Experimental mitigation of the effects of the limited size aperture or misalignment by singular-value-decomposition-based beam orthogonalization in a free-space optical link using Laguerre-Gaussian modes

KAI PANG[1*], HAOQIAN SONG[1], XINZHOU SU[1], KAIHENG ZOU[1], ZHE ZHAO[1], HAO SONG[1], AHMED ALMAIMAN[1,2], RUNZHOU ZHANG[1], CONG LIU[1], NANZHE HU[1], SHLOMO ZACH[3], NADAV COHEN[3], BRITTANY LYNN[4], ANDREAS F. MOLISCH[1], ROBERT W. BOYD[5], MOSHE TUR[3], ALAN E. WILLNER[1]

[1] Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089, USA
[2] King Saud University, Riyadh 11362, Saudi Arabia
[3] School of Electrical Engineering, Tel Aviv University, Ramat Aviv 69978, Israel
[4] Naval Information Warfare Center, Pacific, San Diego, CA 92152, USA
[5] The Institute of Optics, University of Rochester, Rochester, New York 14627, USA
**Corresponding author: kaipang@usc.edu*

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Limited-size receiver apertures and transmitter-receiver (Tx-Rx) misalignments could induce power loss and modal crosstalk in a mode-multiplexed free-space link. We experimentally demonstrate the mitigation of these impairments in a 400-Gbit/s 4-data-channel free-space optical link. To mitigate the above degradations, our approach of singular-value-decomposition-based (SVD-based) beam orthogonalization includes: (i) measure the transmission matrix H for the link given a limited-size aperture or misalignment; (ii) perform SVD on the transmission matrix to find the U, Σ, and V complex matrices; (iii) transmit each data channel on a beam that is a combination of Laguerre-Gaussian (LG) modes with complex weights according to the V matrix; and (iv) apply the U matrix to the channel demultiplexer at the receiver. Compared with the case of transmitting each channel on a beam using a single mode, our experimental results when transmitting multi-mode beams show that: (a) with a limited-size aperture, the power loss and crosstalk could be reduced by ~8 and ~23 dB, respectively; and (b) with misalignment, the power loss and crosstalk could be reduced by ~15 and ~40 dB, respectively.

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Space-division-multiplexing (SDM) has the potential to further increase the total capacity of a free-space optical (FSO) link, since multiple independent data-carrying beams can be simultaneously transmitted [1]. One subset of SDM is mode-division-multiplexing (MDM), in which each data-carrying beam occupies a different orthogonal spatial mode from a modal basis set [2,3]. With MDM, little inherent crosstalk is induced when multiplexing at the transmitter (Tx) aperture, spatially co-propagating, and demultiplexing at the receiver (Rx) aperture [4].

One example of an orthogonal modal set is Laguerre-Gaussian (LG) beams that form a 2-dimensional $\{l, p\}$ set, in which: (i) l represents the number of 2π phase shifts in the azimuthal direction of the phasefront, and (ii) $p+1$ represents the number of radial intensity nodes (e.g., rings) [5]. An MDM link can multiplex multiple beams, each composed of a single LG mode [4, 6,7, 8]. Unfortunately, such MDM FSO links can suffer from degrading effects that can be induced by a limited-size receiver aperture or misalignments between the transmitter and receiver apertures. These two issues can cause deleterious signal-power loss and modal-power-coupling crosstalk in MDM links in which each channel is transmitted on a beam using a single LG mode [9-14].

In general, approaches that were utilized to reduce the modal crosstalk caused by a limited-aperture receiver include the following: (i) Each data channel is transmitted on a beam composed of one carefully-selected orbital-angular-momentum (OAM) mode such that the received channels on different beams remain orthogonal [12,13]; and (ii) Carefully choose specific modes at the

