Fundamental System-Degrading Effects in THz Communications Using Multiple OAM beams With Turbulence

Zhe Zhao[1]*, Runzhou Zhang[1], Hao Song[1], Kai Pang[1], Ahmed Almaiman[1], Huibin Zhou[1],Haoqian Song[1], Cong Liu[1], Nanzhe Hu[1], Xinzhou Su[1], Amir Minoofar[1], Shlomo Zach[2], Nadav Cohen[2], Moshe Tur[2], Andreas Molisch[1], and Alan E. Willner[1]

1. Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089, USA.
2. School of Electrical Engineering, Tel Aviv University, Ramat Aviv 69978, ISRAEL

Corresponding email*: zhezhao@usc.edu

Abstract— **We explore and find the fundamental system-degrading effects when using multiple orbital-angular-momentum (OAM) beams in a THz communications link under atmospheric turbulence in simulation. Unlike optical links with relatively small divergence effects, the crosstalk performance of THz OAM links is dependent on divergence-related parameters, including OAM mode order, frequency, and beam waist. Simulation results show: (i) for the cases with the same ratio of beam diameter to the Fried parameter (*D/r_0*), the signal power increases and the crosstalk (XT) decreases when increasing the divergence-related parameters; and (ii) for the cases with the same atmospheric structure constant Cn^2, the signal power decreases and the XT increases when increasing the divergence-related parameters. Moreover, for building a link where OAM +4 is transmitted with the parameters: (i) beam waist of 0.1 m and link distance of 200 m, and (ii) beam waist of 1 m and link distance of 1 km, the XT from neighbouring mode remains less than -15 dB when carrier wave frequency is < 1 THz and 0.1 THz, respectively. In addition, simulation results also show that: (i) limited aperture size of the system has high influence on the XT performance under both weak and strong turbulence; and (ii) displacement of the system has high influence on the XT performance under no and weak turbulence.**

Keywords—Atmospheric turbulence; orbital angular momentum; millimeter wave; spatial-division multiplexing

I. Introduction

There is a rising interest in the potential of using mode-division-multiplexing (MDM) to increase total transmission capacity and spectral efficiency in a point-to-point communications link [1-6]. Mode multiplexing is a subset of spatial multiplexing, in which multiple data-carrying orthogonal beams are efficiently multiplexed at the transmitter aperture, spatially co-propagating through a free-space medium, and demultiplexed at the receiver aperture [7]. All this would occur with little inherent crosstalk due to the orthogonality among the different beams [8].

One example of a modal basis set in which different beams are structured to be orthogonal is orbital-angular-momentum (OAM), which is a subset of Laguerre Gaussian ($LG_{l,p}$) beams [7]. An $LG_{l,p}$ beam carrying OAM has a phasefront that "twists" in a helical fashion as it propagates, such that the mode order l represents the number of 2π phase shifts occurring in the azimuthal direction, and $1+p$ represents the number of radial intensity nodes (rings/spots) [7]. Moreover, the intensity profile of an OAM beam is a vortex, such that there is an intensity ring with a null in the center.

The use of multiple OAM beams in a communications link has been reported for optical and millimeter-wave systems [1-6]. In general, there are multiple possible fundamental issues that could: (a) degrade the orthogonality among the different beams, thereby causing coupling of power to other modes and resulting in deleterious system crosstalk, and (b) reduce the signal power that can be recovered from the uniquely shaped vortex beam [9-16]. For optical systems in which the carrier wave is ~200 THz, a key limitation is the interaction of the beam with atmospheric turbulence [10-12]. On the other hand, millimeter-wave systems with carrier waves below 100 GHz tend to be significantly limited by beam divergence and signal power loss [13-15].

There is growing importance within the communications community of using the THz regime for high-capacity links [15-20]. Therefore, a laudable goal would be to explore the use of OAM-multiplexed links in the THz regime. Specifically, a key question would be to determine the fundamental system-degrading effects for a THz OAM link under atmospheric turbulence that might be different than what would be experienced by optical or mm-wave systems.

In this paper, we explore and find the fundamental system-degrading effects when using multiple OAM beams in a THz communications link under atmospheric turbulence in simulation. We analyze the crosstalk (XT) and signal-to-interference ratio (SIR) performance of various systems when varying frequency, OAM order, and beam waist, from 0.1-10 THz, +1 to +10, and 0.1-3 m, respectively. For the cases with the same ratio of beam diameter to the Fried parameter (D/r_0), the signal power increases and the XT decreases when increasing the divergence-related parameters. However, for the cases with the same atmospheric structure constant Cn^2, the signal power decreases and the XT increases as the mode order, frequency, and beam waist increases, which induces larger values of D/r_0. Moreover, for building a link where OAM +4 is transmitted with the parameters: (i) beam waist of 0.1 m and link distance of 200 m, and (ii) beam waist of 1 m and link distance of 1 km, the XT from neighbouring mode remains less than -15 dB when carrier wave frequency is < 1 THz and 0.1 THz, respectively. Simulation results show: (i) limited aperture size of the system has high influence on the XT performance under weak and strong turbulence; and (ii) displacement of the system has high influence on the XT performance under no and weak turbulence.

