Directional ZigZag: Neighbor Discovery with Directional Antennas

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***Abstract*—We introduce a new neighbor discovery method for wireless nodes with adaptive antennas, called "directional ZigZag." Adaptive antennas are capable of electronically changing their gain pattern and in particular form beams and steer them in arbitrary directions. Despite improving range, this makes neighbor discovery more difficult. We consider three cases; namely, nodes transmit with beamsteering and receive in omni-directional mode (DTOR), transmit with omni-directional antenna and receive with beamsteering (OTDR), and use beamsteering (directional antennas) for both transmitting and receiving (DTDR). We show that directional ZigZag detects the neighbors with vanishing probability of error. Furthermore, it is the first algorithm that requires a discovery period that scales on linearly with the average number of neighbors. This is proven through establishing a connection between ZigZag and message passing decoding.**

I. Introduction

In recent years, there has been a surge of interest in device-to-device communications, due to the improved spectral efficiency and flexibility it provides [1]. In such networks, devices transmit information directly between each other (possibly under *control* of a base station (BS), but without sending payload via it), either in a single-hop or via multi-hopping. Directional antennas can greatly enhance the performance of such systems, as they improve the signal-to-noise-ratio (SNR), which is especially important due to the higher pathloss in Device-to-Device (D2D) communications (compared to cellular systems) both at microwave and millimeter-wave frequencies [2]. Furthermore, adaptive antennas strongly reduce interference by receiving mostly in the main beam direction and as a result, increase the spatial reuse by allowing the scheduling of multiple nodes at the same time.

A key requirement for any D2D network is neighbor discovery, *i.e.*, the process of detecting which nodes can communicate with each other directly. Obviously, this is the first task that must be performed in the network formation, since it enables all other operations such as scheduling [3], and routing [4].

Neighbor discovery algorithms can be categorized as "randomized" or "deterministic". In the randomized algorithms [5]–[7], the nodes—due to the half-duplex property—choose to either transmit or listen randomly in each time slot so that each node gets a chance to hear its neighbors as well as be heard by them. In the deterministic algorithms [8]–[11], on the other hand, nodes transmit their identifiers according to a pre-determined transmission pattern.

The early work on neighbor discovery can be viewed as part of medium access control (MAC) protocols and focuses on omni-directional antenna based systems. How to harmonize the directional transmission and reception in the system is an essential research issue as deployment of directional antennas complicates the neighbor discovery. That is, each node should scan its surroundings by advertising its unique identifier in different directions. Further, unlike omni-directional neighbor discovery where the nodes could detect their neighbors without them knowing, the nodes should agree on becoming neighbors. This is due to the fact that a node may only transmit to a neighbor when both aim their antenna beams toward one another. Furthermore, fast discovery of neighbors is essential especially in mobile networks, since it must be repeated often enough for sufficient knowledge of the network topology is maintained.

Several approaches have been proposed for directional wireless links by modifying the IEEE802.11MAC protocol [12], [13]. They use the Directional Virtual Carrier Sensing (DVCS) concept that extends the IEEE802.11 Distributed Coordinated Function (DCF) to directional wireless networks. However, these protocols require the receiver in omni-mode to receive Request to Send (RTS) control packets. A major challenge is the fact that if one link end uses a beamwidth that is larger than during actual communications (as is the case in the 802.11 MAC), then not all viable neighbors may be detected.

Furthermore, the efficiency of those algorithms is very limited as they rely on random transmission and reception and thus their performance deteriorates as the network becomes denser and denser.

In this paper, we take a different approach, namely extend our highly efficient omni-directional discovery method [11] and introduce "*directional ZigZag*" for devices with beamsteering antennas. The main idea of ZigZag is that the nodes may cancel the known identifiers from received signals to retrieve identifiers of other neighbors, *i.e.*, perform a sort of iterative interference cancellation. The challenge consists of designing the transmission patterns so that the discovery time remains short while all nodes manage to detect all their neighbors with high probability.

A. Our contribution

To the best of our knowledge, we offer the first neighbor discovery with directional antennas where the discovery period (in slots) is on the order of the average number of neighbors. This is a *significant* improvement over the previous schemes [6], [12], [14].[1]

Further, our algorithm is the first deterministic algorithm for directional antennas and there exist explicit constructions for the on-off transmission pattern of the nodes, *e.g.*, Gallager [15] or PEG [16] construction.