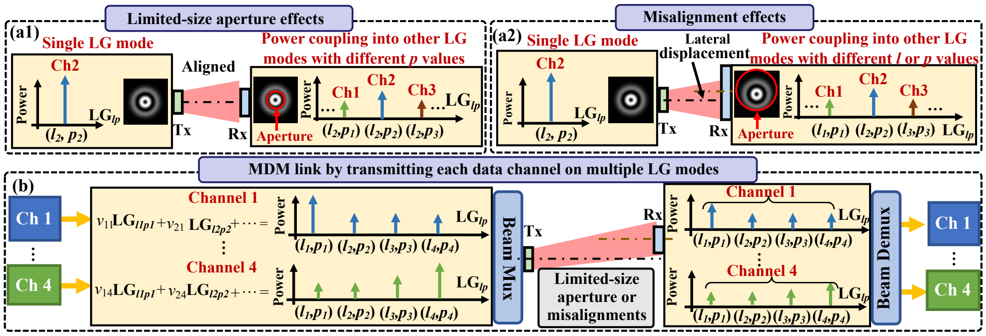

Figure 1 (a) Concept of (a1) limited-size aperture and (a2) misalignment effects on an FSO link using LG modes. (b) Concept diagram of transmitting each data channel on a designed beam that is combination of multiple LG modes (multi-mode beam) to mitigate the effects of the limited-size aperture or misalignments an MDM link. Ch: Channel; Tx: Transmitter; Rx: Receiver.

transmitter and carefully choose receiver aperture to reduce the crosstalk such that cross-coupling does not significantly appear at the other channel [14].

In our approach, each data channel is transmitted on a single beam composed of a unique set of LG modes. By carefully designing the modal complex weights, all data channels could be spatially tailored and remain orthogonal at the receiver. This could simultaneously mitigate power loss and crosstalk induced by limited-size apertures or Tx-Rx misalignments.

In this Letter, we experimentally demonstrate the mitigation of the power loss and crosstalk induced by the limited-size aperture or misalignments in a 400-Gbit/s four-channel FSO link. In our approach, we follow these steps: (i) measure the transmission matrix **H** for the link that has a limited-size aperture or misalignments (ii) perform SVD on the transmission matrix to find the U, Σ, and V complex matrices; (iii) transmit each data channel on multi-mode beam with weights according to the V matrix; and (iv) apply the U matrix to the receiver demultiplexer. Experimental results when transmitting multi-mode beams instead of single-mode beams show that: (a) with a limited-size aperture, the power loss and crosstalk could be reduced by ~8 and ~23 dB, respectively; and (b) with misalignments, the power loss and crosstalk could be reduced by ~15 and ~40 dB, respectively.

In a perfectly-aligned LG mode multiplexed FSO link, the aperture of the Rx is large enough to receive the whole transmitted beam with little power loss and crosstalk. However, if the Rx aperture size is limited, only a part of the power would be collected. Moreover, this issue could also cause power coupling from the desired LG mode to other LG modes with the same *l* value but different *p* values, thereby increasing both power loss and crosstalk, as shown in Fig. 1 (a1) [10]. In addition, when there is a Tx-Rx misalignment, the power on the desired LG mode would be coupled to other LG modes with different *l* or *p* values. This could induce both power loss and crosstalk, as shown in Fig. 1 (a2) [9].

Figure 1 (b) presents the concept of transmitting each data channel on a multi-mode beam to mitigate effects of the limited-size aperture or misalignments in an MDM link. Generally, the mode coupling between a set of LG modes in a given link can be described as a complex transmission matrix **H** [8]. Our approach is based on SVD of **H** ($\mathbf{H} = \mathbf{U} \cdot \sum \cdot \mathbf{V}^*$), which is usually utilized to find orthogonal basis at the Tx and Rx with no interference between data channels [15-17]. At the Tx, the multi-mode beams are generated by complex combinations of multiple LG modes, of which the complex weights are given by the orthogonal column vectors of the **V**. After passing through a given link with a limited-size aperture or misalignments, the resulting beams on different channels are still composed of multiple LG modes, of which the complex weights are the orthogonal row vectors multiplied by singular values in $\sum$ matrix. This means that these beams are still orthogonal to each other, and thus can be demultiplexed with little crosstalk based on the orthogonal row vectors of the inverse of the **U** matrix. Besides, intensity profiles of the transmitted beams would be spatially shaped, which might also reduce the power loss caused by the limited-size aperture or misalignment.

