Analysis of Urban Millimeter Wave Microcellular Networks

(Invited Paper)

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***Abstract*—Millimeter wave (mmWave) networks are sensitive to blockages due to buildings in urban areas. This is critical for vehicle-to-infrastructure networks which are cellular networks designed to support emerging vehicular applications. Motivated by measurement and ray tracing results in urban microcells, instead of characterizing the pathloss by Euclidean distance, we calculate it by the weighted sum of segment length along the propagation path, i.e., Manhattan distance, and a certain corner loss at the intersections along the path. We analyze network performance by modeling the urban microcell network by a Manhattan Poisson line process. Our results show significant differences between Manhattan and Euclidean distance-based pathloss models. Assuming the receiver is associated with the base station (BS) with the smallest pathloss, we derive closed-form expression of the distribution of the associated link pathloss. We obtain the coverage probability and reveal the impacts of interference from the LOS and NLOS BSs. It is shown that in this scenario the interference from a NLOS parallel street is negligible.**

I. INTRODUCTION

Vehicle-to-Infrastructure (V2I) communication has applications in safety, traffic efficiency, and infotainment. Safety applications can make use of gigabit-per-second data rates exchange of raw sensor data between vehicles and infrastructure [1][2], which cannot be supported by conventional vehicular communication technologies, e.g., dedicated short-range communication [2][3]. This motivates a cellular architecture for V2I communication where base stations are located along streets and provide service to vehicles. The use of millimeter wave (mmWave) in cellular communication provides access to high bandwidth communication channels, leading to the potential for higher data rates and thus serves as a viable approach to enable such high-rate transmissions [2].

Urban millimeter wave cellular networks have been widely studied and analyzed over the past few years. Because both streets and buildings are densely distributed, as shown in Fig. 1, the effects of shadowing, building blockages, and reflections should be taken into account while modeling the propagation channel. Most prior analytical work is primarily based on Euclidean distance-based pathloss models [4][5]. In [6], buildings were modeled with random sizes and directions, and the blockage probability was computed as a function of the Euclidean length of the link. A correlated shadowing model for urban wireless networks was proposed and analyzed in [7] under an urban Manhattan network. Using a shadowing based NLOS pathloss model, blockages were modeled by the number of streets the propagation link crosses. Recently, a spatially consistent pathloss model for urban mmWave channels in microcells was proposed in [8]. It was shown through the analysis of ray tracing data that (i) in urban microcells, the Euclidean distance need not be a good measure for the pathloss, and (ii) the pathloss exponent in urban street canyons changes from streets to streets and is a function of the street orientation. The pathloss exponents for each street were obtained by straight-line fit of ray tracing data. Prior analysis based on Euclidean distance blocking functions as in [6] therefore does not necessarily apply for urban microcells.

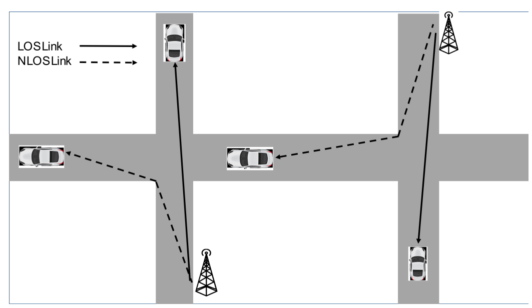

Fig. 1. An example of urban vehicular network. The gray area represents streets while other areas are buildings in the urban network. The solid black line is LOS downlink, the dashed black line represents the NLOS downlink. The severe blockage effects in mmWave communication make penetration effects negligible.