Perhaps most importantly, we provide bounds on the probability of missed detection. Finally, we verify our findings with simulations for both the ideal setting and more realistic one.

The remainder of this manuscript is organized as follows: In section II we discuss the model and assumptions we consider. Then in section III, we describe the directional ZigZag algorithm and state the main theorem. Section IV considers the practical aspects of using ZigZag in more realistic environment. Finally, in section V we verify our findings with simulations.

II. System Model

We consider a network with n devices with the following assumptions:

- Each node is assigned a unique identifier (ID).
- Each node can either transmit or listen (half-duplex assumption).
- All nodes know when the discovery starts and transmit synchronously (synchronous model). This can be achieved in a D2D system, *e.g.*, through a beacon signal sent by the BS.
- All nodes are equipped with electronically steerable antenna arrays that can be used to either provide directivity or enable an omni-directional mode.
- The directional beampatterns are "flat-top", *i.e.*, the antenna gain pattern in directional mode is a circular sector with angle w and radius equal to the transmission/reception range.
- All nodes transmit with the same power P_t, and use beam width w_t and w_r while transmitting and receiving, respectively. Antenna gains g_{w_t} and g_{w_r} are associated with these beam widths.
- A noiseless model, *i.e.*, energy at the receiver is offset by the thermal floor such that 0 corresponds to just noise. Thus, given the large enough transmission power, reading any energy by the receiver means the presence of transmission.
- Beam directions and directions in which nodes can be discovered are frequency-independent. This assumption is well fulfilled in frequency-flat channels as well as in the case that the adaptive antennas perform analogue beamforming in a narrowband system.

Then, the goal is for each node to detect all its neighbors. The area covered by antennas depends on the beam width of both transmitter and receiver antennas. As a result, the neighborhood and its area depends on the antenna gains.

Definition 1: The neighborhood of a node is the set of nodes that can be heard with power greater than a certain threshold Γ which depends on the communication channel and purpose of the communication, *i.e.*,

$$\mathcal{N}(v_i, g_t, g_r) = \{v_j : \max_{\sigma_i,\sigma_j} P_t g_t g_r |h_{v_i(\sigma_i,w_t),v_j(\sigma_j,w_r)}|^2 \geq \Gamma\}, \tag{1}$$

where $h_{v_i(\sigma_i,w_i),v_j(\sigma_j,w_j)}$ is the channel gain between nodes v_i and v_j when they point their beams to directions σ_i, σ_j with width w_r, w_t, respectively; g_t, g_r are the antenna gains for transmitter and receiver, respectively; and the maximum is taken over all possible directions.
Let w_t^* and w_r^* be the minimum formable beam width for a receiver and transmitter antenna, and g_t^*, g_r^* be the corresponding gains. Then the maximum neighborhood is achieved when both transmitters and receivers use minimum beamwidths g_t^* and g_r^*. Similarly, the "omni-neighborhood" is when $g_t = g_r = 1$.

As shown by (1), the neighborhood relation is reciprocal if nodes use similar antenna patterns. Further, as shown by (1), nodes require to record the angle on which the transmission/reception to/from the channel is most powerful and thus the profile of a neighbor includes both its ID and angle. Furthermore, each two neighbors must agree on being neighbors and setting some time for future contact. The protocol by which nodes decide on the future communication, which could be either directly between nodes, or supported by the BS, is out of the scope of the current paper.

A node that uses beam antenna needs to scan all its surroundings. For an antenna with beam width w, the number of beams required to scan in 2-dimension is $\Sigma^2 = \lceil \frac{2\pi}{w} \rceil$, while in 3-dimension $\Sigma^3 = \lceil \frac{2}{1-cos(\frac{w}{2})} \rceil$ beam directions [17] are needed.

Many works on directional neighbor discovery focus on using directional transmitters and omni-directional receivers (DTOR). While this case keeps the interference low, it suffers from complications of forming the transmitting antenna beam. On the other hand, it is easy to form receiving antennas with small beam patterns. Hence, for directional ZigZag, we also consider the case with

[1]Some of these work have not stated the expected discovery period in a closed form manner and thus we judged the performance based on the simulations presented in the paper.

omni-directional transmitting and directional receiving (OTDR). Note that this method suffers from more interference compared to the previous case since nodes broadcast their ID in all direction and disturb the surrounding receivers. Finally, we consider directional antennas on both transmitter and receiver (DTDR).