II. System Configuration

A. Concept and Simulation Model

Figure 1 shows a schematic of a THz communication link using OAM multiplexing. The multiplexed OAM beams diverge and lose power when propagating through free space. The divergence effect is dependent on the beams' frequency, beam size and OAM mode order [9]. The power loss is dependent on many factors, such as atmospheric absorption [17,20]. Moreover, after propagating through atmospheric turbulence, the phasefront of transmitted OAM beam might also be distorted, thus leading to power leakage to the other OAM beams. Such power leakage results in crosstalk among different channels when the transmitted OAM beams are demultiplexed and converted back to Gaussian-like beams at the receiver side. Besides, a limited receiver aperture size induces additional power loss and misalignment between the transmitter and receiver could further increase the inter-channel crosstalk [9].

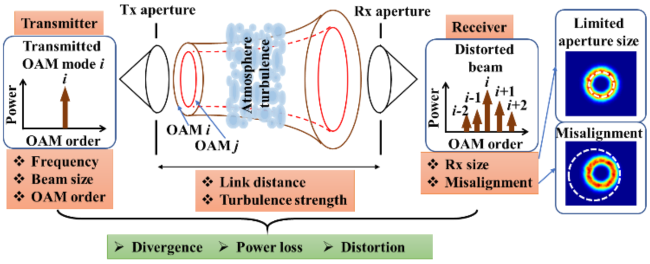

Figure 1 Concept of THz communication links using OAM multiplexing. An OAM beam propagates through turbulence, which leads to the distortion of its phasefront. At the receiver side, such a beam evolves to a beam with both distorted amplitude and phase profiles. A receiver with limited aperture size and misalignment could further degrade the system performance. Tx: transmitter; Rx: receiver.

To study the influence of atmospheric turbulence on THz OAM beams, we first build a model to emulate the turbulence. According to Kolmogorov turbulence theory, the refractive index fluctuation caused by atmospheric turbulence can be described by a structure function D_n, which is determined by [10]

$$D_n(\Delta r) = 6.88\left(\frac{\Delta r}{r_0}\right)^{5/3} \tag{1}$$

where Δr is the distance between two points in space, and r_0 is the Fried parameter, or the coherence length. Assuming the value of C_n^2 keeps unchanged over propagation path, r_0 can be calculated from the atmospheric structure constant C_n^2 by the formula [10]

$$r_0 = (0.423k^2 C_n^2 L)^{-3/5} \tag{2}$$

where $k = 2\pi/f$ is the wave number and L is the propagation length. The values of C_n^2 are around 1×10^{-13} and 1×10^{-11} $m^{-2/3}$ which represent weak and strong turbulence conditions for THz beams, respectively [17].

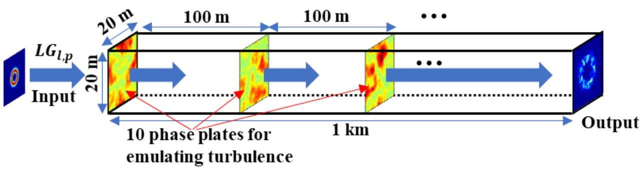

Figure 2 Schematics of the simulation model for studying the atmospheric turbulence effects on THz OAM beams. 10 random-generated phase plates with equal spacing of 100 m are used to emulate the turbulence for a link of a distance up to 1 km.

Based on Eqs. (1) and (2), the turbulence can be simulated by using phase plates, which can be described as N-by-N matrices of random phase numbers with statistics that match Kolmogorov turbulence theory [10,17]. Our simulation model of a single OAM or an OAM-multiplexed THz communication link under atmospheric turbulence is depicted in Fig. 2. The outer scale and inner scale of the turbulence are set as 100 m and 0.01 m, respectively [10,17]. The outer scale of the turbulence represents the upper limit of distance between neighbouring phase plates for emulating turbulence. In order to save the simulation time cost, we use 10 phase plates, with each emulating 100-m turbulence distortion, to study the turbulence effect on THz OAM beams in a 1-km free-space link. We assume that the atmospheric structure constant C_n^2 remains the same at different distances. The input OAM beams are a subset of $LG_{l,p}$ beams with zero p values. The input beams are projected onto the 1-st phase plate and then propagate in free space for 100 m. Such processes of the projection onto the n-th phase plates and 100-m free-space propagation are repeated 10 times to simulate 1-km free-space propagation through turbulence. The size of the simulation screen is 20 m, and N is 2000. The parameter definitions in the model are listed in Table I.

TABLE I. PARAMETERS

D	Transmitted beam diameter
f	Transmitted beam frequency
w_0	Transmitted beam waist
l	Transmitted beam OAM order
Cn^2	Atmospheric structure constant
r_0	Fried parameter

B. *Assumptions and definitions*

For the convenience of analysis, the following assumptions are made:

(i) We note that the atmospheric attenuation could be larger than 100 dB/km for THz waves with f > 1 THz [16]. Even though the loss is one key issue for THz link, we assume that there is no absorption loss during the free-space propagation. This assumption will help us to understand the turbulence effects on THz OAM beams in the regime of OAM spectrum (*i.e.*, the received power distribution on different OAM orders) outside the problem of absorption loss. If we consider the atmospheric attenuation, the power of THz waves on different OAM orders after free-space propagation would decrease while there would be almost no influence on the SIR performance for each OAM channels;

(ii) Each transmitted OAM beam has the same input power of 0 dBm;

(iii) The transmitter aperture is always larger than the input beam size, and we assume that it does not affect the intensity and phase profiles of the input beam. If the transmitter aperture is less than the input beam size, the mode purity of the transmitted OAM beams would decrease;

(iv) We only consider the case of a single-polarized system. Since there is no obvious power coupling between orthogonal polarization, most of the results could be extended to the case of the double-polarized system.