In this paper, we analyze the ergodic network coverage in urban microcellular networks. The analysis is based on (i) modeling the urban street structure as a Manhatten Poisson line process, and (ii) approximating the pathloss model of [8]. This allows us to retain the key physical features of mmWave propagation in urban microcells while retaining mathematical tractability. We analyze the performance of this mmWave microcellular network where the base stations are randomly located along those lines. Based on previous measurement results in [8], we propose a pathloss model that is a function of the segment lengths along the propagation link (Manhattan distance) instead of Euclidean distance, as shown in Fig. 1. Using this model, we derive the distribution of the associated link pathloss and the SINR. We compare our approach to other analyses of mmWave cellular networks using blocking functions and Euclidean distance performance measurements and find significant differences that justify using our more realistic model. A result of our model is that coverage is primarily determined by the base stations that are located on the same street and cross street; interferers from NLOS parallel streets contribute negligible interference.

II. System Model

A. Pathloss Model

In this paper, the pathloss model considered is for the an urban microcell download scenario. The dynamics of the vehicle movement, and the shadowing variations within each street, are not explicitly included. Based on ray tracing and measurement results, [8] developed a pathloss model such that the total pathloss consists of contributions from different segments of the propagation path, with an extra corner loss when the signal couples into a street with a different direction. Ray tracing showed that the pathloss exponent varies along individual streets with different locations and direction. In this paper, we use a slightly modified form of the model, which among other simplifications assumes that all of the NLOS streets share the same pathloss exponent α_N, while the pathloss exponent for LOS street is α_L. We denote the distance of the LOS segment as d_L and the set of length of NLOS segments as $\mathcal{D}_N$. Δ is the corner loss and M is the number of corners the propagation path goes through. The pathloss, which is a function of the Manhattan distance of the propagation link, can be characterized as follows

$$PL_{\text{dB}}(d_L, \mathcal{D}_N) = 10\alpha_L \log_{10} d_L + 10 \sum_{\tilde{d}\in\mathcal{D}_N} \alpha_N \log_{10} \tilde{d} + M\Delta, \tag{1}$$

This pathloss model bears some resemblance to that in [9]. We illustrate the segment distances involved in the pathloss model in Fig. 2.

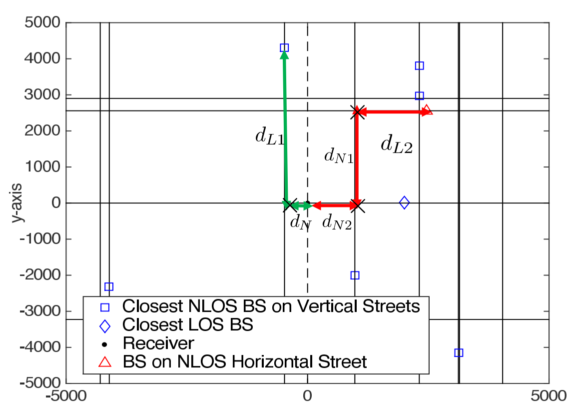

Fig. 2. An illustration of our proposed pathloss model. The red line denotes the propagation link of a NLOS BS on horizontal streets and the green line is the propagation path of a NLOS BS on vertical streets. We use $\times$ to represent one corner loss. For propagation link from a BS on NLOS vertical street, d_{L1} is the length of LOS segment, $\mathcal{D}_N = \{d_N\}$. For BS on NLOS horizontal street, d_{L2} is the LOS segment and $\mathcal{D}_N = \{d_{N1}, d_{N2}\}$.

B. Network Model

1) Manhattan Poisson Line Process Model: We model the system as Manhattan network [10–12], which is similar to [7]. Consider the Manhattan network on the two dimensional Euclidean plane consisting of horizontal and vertical streets, as shown in Fig. 2. Both the vertical and horizontal streets are modeled as homogeneous Poisson point processes (PPP), respectively denoted as Ψ_{sv} and Ψ_{sh}, and with intensity λ_{sv} and λ_{sh}. Without loss of generality, we assume the receiver o is located at the origin, on a horizontal street. We classify the base stations (BSs) into three categories, respectively the LOS BS denoted as ϕ_L, the NLOS BS on vertical streets ϕ_V and NLOS BS on horizontal streets ϕ_H. BSs on vertical and horizontal streets are also modeled as independent homogeneous PPPs, and with intensity given by λ_{tv} and λ_{th}, respectively.