III. Directional ZigZag Discovery

In this section we discuss our directional discovery algorithm. We start by briefly explaining the omni-directional ZigZag [11], and then proceed by modifying it to the directional case.

A. Omni-directional ZigZag

We introduced the ZigZag neighbor discovery in [11]. The method relies on using a transmission pattern which is known by all nodes for transmitting the unique identifier (ID) of the nodes, which in return enables the nodes to cancel the known ID's from other transmissions and recover more IDs.

ZigZag involves successive cancellation of already detected neighbors. If node v_i finds a timeslot ("chunk") in which it hears only the transmission of node $v_j \in \mathcal{N}(v_i)$ and thus can recover its ID, then it will be able to remove all other transmissions of node v_j from other slots, such that it will be able to create more free timeslots to detect other neighbors, even in the presence of collisions.

To theoretically estimate the performance of ZigZag, we consider the collision model. In the collision model, node v_i detects its neighbor v_j when it is the only neighbor transmitting toward v_i. That is, there exists a slot c such that $A(c, j) = 1$, $A(c, i) = 0$, and $A(c, k) = 0$ for any other node $v_k \in \mathcal{N}(v_i)$. Let us call such slots "*free-chunks*". Further, there is no interference from non-neighbors in the collision model. That is, for node v_i, $h^i_j = 0$ if $v_j \notin \mathcal{N}(v_i)$. Let us clarify this by an example.

Consider the scenario where nodes v_1, v_2, and v_3 are neighbors of v_0. In slot t_1, nodes v_1 and v_2 transmit their identifiers $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. In t_2, nodes v_2 and v_3 transmit their identifiers $\mathbf{s}_2$ and $\mathbf{s}_3$, and finally in slot t_3 node v_3 transmits its identifier $\mathbf{s}_3$. Thus node v_0 receives the following signals on these slots

$$
\begin{aligned}
\mathbf{r}_0[t_1] &= P_t h_{v_0,v_1}[t_1]\mathbf{s}_1 + P_t h_{v_0,v_2}[t_1]\mathbf{s}_2 \\
\mathbf{r}_0[t_2] &= P_t h_{v_0,v_2}[t_2]\mathbf{s}_2 + P_t h_{v_0,v_3}[t_2]\mathbf{s}_3 \\
\mathbf{r}_0[t_3] &= P_t h_{v_0,v_3}[t_3]\mathbf{s}_3.
\end{aligned}
$$

Then v_0 can recover $\mathbf{s}_3$ from slot t_3 (a free chunk) and cancel it from slot t_2 in order to recover $\mathbf{s}_2$. Finally v_0 recovers $\mathbf{s}_1$ by canceling $\mathbf{s}_2$ from slot t_1. Thus, node v_0 can recover the ID and the channel coefficients of all its three neighbors, despite the fact that two out of three slots had collisions. As will be discussed in Sec. III-C, the construction of the transmission patterns ensures free chunks with high probability.

B. Directional ZigZag Algorithm

Here we extend the Zigzag algorithm to the directional case. Apparently, we should introduce a way for each node to scan all its surrounding as both receiver and transmitter. This, however, is not a trivial task. Specifically, this should be done in a manner that for any two nodes v_i and v_j, with high probability, there exists a time slot, in which v_i transmits its ID toward v_j while v_j is receiving in the direction of v_i, and further v_j gets to decode the ID of v_i.

Fix Σ_t and Σ_r according to w_t and w_r, respectively. Let the algorithm runs for Σ phases where $\Sigma = \Sigma_t$ phases for DTOR and DTDR, and $\Sigma = \Sigma_r$ phases for OTDR. Further, define Π_i to be a random ordering (permutation) of Σ indices such that $\Pi_i(\sigma)$ is the σ-th element of Π_i. Furthermore, the transmission pattern of the nodes on the σ-th phase is represented by an $m \times n$ zero-one matrix $\mathbf{A}_\sigma$ where m is the number of times slots for each transmission phase, and n is the number of nodes in the network. . That is, we assign each node a column of $\mathbf{A}_\sigma$ as its transmission pattern on the σ-th phase. Each node knows all the transmission patterns, *i.e.*, $\mathbf{A}_\sigma$. Given the time slot to be the time required to transmit the ID, then $A_\sigma(c, i) = 1$ means that node v_i transmits on the c-th time slot in direction σ. Further, nodes listen to the channel on the zero entries of their transmission pattern to receive identifiers of other nodes.

