and for increasing the data rate of wireless systems by spatial multiplexing. To a first approximation, the achievable increase in data rate is proportional to the number of antenna elements at transmitter and receiver. However, correlation and other channel effects can considerably decrease that capacity.

In order to reduce the hardware complexity, antenna selection is a very promising method. The diversity order is proportional to the number of available antenna elements, while the number of data streams is proportional to the number of RF chains. Furthermore, conventional antenna selection schemes show a beamforming gain that is proportional to the number of RF chains; however, RF preprocessing methods can increase that gain to the gain of a full-complexity system (i.e., number of antenna elements).

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References

- Foschini, G. J., Gans, M. J. (1998): On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Communications* 6: 311–335.
- Gesbert, D., Shafi, M., Da shan Shiu, Smith, P. J., Naguib, A. (2003): From theory to practice: an overview of MIMO space-time coded wireless systems. *IEEE J. Selected Areas Comm.* 21: 281–302.
- Molisch, A. F., Win, M. Z. (2004): Mimo systems with antenna selection – on overview. *IEEE Microwave Magazine*, March 2004.
- Molisch, A. F., Tufvesson, F. (2005): Mimo channel capacity and measurements. In: *Smart antennas – state of the art* (Kaiser, T., ed.). Eurasip publishing.
- Paulraj, A., Gore, D., Nabar, R. (2003): *Multiple antenna systems*. Cambridge: University Press.
- Telatar, I. E. (1999): Capacity of multi-antenna Gaussian channels. *European Trans. Telecomm.* 10 (November–December 1999).
- Winters, J. H. (1987): On the capacity of radio communications systems with diversity in Rayleigh fading environments. *IEEE J. Selected Areas in Comm.* 5: 871–878. ■