(v) Except for Section V, we assume the receiver aperture is always larger than the beam size of the received beam and its center is overlapped with the input beam center. Thus, the receiver aperture has no effect on the intensity and phase profiles of the received beam.

(vi) Each data points in all figures with non-zero C_n^2 represents the average of 50 simulation iterations.

For the convenience of analysis, the following definitions about OAM beams are used:

(i) The input OAM beam with OAM order of l is an $LG_{l,p}$ beam with a zero p value, where its electrical field is given by the following equation [7]

$$LG_{l,p}(r,\theta,z,\omega)=\frac{C_{l,p}^{LG}}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|}\exp\left(-\frac{r^2}{w^2(z)}\right)L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\times \exp\left(-i\left(k\frac{r^2}{2R(z,\omega)}+l\theta+kz-\psi(z,\omega)\right)\right) \quad (3)$$

where $L_p^{|\ell|}$ are the generalized Laguerre polynomials, $C_{l,p}^{LG}$ are the required normalization constants, $w(z)$ is the beam size at a distance of z, $R(z,\omega)=z(1+(z_R(\omega)/z)^2)$ is the wavefront radius, where $z_R(\omega)$ is the Rayleigh range, k is the wave number, ω is the angular frequency, (r,θ,z) represents the cylindrical coordinate, and $\psi(z)$ is the Gouy phase and equals $(|l|+2p+1)\arctan(z/z_R(\omega))$;

(ii) The beam diameter of the input OAM beam with OAM order of l is calculated as $D=w_0\sqrt{|l|+1}$, where w_0 is the waist at the distance of 0 [9];

(iii) We calculate the OAM spectrum of the output beam as the normalized power weight coefficient of each OAM order l, namely, the ratio of the power on each OAM mode to the total power of the received beam, which is given as [7]

$$|C_\ell|^2=|\iint E_1(x,y)E_2^*(x,y)dxdy|^2 \quad (4)$$

where $E_1(x,y)$ is the electric field of the output beam at the receiver aperture, and $E_2(x,y)$ is the electric field of a pure $LG_{\ell,0}$ beam after propagating from the transmitter aperture to the receiver aperture, where its beam waist w_0 is set as the same as that of the input OAM beam.

III. System Performance Dependence on Divergence Related Parameters

In this section, we analyze signal power and crosstalk performance of THz OAM links under atmospheric turbulence when varying the divergence related parameters of the input OAM beam. Specifically, we analyze the OAM spectrum at the receiver aperture for OAM beams with different frequencies, beam waists, and OAM orders.

A. *Turbulence-induced distortion v.s. OAM mode order*

We first explore the dependence of turbulence-induced distortion of OAM spectrum on the transmitted OAM order when THz OAM beams propagate through atmospheric turbulence. Previous work has shown that for optical OAM-multiplexed links, where the divergence effect is relatively small, (i) the turbulence-induced distortion effect does not depend on the OAM order too much, and (ii) the larger the ratio of the transmitted beam diameter to the Fried parameter D/r_0, the higher the turbulence-induce distortion effect (*i.e.*, the signal power loss on the transmitted OAM mode and the power leakage to the other OAM modes become larger) [12]. However, for a 1-km THz link where the divergence effect is relatively large, as shown in Fig. 3, the distortion effect on the received OAM spectrum becomes larger when the transmitted OAM order increases. Take OAM +1 and OAM +10 as examples, when the transmitted OAM order increases from +1 to +10: (i) the normalized signal power, namely, the received power on the transmitted mode decreases from -2.9 dB to -8.0 dB, and (ii) the crosstalk from right (higher) nearest mode (XT1) and the crosstalk from right 2nd-nearest mode (XT2) increases from -7.7 dB to -1.9 dB and from -15.2 dB to -4.2 dB, respectively. This could be because that even though the input beams have the same beam diameter of D = 0.17 m at the transmitter side, the OAM beam with a higher OAM order diverge faster and thus has a larger beam diameter after propagating in free space for 1 km. Therefore, the value of D/r_0 evolves higher when the higher-order OAM beam propagates in free space due to higher divergence effect. Thus, it leads to a larger distortion effect on the OAM spectrum.

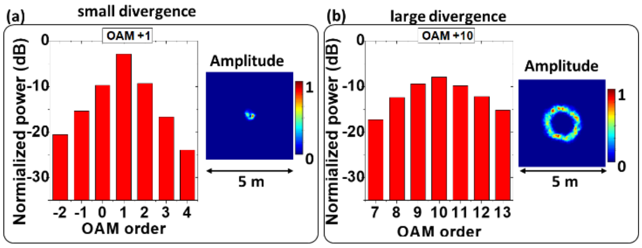

Figure 3 Normalized power distribution on different OAM modes when transmitting (a) OAM +1 with small divergence effect, and (b) OAM +10 with large divergence effect through a 1-km link. Parameters are set as $C_n^2=1\times10^{-12}$ $m^{-2/3}$, D = 0.17 m and f = 5 THz, where D/r_0 = 0.107 at the transmitter side.