Figure 2 shows the experimental setup of a four-channel MDM FSO link. At the Tx, a 100-Gbit/s quadrature phase shift keying (QPSK) signal at 1550 nm is amplified by an erbium-doped fiber amplifier (EDFA) and subsequently equally split into four branches by a 1×4 coupler. All of the four copies are delayed by single mode fibers with different lengths to decorrelate the data sequences. Subsequently, each branch is sent into a collimator that generates a collimated Gaussian beam with a diameter of 3 mm. The Gaussian beams are sent to two spatial light modulators (SLMs) loaded with different phase holograms on each half of the screen to create specific beams. These four outputs are multiplexed and coaxially propagate in free space over ~1 m. At the Rx, SLM 3 is loaded with a specific phase pattern to convert one of the incoming beams to a Gaussian-like beam. Finally, it is coupled to an SMF for coherent detection and bit error rate (BER) measurements.

The process of the proposed mitigation method includes the following steps: (i) Measure the complex transmission matrix **H** for a set of LG modes using the method in [8]. For an MDM link with a given limited-size aperture or misalignment, the amplitude and phase of each element in **H** represent the power coupling between two LG modes. By transmitting and receiving single LG modes (i.e., probe beams) separately, the amplitude of each element is obtained by direct power measurement. In addition, its phase is calculated by sequentially loading four different phase masks on the SLM at the Tx and measuring four corresponding power values at the Rx with a specific phase pattern; (ii) Calculate the SVD of **H** by $\mathbf{H} = \mathbf{U} \cdot \sum \cdot \mathbf{V}^*$. (iii) At the Tx, utilize different columns of matrix V to design mutually orthogonal multi-mode beams, each of which is a complex combination of multiple LG modes and carries one of the four data channels. Subsequently, we construct the phase patterns on the SLM (1 and 2) for orthogonal multi-mode beams using the approach in [8]. (iv) At the Rx, utilize different rows of the inverse matrix U to construct the phase patterns on SLM 3 that can convert the incoming multi-mode beam back to a Gaussian-like beam [8]. As an example, when the radius of the receiver aperture is 1.0 mm, the Tx and Rx SLM patterns for single-mode

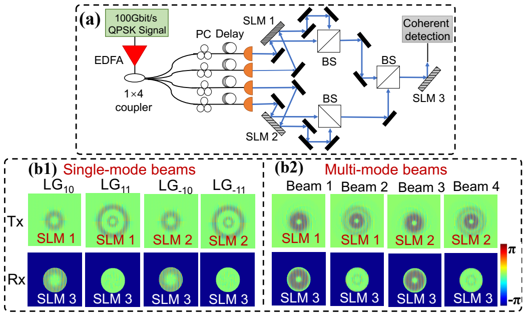

Fig. 2. (a) Experimental setup of a four-channel MDM FSO system. QPSK: quadrature phase-shift keying; EDFA: erbium-doped fiber amplifier; PC: polarization controller; Col.: collimator; SLM: spatial light modulator; BS: beam splitter. (b) The Tx and Rx SLM patterns for (b1) single-mode beams, and (b2) designed multi-mode beams. Here, the radius of receiver aperture is 1.0 mm.

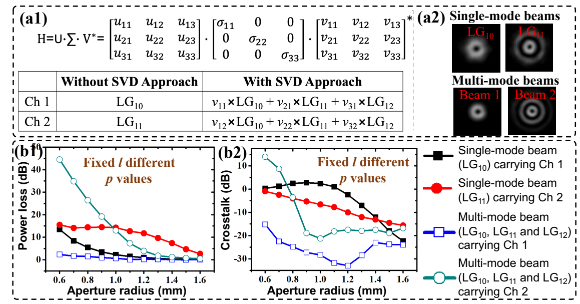

Fig. 3. (a1) The complex weights of LG modes carrying data channel 1 and 2 without and with SVD approach; (a2) Measured intensity profiles of single-mode beams and multi-mode beams when aperture radius is 1.0 mm. (b) Measured limited-size-aperture induced (b1) power loss and (b2) crosstalk under various aperture radii when transmitting data channels on single-mode beams or multi-mode beams (LG_{10}, LG_{11} and LG_{12} modes). For each aperture radius, the two multi-mode beams are unique due to different transmission matrix.

beams and multi-mode beams are shown in Fig. 2 (b1) and Fig .2 (b2), respectively. We note that the Tx and Rx SLM patterns should be different under different aperture sizes or misalignments due to different transmission matrix.