2) Signal-to-Interference-plus-Noise Ratio (SINR) : We assume all links experience independent Rayleigh fading with mean 1, $h \sim \exp(1)$, since the main focus of the paper is to propose an alternate distance-dependent pathloss model and the assumption of same small scale fading is made to simplify SINR coverage analysis. The corner gain is $c = 10^{-\Delta/10}$, the transmit power is normalized, i.e., $P_B = 1$ and the noise is denoted by N_0. In this paper, we focus on the standard power law attenuation function $\ell(x)$, specifically we apply diffrent pathloss exponents to LOS and NLOS links, respectively, which is $\ell_L(x) = x^{-\alpha_L}$ for LOS links and $\ell_N(x) = x^{-\alpha_N}$ for NLOS links. Denote the associated link pathloss as u.

We denote the set of LOS link distances x_L from receiver o to the LOS BSs as Φ_L. We use Φ_V to represent the set of lengths of the NLOS and LOS segments $(x_V, y_V) = (d_N, d_{L1})$ constituting the propagation path from the BSs on vertical streets (see Fig. 2). Similarly, Φ_H denotes the set of distance $(x_H, y_H, z_H) = (d_{N1}, d_{N2}, d_{L2})$ which are the Manhattan distances of NLOS and LOS segments of the propagation link from the NLOS BSs on the horizontal streets. To formulate the SINR, we first give the following two assumptions.

Assumption 1. *The receiver is associated with the BS with the smallest pathloss.*

Assumption 2. *The probability that the receiver is associated with a NLOS BS on a horizontal street is negligible.*

Let u denote the pathloss from the receiver to the serving BS. Conditioned on u, we have

$$\text{SINR} = \frac{hu}{N_0 + I_{\phi_L}(o) + I_{\phi_V}(o) + I_{\phi_H}(o)}, \tag{2}$$

where

$$I_{\phi_L}(o) = \sum_{x_L\in\Phi'_L} h\ell_L(x_L), \tag{3}$$

$$I_{\phi_V}(o) = \sum_{(x_V,y_V)\in\Phi'_V} h\ell_N(x_V)\ell_L(y_V)c, \tag{4}$$

and

$$I_{\phi_H}(o) = \sum_{(x_H,y_H,z_H)\in\Phi'_H} h\ell_N(x_H)\ell_N(y_H)\ell_L(z_H)c^2. \quad (5)$$

Given Assumption 1, we further have the following constraints on the set Φ'_L, Φ'_V and Φ'_H in (3-5)

$$\Phi'_L = \left\{x \in \Phi_L \middle| \ell_L(x) < u\right\}, \quad (6)$$

$$\Phi'_V = \{(x,y) \in \Phi_V \Big| \ell_N(x)\ell_L(y)c < u\}, \quad (7)$$

$$\Phi'_H = \left\{(x,y,z) \in \Phi_H \middle| \ell_N(x)\ell_N(y)\ell_L(z)c^2 < u\right\}. \quad (8)$$

III. Performance Analysis

A. Distribution of the Associated link Pathloss

Given Assumption 1 and 2, we derive the cumulative density function (CDF) of the associated link pathloss u via the following lemma.

Lemma 1. *The CDF $F(u)$ of associated link pathloss is*

$$F(u) = \exp\left(-2\lambda_{th}u^{-\frac{1}{\alpha_L}}\right)\exp\left(-Cu^{-\frac{1}{\alpha_N}}\right), \quad (9)$$

where

$$C = 2\lambda_{sv}\Gamma\left(1-\frac{\alpha_L}{\alpha_N}\right)\left(2\lambda_{tv}c^{\frac{1}{\alpha_L}}\right)^{\frac{\alpha_L}{\alpha_N}}. \quad (10)$$