To further evaluate the influence of the mode-order-induced divergence effect on the turbulence distortion effect, we compare the performance of two links with large and small divergence effects. Figures 4(a) and 4(b) represent links of distances of 1 km and 100 m, respectively. We can see from Fig. 4(a) that the signal power decreases with the transmitted OAM order while XT1 and XT2 increase with the transmitted OAM order for a 1-km link. However, the signal power, XT1 and XT2 all fluctuate around their average values within a range of less than 5 dB for a 100-m link. The reason might be that: for a 100-m link, the OAM beam slowly diverges such that different transmitted OAM beams still have a similar D/r_0 at the receiver side.

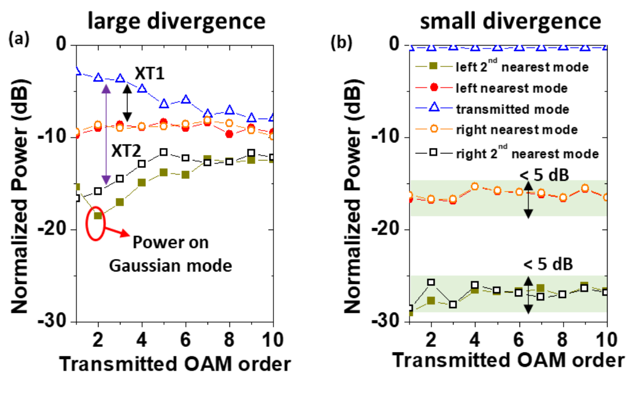

Figure 4 Effects of varying the transmitted OAM order on the normalized power distribution on different OAM modes for a (a) 1-km link with large divergence effect, and (b) 100-m link with small divergence effect. The OAM order the transmitted beam is varied from +1 to +10. Parameters are set as C_n^2 = 1 × 10^{-12} m$^{-2/3}$, D = 0.17 m and f = 5 THz at the transmitter side. The legends in (a) and (b) are the same. XT1: the crosstalk from right (higher) nearest mode; XT2: the crosstalk from right 2nd-nearest mode.

B. *Turbulence distortion effect v.s. frequency*

We then investigate the effects of varying frequency on the turbulence-induced distortion of OAM spectrum. Simulation results for transmitting OAM +4 show that : (i) for a THz link where the input beam has the same value of D/r_0 (similar to the cases where the input beams with different frequencies propagate through the emulated phase plates with the same phase patterns), the higher frequency, the smaller the turbulence-induced distortion effect (Fig. 5(a)); (ii) however, for a THz link where the atmosphere has the same value of C_n^2 (similar to the cases where the input beams with different frequencies propagate through the same physical atmospheric layer), the higher frequency, the larger the turbulence-induced distortion effect (Fig. 5(b)). When the frequency increase from 0.03 THz to 10 THz for a 200-m link: (i) as shown in Fig. 5(a), the signal power increases by ~ 8 dB and XT1 decreases by ~ 5 dB, where the OAM beam has the same value of $D/r_0 = 0.224$ at the transmitter side; (ii) in comparison, as shown in Fig. 5(b), the signal power decreases by ~ 11 dB and XT1 increases by ~ 38 dB, where the OAM beam propagates through turbulence with the same value of $Cn^2 = 1 \times 10^{-11}$ m$^{-2/3}$. This could be because that for the case with the same value of D/r_0, the turbulence-induced distortion effect is mainly dependent on the beam divergence effect. Therefore, the OAM beam of a higher frequency diverges slower during free-space propagation, leading to a smaller distortion effect. However, for the case with the same C_n^2, the OAM beam propagates through free space with the same turbulence condition. Eq. (2) shows that the Fried parameter r_0 decreases with frequency. Although higher frequency leads to a small divergence effect, the value of D/r_0 still increases with frequency, thus leading to a larger distortion effect. We note that: (i) for a 200-m link with a transmitted beam waist of 0.1 m, XT1 is < -15 dB when frequency is < 1 THz (Fig. 5(b)); and (ii) for a 1-km link with a transmitted beam waist of 1 m, XT1 is < -15 dB when frequency is < 0.1 THz (Fig. 5(c)).