First, we evaluate the effect of the Rx aperture size on the MDM link when transmitting data channels 1 and 2 (Ch 1 and 2) on a single LG_{10} and LG_{11} mode, respectively, as shown in Fig. 3(a1). It should be noted that the "Ch 1" and "Ch 2" in the following figures carry the same meaning as the ones in Fig. 2. At the Tx, we generate single LG_{10} or LG_{11} beam with a beam radius of 0.7 mm by using the SLM. After propagating through the free space, LG_{10} and LG_{11} beams have beam size of ~1.3, and ~1.8 mm at the Rx, respectively. The phase patterns on SLM-3 have circular aperture shapes, whose radii is defined as aperture sizes in our experiment. As expected, the power loss increases as the aperture size decreases, as shown in Fig. 3 (b1). In addition, we also measured channel crosstalk that refers to the power coupled from the other modes over the power on the desired mode. We see that the crosstalk for both LG beams become higher when decreasing the aperture size. This is because a limited-size aperture blocks a part of the LG beams, and thus degrading the orthogonality between LG modes with different *p* values [10]. In our approach, the data channel is transmitted on a multi-mode beam. Since the limited-size aperture affects the beam profiles in the radial direction, multiple LG modes with different *p* values are utilized to tailor the profiles of multi-mode beams in the radial direction, as shown in Fig. 3 (a1). However, we find that there would be a relatively high power loss if we only use the same two LG modes (LG_{10}, and LG_{11}) as the case of single-mode beams. Therefore, an extra LG mode (LG_{12}) is utilized in the generation of multi-mode beams. Theoretically, the power loss of the multi-mode beams is related to the diagonal elements in the ∑ matrix [15]. We see that power loss for both channels is reduced with an aperture radius of 1.0-1.6 mm. In addition, we also find that when the aperture radius is less than 0.8 mm, the power loss of the multi-mode beam carrying data channel 2 would become even larger. This might be due to the variation of **H** with the aperture radius. As the aperture radius decreases, the power coupling from the desired single LG mode to the other LG mode gradually increases and becomes similar to the power on the desired LG mode [10]. This results in a smaller difference between the values of diagonal elements and off-diagonal elements in **H**. Consequently, this would induce a decrease in the value of the second diagonal element in the ∑ matrix after SVD calculation of **H**, which causes an increase in the power loss for the second multi-mode beam.

Moreover, the crosstalk between the two channels is also reduced, which is due to that the orthogonality between the two beams is not degraded by the limited-size aperture. We also note that the crosstalk for channel 2 increases when the aperture radius is less than 0.8 mm. This is mainly due to a large power loss of the multi-mode beam carrying channel 2 under a small aperture size, which will cause a higher crosstalk that refers to the ratio of power coupled from the other multi-mode beam to the power of desired multi-mode beam. Moreover, it could be expected that when using additional LG modes with higher *p* values, the performance of these two channels might be further improved. This might be due to more sophisticated control of the multi-mode beam's spatial profile in the radial direction by using additional LG modes with higher *p* values [18]. As an example, the experimental intensity profiles of single-mode beams and multi- mode beams with an aperture radius of 1.0 mm are shown in Fig. 3 (a2). The intensity profiles of multi-mode beams vary with the aperture radius according to the SVD calculations of transmission matrices for different aperture radii. This results in different power losses with the aperture radius, as shown in Fig. 3 (b1). We can see that when the aperture radius is 1.0 mm, both multi-mode beams are smaller than single-mode beams, which results in lower power losses.

As one kind of the misalignments in an MDM system, the influence of horizontal displacements on system performance is also investigated. As shown in Fig. 4 (a), when transmitting data channels on single-mode beam with different *p* values (LG_{10}, LG_{11}), both the power loss and crosstalk become larger with horizontal displacements. In our approach, the same two LG modes (LG_{10}, LG_{11}) are utilized to generate the multi-mode beams. Compared with the case of single-mode beams, the power loss for both channels is reduced within a displacement range of 0.4-0.7mm. In addition, the crosstalk for both multi-mode beams remains at a relatively low level (<-27 dB) with the displacement. Furthermore, we also explore the effects of displacements on LG modes with different *l* values (LG_{10}, LG_{-10}), as shown in Fig. 4 (b). It is observed that the power loss and crosstalk of LG_{10}, LG_{-10} modes increase with the displacement. For both channels, the power loss can be reduced when using the multi-mode beams (composed of LG_{10}, LG_{-10} modes) with a displacement range of 0.8-1.1 mm. Moreover, the crosstalk for both channels stays at a relatively low level (<-17 dB) with