For OTDR, on phase σ, each node chooses a direction $\Pi_i(\sigma)$ where $\sigma \in \{1, \dots, \Sigma_r\}$ for its receiver to use during the zero entries of the pattern $\mathbf{a}_i$. On the one entries of the transmission pattern, a node transmits omni-directionally to its surrounding and the discovery has Σ_r phases. For DTOR, on the other hand, we do the reverse. That is, on one entries, node v_i transmits in direction $\Pi_i(\sigma)$ for $\sigma \in \{1, \dots, \Sigma_t\}$ and use an omni-directional pattern for receiving on zero entries. For DTDR, we use the system similar to OTDR. On one entries of the pattern, however, nodes randomly scan the surrounding. That is, they transmit in all directions $\sigma \in \{1, \dots, \Sigma_t\}$ in a pseudo-random order which enable the neighbors to hear them. Thus, if we denote the total discovery period with $M = \Delta m$, then $\Delta = \Sigma_r, \Sigma_t$ for DTOR and OTDR, respectively; and $\Delta = \Sigma_r \Sigma_t$ for DTDR.

Let us abuse the notation and use $\mathbf{h}^i_\sigma$ to denote the vector of channel power gains whose elements $h^i_\sigma(j) = |h_{v_i(\Pi_i(\sigma), w_r), v_j(\Pi_j(\sigma), w_t)}|^2$. Due to the half-duplex condition, where node v_i can either transmit or receive, node v_i listens to the channel on slots c for which $A_\sigma(c, i) = 0$. Let $\mathbf{A}^i_\sigma$ denote matrix $\mathbf{A}_\sigma$ whose rows $\mathbf{a}^c$ where $A_\sigma(c, i) = 1$ are omitted. Then, the signal received by the i^{th} node during the σ^{th} phase by $\mathbf{y}^i_\sigma$ is

$$\mathbf{y}^i_\sigma = P_t g_{w_t} g_{w_r} \mathbf{A}^i_\sigma \mathbf{h}^i_\sigma. \tag{2}$$

We may use a different transmission pattern for each phase or use the same transmission pattern Σ times. We can also use the same transmission pattern, but assign the columns, *i.e.*, patterns, randomly to different nodes for each phase.

As pointed out before, in neighbor discovery two neighboring nodes must agree on neighbor relationship and choose a future time for their future rendezvous. To address this issue, we assume each node runs the ZigZag at the end of each phase σ and detects all the neighbors in a particular direction (or as in DTOR, announces itself to the neighbors in particular direction). Then, from the second phase, each node includes the index of the detected neighbors with the rendezvous in its ID. Thus, the detected neighbors can hear them and agree on the future communication. The only concern is when two neighbors point their antennas at each other during the same phase which happens with probability $\frac{1}{\Sigma_r \Sigma_t}$. Note that, in practical systems, for small enough beam width and due to multi-path fading, a neighbor can be heard in several directions, which decreases the probability of one-sided agreement greatly. Alternatively, the neighbor lists can be communicated to the BS, which then in a broadcast lets all devices know their neighbors and the associated pattern directions. The discussion on how the future rendezvous is scheduled depends on the MAC protocol and is beyond the scope of current manuscript.

C. The transmission pattern

Our goal is to design the matrices $\mathbf{A}_\sigma$ so that when nodes transmit according to $\mathbf{A}_\sigma$, free-chunks exist with high probability for all nodes. A formal equivalence between ZigZag decoding and message passing (MP) decoding for LDPC codes over binary erasure channels (BEC) is shown in [11] (see also [18], [19]). This connection allows us to use the parity-check matrix of "good" regular LDPC codes with large girths[2]. It is well-known in coding literature that a code of length n is considered to have a large girth g if it satisfies the log relationship

$$g = \Theta\left(\frac{\log(n)}{\log((d_r-1)(d_\ell-1))}\right), \tag{3}$$

where d_ℓ, d_r represent the degree of nodes and measurements. Many constructions such as Gallager's [15] or progressive edge growth method [20] satisfy (3). On the other hand, it is known [21] that cyclic and quasi cyclic codes do not satisfy (3) and thus cannot be used for our purpose.