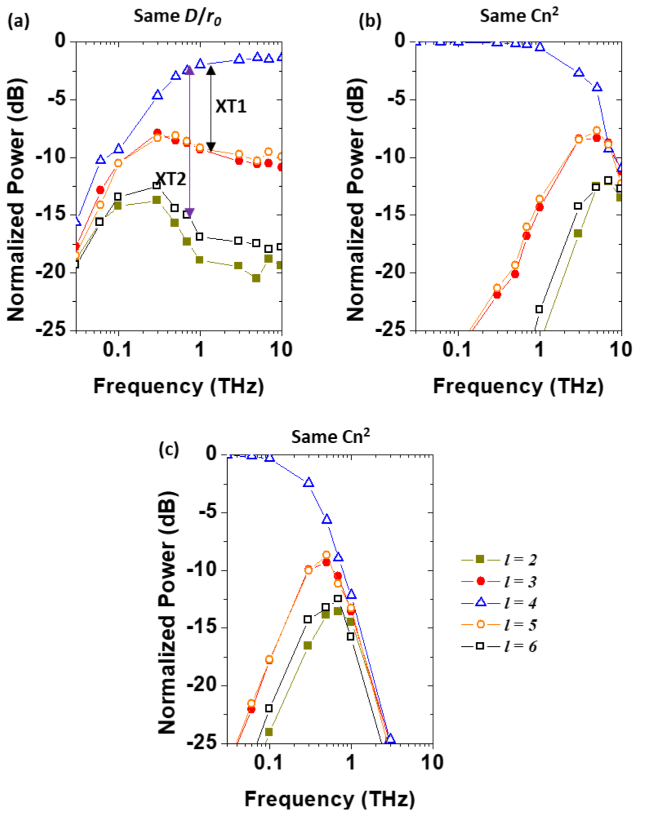

Figure 5 Effects of varying the frequency on the normalized power distribution on different OAM modes for a (a) 200-m link with $D/r_0 = 0.224$, $w_0 = 0.1$ m at the transmitter side, (b) 200-m link with $C_n^2 = 1 \times 10^{-11}$ m$^{-2/3}$, $w_0 = 0.1$ m and (c) 1-km link with $C_n^2 = 1 \times 10^{-11}$ m$^{-2/3}$, $w_0 = 1$ m at the transmitter side. OAM +4 is transmitted for all the cases. The legends in (a) to (c) are the same.

C. Turbulence distortion effect v.s. beam waist

We also study the effects of varying the beam waist on the turbulence-induced distortion of OAM spectrum. Simulation results show: (i) for a THz link where the input beam has the same value of D/r_0, the larger the beam waist, the smaller the turbulence-induced distortion effect; (ii) however, for a THz link where the atmosphere has the same value of C_n^2, when increasing the beam waist, the turbulence-induced distortion effect becomes smaller at first and then becomes larger for beam waist of > 0.3 m. Figure 6(a) can be still explained as that OAM beams with larger beam waist diverge slower, leading to a smaller distortion effect. Figure 6(b) might be explained by the comprehensive effects of two factors: (i) w_0-related divergence effect, and (ii) the value of D/r_0 linearly increases with w_0. For the case where w_0 is < 0.3 m, increasing the beam waist quickly slows down the divergence effect. Even though D/r_0 linearly increases with w_0 at the transmitter side, the comprehensive effects still decrease the value of D/r_0 as the THz beam propagates. As a result, when w_0 increases from 0.1 m to 0.3 m, the signal power increases by ~ 0.7 dB and XT1 decreases by ~2.2 dB. For the case of $w_0 > 0.3$ m, the distortion effect is mainly dependent on the linear relation between D/r_0 and w_0, such that a larger w_0 leads to a larger distortion effect.

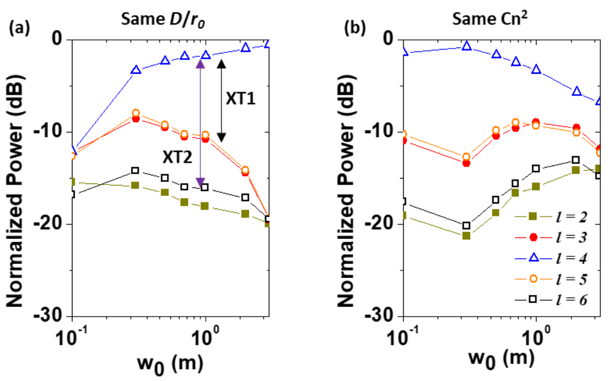

Figure 6 Effects of varying the transmitted beam waist on the normalized power distribution on different OAM modes for (a) a 500-m link with $D/r_0 = 0.224$ at the transmitter side, and (b) a 500-m link with $C_n^2 = 1 \times 10^{-11}$ m$^{-2/3}$. OAM +4 at $f = 0.5$ THz is transmitted. The legends in (a) and (b) are the same.

IV. System Performance under Different Turbulence Conditions

THz OAM-multiplexing links of different distances might have different system performance under different turbulence conditions. In this section, we investigate the crosstalk and SIR performance of single-OAM and multiple-OAM THz links under different turbulence conditions.

A. Turbulence distortion effect v.s. the atmospheric structure constant

We first explore the effect of varying the atmospheric structure constant on the system performance. Figure 7 shows that when transmitting OAM +4 with a beam waist $w_0 = 1$ m through a 1-km link, the 0.1 THz OAM beam is distorted only a little bit (XT1 < -15 dB) under strong turbulence condition ($C_n^2 = 1 \times 10^{-11}$ m$^{-2/3}$), while the 10 THz OAM beam experience a large distortion effect even under weak turbulence condition ($C_n^2 = 1 \times 10^{-13}$ m$^{-2/3}$). Moreover, the distorted OAM spectra in

Fig. 7(a) and 7(c) resemble those in Fig. 7(b) and (d), respectively. The reason might be that they have the same value of D/r_0 at the transmitter side, and such value does not change to much due to the small divergence effect during free-space propagation.