Proof. Proof is omitted here to save space. □

B. Blockage Probability

We compare our model with two different Euclidean distance based pathloss models. The first model computes the pathloss directly by Euclidean distance d, where $PL_{\mathrm{dB}}(d) = 10\tilde{\alpha}\log_{10} d + \Delta_1$, and $\tilde{\alpha}$ is the pathloss exponent, Δ_1 is the offset for straight-line fit of this Euclidean pathloss model. In [6], a key parameter for characterizing coverage in mmWave wireless networks is the distance dependent blockage probability. In the urban microcell downlink scenario, we define it as $p_{\mathrm{B}}(d)$ and apply different pathloss exponents $\tilde{\alpha}_L$ and $\tilde{\alpha}_N$ for unblocked (LOS) and blocked (NLOS) links. The pathloss is calculated by $PL_{\mathrm{dB}}(d) = (1-\mathbb{I}(p_{\mathrm{B}}(d)))\left(10\tilde{\alpha}_L\log_{10} d + \Delta_2^L\right) + \mathbb{I}(p_{\mathrm{B}}(d))\left(10\tilde{\alpha}_N\log_{10} d + \Delta_2^N\right)$, where $\mathbb{I}(x)$ is the Bernoulli function with parameter x, Δ_2^L and Δ_2^N are respectively the offsets for the LOS and NLOS case.

We provide the following lemma giving the blockage probability in [6] with respect to the Euclidean distance under MPLP model.

Lemma 2. *The blockage probability of a propagation link from a BS at Euclidean distance d is*

$$p_B(d) = 1 - \frac{1-\exp(-2d(\lambda_{sh}+\lambda_{sv}))}{2d(\lambda_{sh}+\lambda_{sv})}. \quad (11)$$

Proof. Proof is omitted for space reasons. □

C. Coverage Analysis and Simplification

Conditioning on associated link pathloss u, we denote the coverage probability as $p_c(u,T)$

$$p_c(u,T) = \mathbb{P}(\mathrm{SINR} > T|u). \quad (12)$$

We can then formulate the coverage probability as

$$\begin{aligned} p_c(u,T) &= \mathbb{P}(\mathrm{SINR} > T|u) \\ &= \mathbb{E}\left[\exp\left(-Tu^{-1}\left(N_0 + I_{\phi_L}(o) + I_{\phi_V}(o) + I_{\phi_H}(o)\right)\right)\right] \\ &= \exp(-TN_0u^{-1})\mathcal{L}_{I_{\phi_L}}(Tu^{-1})\mathcal{L}_{I_{\phi_V}+I_{\phi_H}}(Tu^{-1}) \\ &\approx \exp(-TN_0u^{-1})\mathcal{L}_{I_{\phi_L}}(Tu^{-1})\mathcal{L}_{I_{\phi_V}}(Tu^{-1}), \quad (13) \end{aligned}$$

where $\mathcal{L}_{(\cdot)}$ is the Laplace transform of $(\cdot)$.

Note that since the interference from the NLOS vertical and horizontal streets are correlated due to the shared paths, the Laplace transform of the total interference of these two parts of cannot be split up just by direct multiplication. If the interference from the NLOS horizontal interferers can be completely neglected (which will be proved analytically in Section III-D) then, the coverage probability can be given in the following theorem.

Theorem 1. *Conditioning on the associated link pathloss as u, the coverage probability can be given by*

$$p_c(u,T) = \exp(-C_1u^{-1})\exp(-C_2u^{-\frac{1}{\alpha_L}})\exp\left(-C_3u^{-\frac{1}{\alpha_N}}\right), \quad (14)$$

where the constants are respectively

$$\begin{aligned} &C_1 = TN_0, \quad C_2 = 2\lambda_{th}\varrho, \\ &C_3 = C\varrho^{\frac{\alpha_L}{\alpha_N}}, \quad \varrho = \int_1^\infty \frac{1}{1+T^{-1}\mu^{\alpha_L}}dx, \quad (15) \end{aligned}$$

and C in C_3 is given in (10).