Let k denote the average number of neighbors a node has. Further, let k^* denote the average number of neighbors whose ID can be heard during a phase which we call *effective* number of neighbors. For example, if nodes are distributed uniformly over the field, and each node, independent of the antenna patternm (see the discussion in Sec. III-D), has k neighbors in average, then k^* is $k/\Sigma_t, k/\Sigma_r, k/(\Sigma_t\Sigma_r)$ for DTOR, OTDR, and DTDR, respectively.

For the transmission pattern of each phase, we build a (d_ℓ, d_r)-regular LDPC code satisfying (3) with the following degrees and number of rows:

$$\begin{aligned} m &= \lambda k^* \\ d_r &= \lambda_r n/k^* \\ d_\ell &= \lambda_\ell, \end{aligned} \tag{4}$$

where $\lambda, \lambda_r, \lambda_\ell$ are constants which must satisfy $\lambda_\ell = \lambda \cdot \lambda_r$ by hand-shaking lemma. Apparently, $\lambda \geq 1$. Let us point out again that the number of rows of the transmission pattern m is the period of one phase (in slots) for DTOR and OTDR, while for DTDR the phase period is $m\Sigma_t$.

D. Performance Analysis

Our main result is Theorem 1 which bounds the probability of error of Directional ZigZag algorithm for finite setup and shows the vanishing probability of error asymptotically when $k = pn$ is linear in n.We are ready to present the bounds on the performance of Directional ZigZag with DTOR and OTDR.

Theorem 1: Consider a random network with n nodes where each node has on average $k = \mathbb{E}[\,|N(v, g_t, g_r)|\,] = pn$ neighbors, where p depends on the employed antenna pattern and transmission power.Further, assume nodes broadcast their unique identifiers according to an on-off transmission pattern specified by the columns of the parity-check matrix of a large girth regular LDPC code with parameters (4). Then assuming a collision model, the probability of a node not detecting a neighbor for Directional ZigZag discoveries; namely, DTOR, OTDR, and DTDR is bounded by

$$P^{(1)}_{\text{error}} \leq \gamma^{(d_\ell - 1)\frac{(d_\ell-2)^{T/2-1}-1}{d_\ell - 3}}, \tag{5}$$

for all $p < \lambda_r/d_r$, when a) $\lambda_r \leq \Sigma_t$ for DTOR; b) $\lambda_r \leq \Sigma_r$ for OTDR; c) $\lambda_r \leq \Sigma_t\Sigma_r$ for DTDR, where for the girth g, $T < g/2$, and Σ_r, Σ_t are number of receiving and transmitting directions. Further, the probability of detecting the neighbors goes to one as n and respectively T grows.

The proof is similar to the one presented in [11] and is omitted due to space constraints. We refer the interested readers to the longer version of this paper available online at [22].

Note that the neighborhood depends on the antenna gains and hence $k = \mathbb{E}[\,|N(v, g_t, g_r)|\,]$ varies for methods DTOR, OTDR, and DTDR as the antenna gains are different. For example, DTDR has the largest neighborhood area among the three methods as both transmitter and receiver use directional antennas.

As shown by Theorem 1, all three schemes require time on the order of the average number of neighbors k. The length of one phase of DTOR and OTDR, m is on the order of $k^* = pn/\Sigma$, while for DTDR, a period of a phase is $\Sigma_r m$ and m scales as $k^* = pn/(\Sigma_r\Sigma_t)$. Thus, the discovery period $M = \Sigma m = \lambda pn = \lambda k$ for all methods. Our simulations show that λ is around 2. Thus, to detect the same number of neighbors k, DTDR requires

[2]Regular means that all nodes have the same degree d_ℓ and all checks have the same degree d_r.