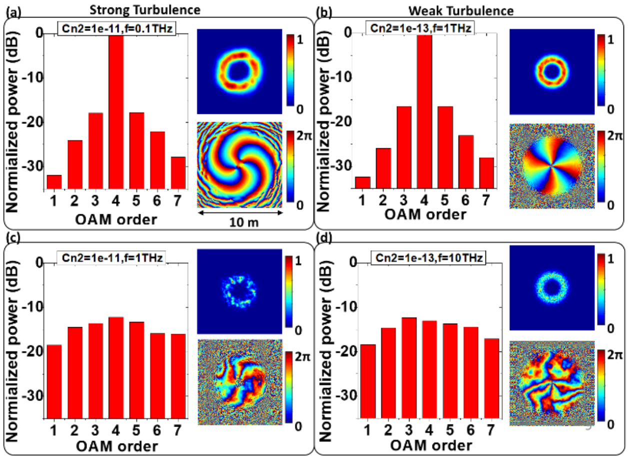

Figure 7 Normalized power distribution on different OAM modes when transmitting OAM +4 with the same beam waist w_0 = 1 m through a 1-km link. Parameters are set as (a) C_n^2 = 1 × 10^{-11} $m^{-2/3}$, f = 0.1 THz, (b) C_n^2 = 1 × 10^{-13} $m^{-2/3}$, f = 1 THz, (c) C_n^2 = 1 × 10^{-11} $m^{-2/3}$, f = 1 THz, (d) C_n^2 = 1 × 10^{-13} $m^{-2/3}$, f = 10 THz.

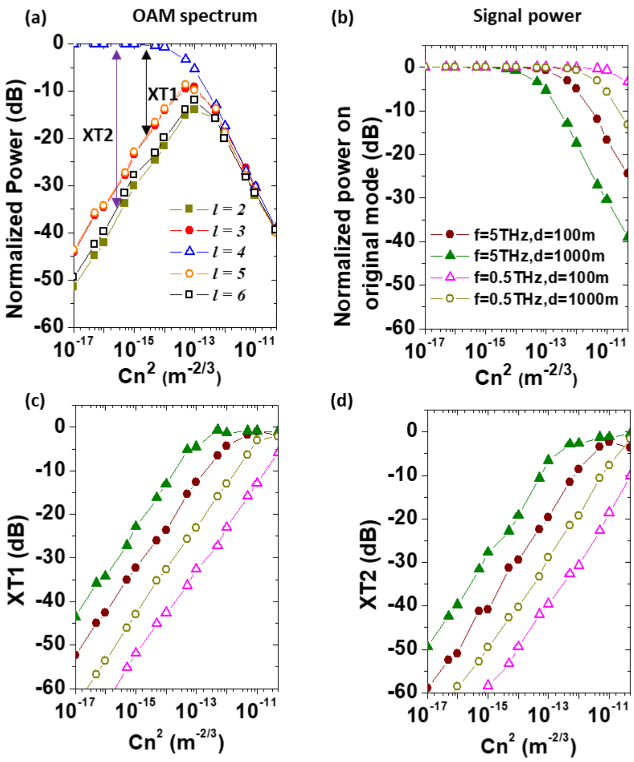

Figure 8 (a) Effects of varying C_n^2 on the normalized power distribution on different OAM modes for a 1-km link with w_0 = 1 m and f = 5 THz, and effects of varying C_n^2 on (b) the signal power, (c) XT1, and (d) XT2 for links with w_0 = 1 m and other parameters labeled in (b). XT1: crosstalk to the neighboring mode; XT2: crosstalk to the 2nd-neighboring mode. OAM +4 is transmitted. The legends in (b) to (d) are the same.

Figure 8 shows the signal power and crosstalk performance of THz-OAM links with different distances and carrier frequencies. Figure 8(a) show the case of a 1-km link where an OAM =4 with w_0 = 1 m and f = 5 THz is transmitted. The signal power on the transmitted OAM mode decreases by ~ 40 dB and the both XT1 and XT2 increases by ~ 40 dB, when C_n^2 increases from 5 × 10^{-18} to 5 × 10^{-11} $m^{-2/3}$. This could be because that the value of r_0 decreases when increasing C_n^2, which results in a larger value of D/r_0 such that the distortion effect becomes stronger. Moreover, the results in Figs. 8(b) to 8(d) show that for the cases when propagating an OAM +4 with w_0 = 1 m through: (i) a 1-km link at f = 5 THz, (ii) a 100-m link at f = 5 THz, (iii) a 1-km link at f = 0.5 THz, and (iv) a 100-m link at f = 0.5 THz, XT1 can keep < -15 dB when C_n^2 < 5 × 10^{-14} $m^{-2/3}$, 5 × 10^{-13} $m^{-2/3}$, 5 × 10^{-15} $m^{-2/3}$, and 5 × 10^{-12} $m^{-2/3}$, respectively.

B. Turbulence distortion effect v.s. link distance

Changing the link distance can also vary the turbulence distortion effect on the system performance of THz OAM links. We also simulate various sets of THz OAM links with different link distances. Figure 9(a) shows the effects of varying the link distance on the normalized power distribution on different OAM modes for links with C_n^2 = 1 × 10^{-11} $m^{-2/3}$. OAM +4 is transmitted with w_0 = 1 m and f = 0.5 THz at the transmitter side. When increasing the link distance from 100m to 1 km: (i) the signal power decreases by ~ 5 dB; and (ii) XT1 and XT2 increases by ~ 9 dB and ~ 15 dB, respectively. Figures 9(b) to 9(d) show that the system performance of 5 other sets of links has similar dependence on the link distance, as that shown in Fig. 9(a). In addition, the curves for links with (i) C_n^2 = 1 × 10^{-11} $m^{-2/3}$, f = 0.5 THz, and (ii) C_n^2 = 1 × 10^{-13} $m^{-2/3}$, f = 5 THz resemble to each other. It could be also explained by that these pairs of links have the same value of D/r_0 at the transmitter side, and such value does not change to much due to the small divergence effect during free-space propagation.