Proof. The proof follows the idea of conditioning on the associated link pathloss u and combining constraints in (3-5) first, details of which are omitted here to save space. □

The coverage probability $P_c(T)$ can be derived by averaging over the associated link pathloss as

$$P_c(T) = \int_0^\infty f_U(u)p_c(u,T)du, \quad (16)$$

where $f_U(u)$ is the probability density function of U (obtained from (9)) and $p_c(u,T)$ is given in (14).

D. The Effect of LOS and NLOS Interferers

In the case when the receiver is associated with a LOS BS, we provide a rigorous proof for our previous assumption that interference from NLOS BS on horizontal streets is negligible. In addition, by utilizing Jensen's inequality, we compare the different effects of interference from LOS BSs ϕ_L and BS on vertical streets ϕ_V.

Assuming that the receiver is associated to a LOS BS at distance r, i.e., $u = r^{-\alpha_L}$, we derive the Laplace transform of NLOS horizontal interferers ϕ_H in the following proposition.

Proposition 1. *The Laplace transform of the interference from the NLOS horizontal streets I_{ϕ_H} is given by*

$$\begin{aligned}\mathcal{L}_{I_{\phi_H}}(Tu^{-1}) &= \mathcal{L}_{I_{\phi_H}}(Tr^{\alpha_L}) \\ &\approx 2\lambda_{sv}\sqrt{\frac{2\kappa}{\lambda_{sv}}}K_1\left(2\sqrt{2\kappa\lambda_{sv}}\right), \end{aligned} \tag{17}$$

where

$$\kappa = 2\lambda_{sh}\Gamma\left(1-\frac{\alpha_L}{\alpha_N}\right)\left[2\lambda_{th}r\varrho c^{\frac{2}{\alpha_L}}\right]^{\frac{\alpha_L}{\alpha_N}}, \tag{18}$$

and $K_1(\cdot)$ is the 1-st order modified Bessel's function of the second kind [13].

Proof. Details of the proof are omitted for space reasons. □

Based on the property of the modified Bessel function, when the argument μ of $K_1(\mu)$ becomes small, we can approximate it as [14]

$$K_1(\mu) \sim \mu^{-1}. \tag{19}$$

In our case, it can be seen that the argument inside the modified Bessel function in (17) is very small since it scales with $\lambda_{\text{sh}}\lambda_{\text{th}}^{\frac{\alpha_L}{\alpha_N}}$. The corner loss term further reduces the value to a large extent, which makes (19) a feasible approximation. Consequently, we show that the integral in (17) reduces to

$$\mathcal{L}_{I_{\phi_H}}(Tr^{\alpha_L}) \approx 2\lambda_{\text{sv}}\sqrt{\frac{2\kappa}{\lambda_{\text{sv}}}}\left(2\sqrt{2\kappa\lambda_{\text{sv}}}\right)^{-1} = 1. \tag{20}$$

With the Laplace transform $\mathcal{L}_{I_{\phi_H}}(Tr^{\alpha_L}) \approx 1$, we can conclude that the correlation between the interference of I_{Φ_H} and I_{Φ_V} and the interference I_{Φ_H} itself is negligible. This justifies the previous assumption of ignoring the NLOS horizontal interferers in Section III-C and the approximation we made in (13).

Next, we compare the interference from LOS BSs and NLOS BSs from vertical streets. The comparison is based on the individual Laplace transform of the interference.