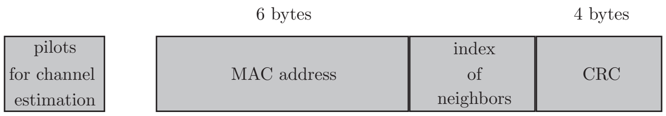

Fig. 1. Node identifiers and the network overhead.

a time (number of slots) similar to DTOR and OTDR. This surprising result is true for large n while for a finite-sized setups we observe an approximate version of this rule. This result —since the neighborhood of DTDR is the maximum neighborhood among the three methods— can make the use of DTOR and OTDR obsolete.

IV. Practical Considerations

Here we want to address some of the practical concerns of the directional ZigZag. Note that in a practical setup, nodes receive interference from non-neighbors. On the other hand, the practical setup, also offers some attributes and advantages compared to the collision model. The identifiers are protected by encoding against the interference and noise. Further, successive interference cancellation allows recovering more than one ID from the received signal in one slot, which improves the performance.

A. ID format

For ZigZag discovery, we consider the packet format shown in Fig. 1 as the node IDs. That is, we use the 6 bytes MAC address as the node identifier. Further, to set the future rendezvous and avoiding one-sided agreements, we assign space for the transmission of indices of the neighbors. The length of this part depends on the protocol in use. Finally, we also add cyclic redundancy check (CRC) of 4 bytes to the ID similar to the OFDM standard, so that the nodes manage to detect the errors when recovering. Further, ZigZag relies on consecutive cancellation of identifiers and thus it requires channel information. As a result, we require to add symbols for channel estimation to the identifiers. These pilot symbols allow us to assume estimation of channel coefficients whenever a receiver manages to decode the signature and use interference cancellation for decoding.

B. Interference Control

The amount of interference depends on the number of nodes transmitting on each slot, *i.e.*, d_r. Thus, by choosing the smallest possible d_r for fixed n and target m, we can reduce the interference.

C. Encoding

As we move beyond the collision model, we should consider the case where ID's are decoded based on the received signal to interference ratio (SIR). We consider an information theoretic recovery model. That is, we assume that the nodes use QPSK modulation with coding rate 1/2 for transmitting their identifiers. For a node to decode the received signals, *i.e.*, identifiers of other nodes, we consider successive interference cancellation (SIC).[3]

[3] We could also consider treating interference as noise recovery but since ZigZag by nature use interference cancellation we found it more appropriate.

In SIC, at each time slot, node v_i tries to decode the strongest ID and if successful, then cancels it from the received signal and continue this until it fails to recover the strongest. More specifically, if the received identifiers are sorted from strongest to weakest, *i.e.*, $t_1|h_{v_1(\sigma_1,w),v_i(\sigma_i,w_i)}| > t_2|h_{v_2(\sigma_1,w),v_i(\sigma_i,w_i)}| > \cdots > t_{v_{n-1}}|h_{v_{n-1}(\sigma_{n-1},w),v_i(\sigma_i,w_i)}|$, then node i on the r^{th} round can decode the identifier of r^{th} node if

$$\log\left(1+\frac{t_r|h_{v_r(\sigma_r,w),v_i(\sigma_i,w)}|^2P_t}{P_N+\sum_{k\in[r+1:n]\setminus\{i,j\}} t_k|h_{v_k(\sigma_k,w),v_i(\sigma_i,w)}|^2P_t}\right)\geq R$$

where $t_j \in \{0,1\}$ is the transmission state that is 1 if v_j is transmitting on that slot, $[1:n]=\{1,\cdots,n\}$, and P_N is the noise power. R denotes the normalized data rate in bits/s/Hz which is 1 for QPSK modulation. Of course, this is too optimistic since the bound is proven for the asymptotic case where length of the identifiers goes to infinity and not the short finite packets we transmit for neighbor discovery. This, however, may be taken into an account by considering a value larger than $R=1$ as the threshold. Note that if the transmitted identifiers on the same slot have well separated receiver powers, then SIC can recover more than one identifier from the same slot.