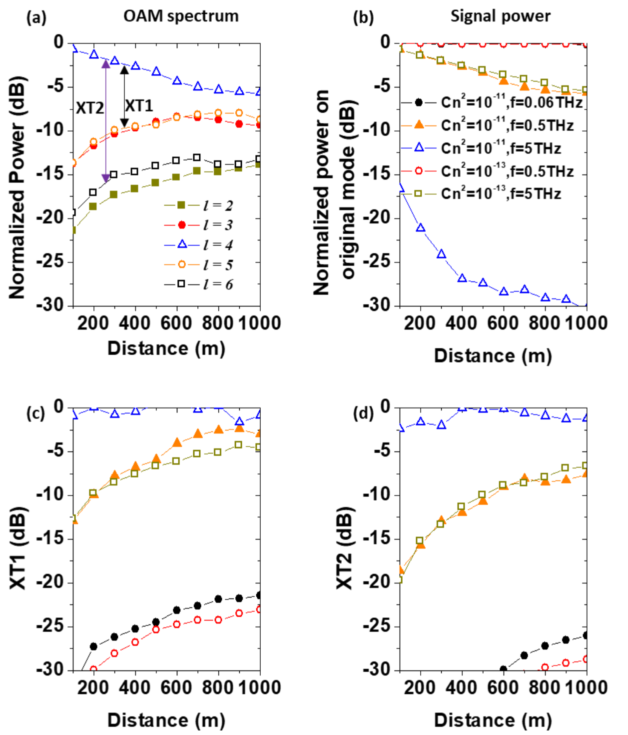

Figure 9 (a) Effects of carrying the link distance on the normalized power distribution on different OAM modes for links with C_n^2 = 1 × 10^{-11} $m^{-2/3}$, w_0 = 1 m and f = 0.5 THz, and effects of link distance on (b) the signal power, (c) XT1, and (d) XT2 for links with w_0 = 1 m and other parameters labeled in (b). OAM +4 is transmitted. The legends in (b) to (d) are the same.

C. Turbulence distortion effect for multiplexed OAM beams

We also simulate THz links, which multiplex various sets of multiple OAM beams, to analyze the SIR performance. Here, the SIR is defined as the ratio of the signal power to the crosstalk from the other channels, where the signal power is the received power on transmitted mode, and the noise is the total power leakage from the other channels to the transmitted mode. Figure 10 shows the cases when (i) multiplexing 3 OAM beams with mode spacing from 1 to 3 (OAM beams [+3,+4,+5]; [+2,+4,+6]; and [+1,+4,+7]); (ii) multiplexing 3 to 7 OAM modes with mode spacing of 1 (OAM beams [+3,+4,+5]; [+2,+3,+4,+5,+6]; and [+1,+2,+3,+4,+5,+6,+7]). For the cases of 3-muxed OAM beams, the SIR decreases by ~ 5-7 dB if the mode spacing increase by 1. This might be because that channel crosstalk between adjacent OAM modes is higher than that with mode spacing of 2 and 3. Moreover, for the cases of 3-to-7-muxed OAM beams with mode spacing of 1, the SIR also decreases with the mode number. For example, SIR decreases by ~ 0.5-1 dB if the mode number increase by 1. This small value change might be explained by that the channel crosstalk from OAM modes with mode spacing of >1 is small (such as < -15 dB) when the turbulence distortion effect is not strong.

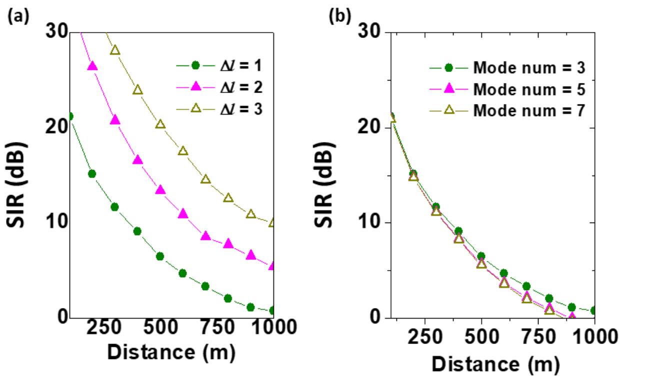

Figure 10 Signal-to-Interference ratio performance for systems with (a) 3-muxed OAM modes with mode spacing from 1 to 3 (OAM beams of [+3,+4,+5]; [+2,+4,+6]; and [+1,+4,+7] are transmitted, respectively), and (b) 3-to-7 -muxed OAM modes with a mode spacing of 1 (OAM beams of [+3,+4,+5]; [+2,+3,+4,+5,+6]; and [+1,+2,+3,+4,+5,+6,+7] are transmitted, respectively). Parameters are set as C_n^2 = 1 × 10^{-11} $m^{-2/3}$, w_0 = 0.1 m and f = 0.5 THz.