The Laplace transform of the interference due to the LOS BSs is $\mathcal{L}_{I_{\phi_L}}(Tu^{-1}) = \mathcal{L}_{I_{\phi_L}}(Tr^{\alpha_L}) = \mathbb{E}_r\left[\exp(-\omega(2\lambda_{\text{th}})r)\right]$ and the Laplace transform of NLOS interference due to BSs on the vertical streets is $\mathcal{L}_{I_{\phi_V}}(Tu^{-1}) = \mathcal{L}_{I_{\phi_V}}(Tr^{\alpha_L}) = \mathbb{E}_r\left[\exp(-\vartheta\lambda_{\text{sv}}(2\lambda_{\text{tv}}r)^{\frac{\alpha_L}{\alpha_N}})\right]$ (given in (15), Theorem 1), where ω and ϑ are respectively two constants independent of the intensity of streets and BSs. Define two convex functions $\varphi_1(r) = \exp(-r)$ and $\varphi_2(r) = \exp(-r^{\frac{\alpha_L}{\alpha_N}})$. Using Jensen's inequality,

$$\mathcal{L}_{I_{\phi_L}}(Tr^{\alpha_L}) \geq \exp\left(-\omega\mathbb{E}_r\left[2\lambda_{\text{th}}r\right]\right). \tag{21}$$

Since r is the distance from the receiver to the closest LOS BS, according to the void probability of PPP, $r \sim \exp(2\lambda_{\text{th}})$ [15], the expectation in the exponent of (21) evaluates to 1.

Similar application of Jensen's inequality on Laplace transform of I_{ϕ_V} results in

$$\mathcal{L}_{I_{\phi_V}}(Tr^{\alpha_L}) \geq \exp\left(-\vartheta\lambda_{\text{sv}}\mathbb{E}_r\left[(2\lambda_{\text{tv}}r)^{\frac{\alpha_L}{\alpha_N}}\right]\right). \tag{22}$$

The expectation term in (22) can be evaluated as follows

$$\begin{aligned}\mathbb{E}_r\left[(2\lambda_{\text{tv}}r)^{\frac{\alpha_L}{\alpha_N}}\right] &= \int_0^\infty (2\lambda_{\text{tv}})^{\frac{\alpha_L}{\alpha_N}} r^{\frac{\alpha_L}{\alpha_N}} \exp(-2\lambda_{\text{th}}r)dr \\ &= \left(\frac{\lambda_{\text{tv}}}{\lambda_{\text{th}}}\right)^{\frac{\alpha_L}{\alpha_N}}\Gamma\left(1+\frac{\alpha_L}{\alpha_N}\right). \end{aligned} \tag{23}$$

Therefore

$$\mathcal{L}_{I_{\phi_V}}(Tr^{\alpha_L}) \geq \exp\left(-\vartheta\lambda_{\text{sv}}\left(\frac{\lambda_{\text{tv}}}{\lambda_{\text{th}}}\right)^{\frac{\alpha_L}{\alpha_N}}\Gamma\left(1+\frac{\alpha_L}{\alpha_N}\right)\right). \tag{24}$$

From (24), it can be seen that the lower bound of Laplace transform of I_{ϕ_V} scales exponentially with the street intensity λ_{sv}, which yields an intuitive result that when street intensity increases, the effects by interferers on vertical streets become more prominent.

IV. Numerical Results

We compare our model and the two models demonstrated in Section III-B in terms of the coverage probability and ergodic capacity in Fig. 3 and Fig. 4. It is shown that the three models show significant difference in coverage probability and ergodic capacity. This motivates us to do further theoretical analysis in our proposed model.

In Fig. 5, we validate our result of associated link pathloss distribution given in Lemma 1. It can be seen that the distribution of the associated link pathloss considering and neglecting the probability of association with a NLOS horizontal BS is the same. This verifies Assumption 2 where we only consider the association case with LOS BSs and NLOS BSs on vertical streets.

Fig. 6 compares the numerical result of coverage probability with and without NLOS interferers on horizontal streets against the theoretical result given in Theorem 1. It is seen that the analytical result matches well with the numerical results. Also, the coverage probability with horizontal NLOS interferers coincides with that without the horizontal NLOS interferers. This verifies the analysis in Section III-D concerning the negligible effects of NLOS interference on parallel streets.