V. Simulation Results

To verify our theoretic results we run directional ZigZag in the collision model and realistic setup. We consider a network with $n = 2000$ nodes. First, we consider the collision model where a node does not receive any interference from non-neighbors. We consider a disk of radius one with the node v_0 at its center. Then, we uniformly distribute k nodes over the disk, *i.e.*, $|\mathcal{N}_{v_0}| = k$ which we vary from 40 to 200. We depict a transmission pattern parameters with the tuple (m, d_ℓ, d_r). We consider the nodes to use the same beam-width for transmission and reception, *i.e.*, $w_t = w_r = w$. We run the simulations for $w = \pi/6$, and $w = \pi/2$, and we are interested in the probability of successful detection of the whole neighborhood by v_0 conditioned on the number of neighbors k, *i.e.*, $\mathbb{P}\left(S=\widehat{S}||S|=k\right)$, where $S, \widehat{S}$ denote the real set of neighbors of node v_0 and its estimate by ZigZag, respectively.

To analyze the performance of ZigZag, let us introduce the discovery period to neighbor size ratio $\rho = M/k$. The value ρ helps us measure in average how many slots we need to discover k neighbors. Note that we already know an lower bound on this ratio through the connection between ZigZag and MP decoding over BEC. That is, we find the maximum p_{DE} for which density evolution converges to zero and compute the corresponding $\rho = \Sigma\frac{d_\ell/d_r}{p}$. Fig. 2 shows ρ for our simulations where dashed lines correspond to $w = \pi/2$ and solid lines to $w = \pi/6$. As depicted, directional ZigZag requires about $2k$ slots to detect k neighbors. The non-smoothness of the curves stems from the limited number of simulation points.

For a more realistic setup where we consider the interference from non-neighbors, we use a model similar to the

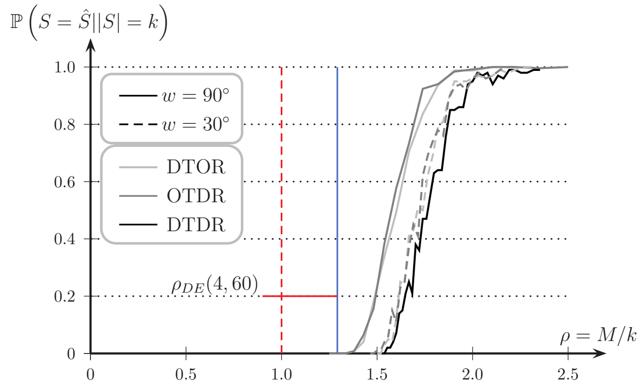

Fig. 2. The ratio $\rho = M/k$ for directional ZigZag in the collision mdoel.

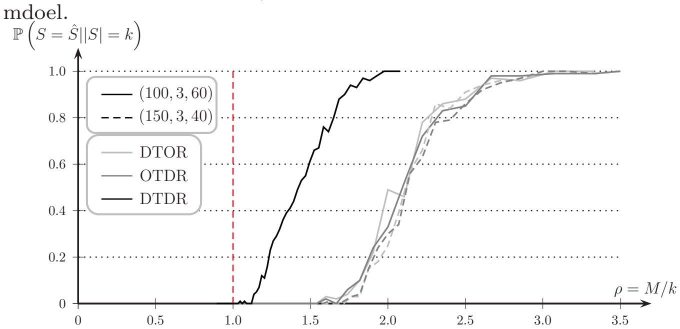

Fig. 3. The ratio $\rho = M/k$ for directional ZigZag in the realistic model and $w = \pi/2$.

one in [6]. That is, we consider a geometric pathloss model $d^{-\alpha}$ for $\alpha = 4$. All nodes use $P_t = 20dBm$ and beamwidth $w = \pi/2$. For our simulation, we choose neighborhood radius $R_n = 150$m for DTDR and $R_n = 107$m for other schemes.

For our simulation, we consider a circular field with radius R_f and locate node v_0 at its center. Further, for different values of k changing from 40 to 200, we uniformly distribute k nodes in the neighborhood of v_0, *i.e.*, inside a circle with radius R_n and center v_0. Then, we uniformly distribute $n - k - 1$ outside the neighborhood of v_0. To preserve the density of the network p, we shrink the radius of the field R_f as we increase k. Specifically, we set $R_f = R_n\sqrt{k/(n-1)}$.

As shown in Fig. 3, the performance of directional ZigZag does not deteriorate much due to interference. For DTOR and OTDR, the ratio of recovery changes from $2k$ to $3k$, while for DTDR it remains unchanged. This is because due to directivity at both ends, the amount of received interference is reduced by factor of Σ^2 which protects the method against the interference.

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