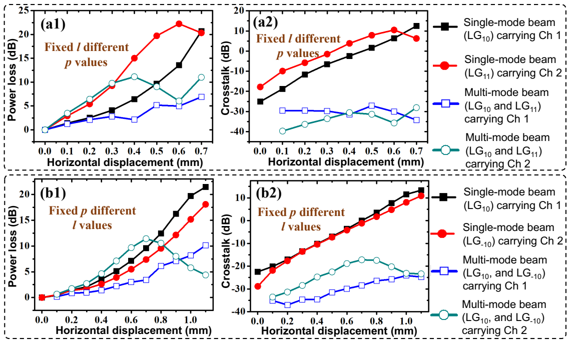

Fig. 4. (a) Measured displacement-induced (a1, b1) power loss and (a2, b2) channel crosstalk with horizontal displacements when transmitting channels on single-mode beams or multi-mode beams.

the displacement. We also find an increase and then a decrease of the power loss for the second beam with the displacement in Fig. 4. This might be due to the change of **H** with the displacement [13]. First, the power coupling from the desired single LG mode to the other LG mode gradually increases and becomes similar to the power of the desired LG mode with the displacement. This results in a smaller difference between the values of diagonal elements and off-diagonal elements in **H**. Consequently, this would induce a decrease in the value of the second diagonal element in the $\sum$ matrix, which causes an increase in the power loss for the second beam. However, as the power coupling continually increases with the displacement, it would be higher than the power in the desired LG mode. Therefore, the difference between the values of diagonal elements and other elements in H will increase. As a result, the value of the second diagonal element in $\sum$ matrix would increase, which causes the power loss of the multi-mode beam to decrease [15].

We subsequently utilize our approach in a four-channel multiplexed link, each of which carries a 100-Gbit/s QPSK signal. Specifically, four single-mode-beams (LG_{10}, LG_{11}, LG_{-10} and LG_{-11}) or four multi-mode beams are transmitted and received. Here, the aperture radius is 1.0 mm and there is no displacement. In the case of single-mode beams, all four channels have high error vector magnitude (EVM) (>50%). This is due to their higher crosstalk caused by the limited-size aperture. However, when using the multi-mode beams, the EVM of the four channels are all at a relatively low level (~22%). Here, first two multi-mode beams are composed of LG_{10}, LG_{11} and LG_{12} modes, while other two multi-mode beams are composed of LG_{-10}, LG_{-11} and LG_{-12} modes. In BER measurements, the power penalties of all four channels are < 2 dB at the 7%-overhead forward error correction (FEC) limit.

In this paper, we utilized the SVD-based beam orthogonalization to mitigate the effects of the limited-size aperture or misalignments. Moreover, we note that the beam size and direction can be further tailored by adding transfer functions of a lens and a linear grating on the transmitter-side SLM, respectively [19, 20]. Therefore, this might be an effective complement to our approach and further improve the system performance. Furthermore, we note that it might also be possible to utilize our approach to mitigate other issues in an MDM link, such as atmospheric turbulence [8].

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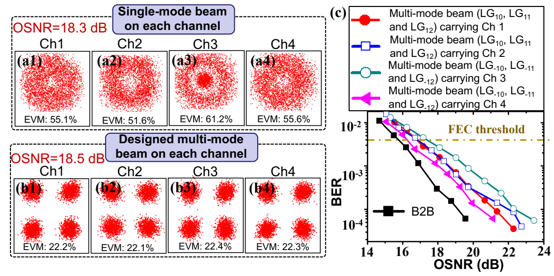

Fig. 5. Measured constellations of four data channels when transmitting each channel on (a) a single-mode beam or (b) a multi-mode beam. (c) Experimental BERs of four data channels with OSNR when using multi-mode beams. B2B: back to back; FEC: forward error correction.

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