V. Conclusion

This paper proposed a new pathloss model under the urban mmWave microcells which computes the pathloss based on Manhattan distance instead of the Euclidean distance. We provided a tractable framework to analyze the new model in terms of the coverage probability. We also compared effects of interference from LOS streets and NLOS streets.

VI. Acknowledgment

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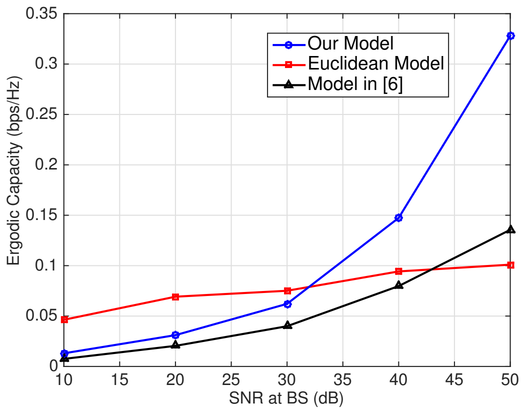

Fig. 3. Comparison of three models in terms of ergodic capacity against the SNR at BS.

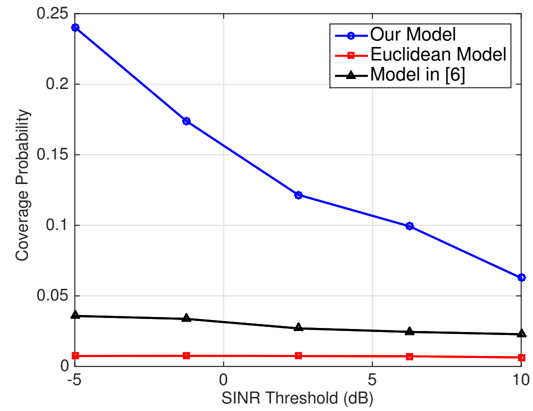

Fig. 4. Comparison of three models in terms of coverage probability against the SINR Threshold T.

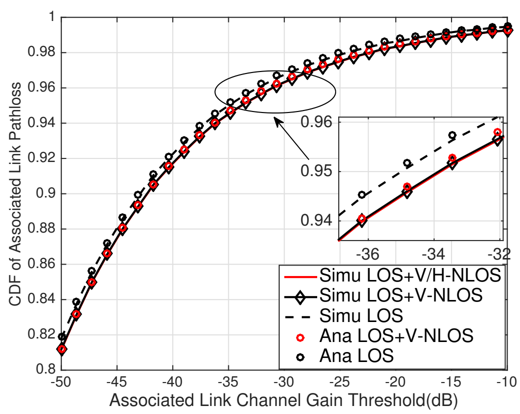

Fig. 5. Comparison of analytical and numerical associated link pathloss distribution. The red solid line, black solid line and black dashed line respectively represent the CDF of the associated link pathloss considering all the interference, the interference of the LOS interferers and NLOS interferers on vertical streets, the LOS interference only.

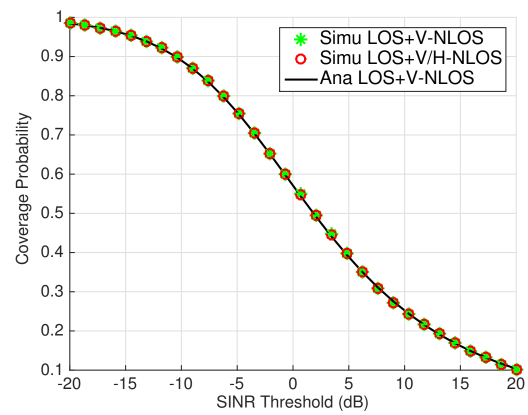

Fig. 6. Comparison of the numerical and analytical coverage probability. The red circle and green star respectively represent the numerical coverage probability w/ and w/o the NLOS horizontal interference.

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