V. PERFORMANCE FOR THE SYSTEMS WITH LIMITED APERTURE SIZE OR DISPLACEMENT

In this section, we analyze the crosstalk performance of various sets of links with: (i) limited aperture size or (ii) displacement between the transmitter (Tx) and receiver (Rx).

A. Signal power and crosstalk analysis for the system with limited aperture size

We first explore the effect of limited receiver aperture size on the crosstalk performance. Figures 11(a) to 11(c) show that the normalized power distributions on different OAM modes depend on the receiver aperture size. We consider the cases for 100-m links with C_n^2 = 0 (no turbulence), 1 × 10^{-13}, and 1 × 10^{-11} $m^{-2/3}$. Other parameters are set as w_0 = 1 m, and f = 5 THz, and the OAM +4 is transmitted. In Fig. 11(a), as the aperture size decreases, the signal power decreases under no turbulence and the crosstalk to the other OAM modes are all < -100 dB in simulation. Moreover, as shown in Figs. 11(b) and 11(c), the aperture size changes the crosstalk performance of those links under turbulence, which shows dependence on the mode order. Specifically, the XT to the left neighbouring (lower) OAM modes decreases with the RX size under turbulence, however the XT to the right neighbouring (higher) OAM modes increases with the RX size under turbulence. As an example, the XT from OAM +3 increases by ~ 4 dB while the XT from OAM +5 decreases by ~ 6 dB, when the Rx size decrease from 2 m to 1 m, as shown in Fig. 11(c). This might be because that lower OAM modes diverge slower such that these modes have smaller beam sizes at the receiver side. As a result, a limited Rx could receive more power on lower OAM modes. By comparing the cases in Figs. 11(b) and 11(c), we observe that the signal power under C_n^2 = 1 × 10^{-11} $m^{-2/3}$ is smaller than that under C_n^2 = 1 × 10^{-13} $m^{-2/3}$, which might be due to a larger value of D/r_0 at the transmitter side.

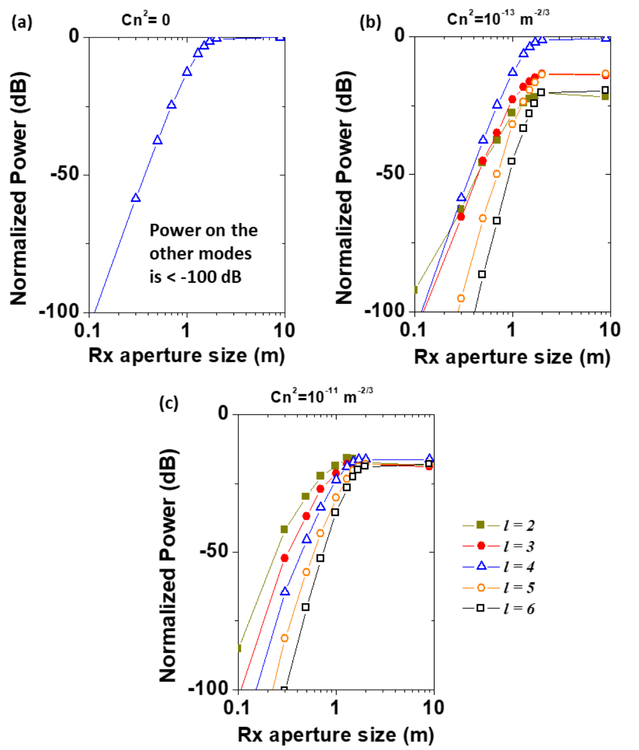

Figure 11 Effects of varying the receiver aperture size on the normalized power distribution on different OAM modes for 100-m links with (a) C_n^2 = 0 $m^{-2/3}$, (b) C_n^2 = 1 × 10^{-13} $m^{-2/3}$, and (c) C_n^2 = 1 × 10^{-11} $m^{-2/3}$. Other parameters are set as w_0 = 1 m, f = 5 THz, and OAM +4 is transmitted. The legends in (a) to (c) are the same.

B. Signal power and crosstalk analysis for the system with displacement

Besides the limited aperture size, the displacement might also occur at the receiver side. This could be considered as an additional lateral shift to the phasefront of the received beam, which might result in channel crosstalk to the system.

We explore the effect of varying the displacement between Tx and Rx on the crosstalk performance. Figures 12(a) to 12(c) show that the normalized power distribution on different OAM modes for 100-m links with C_n^2 = 0, 1 × 10^{-13}, and 1 × 10^{-11} $m^{-2/3}$, respectively. OAM =4 with w_0 = 1 m, and f = 5 THz is transmitted. We assume the Rx aperture size is larger than the received beam. For the cases with no turbulence and weak turbulence (C_n^2 = 1 × 10^{-13} $m^{-2/3}$), a larger displacement leads to lower signal power and higher XT. However, for the cases with strong turbulence (C_n^2 = 1 × 10^{-11} $m^{-2/3}$), the signal power and XT fluctuate in a small range of < 5 dB when the displacement increases from 0 to 0.5 m. One of the reasons for this might be that the power of the received beam is almost averagely leaked to many neighbouring modes under strong turbulence. Even if there is displacement between Tx and Rx, the power leakage to the other modes does not change to much.

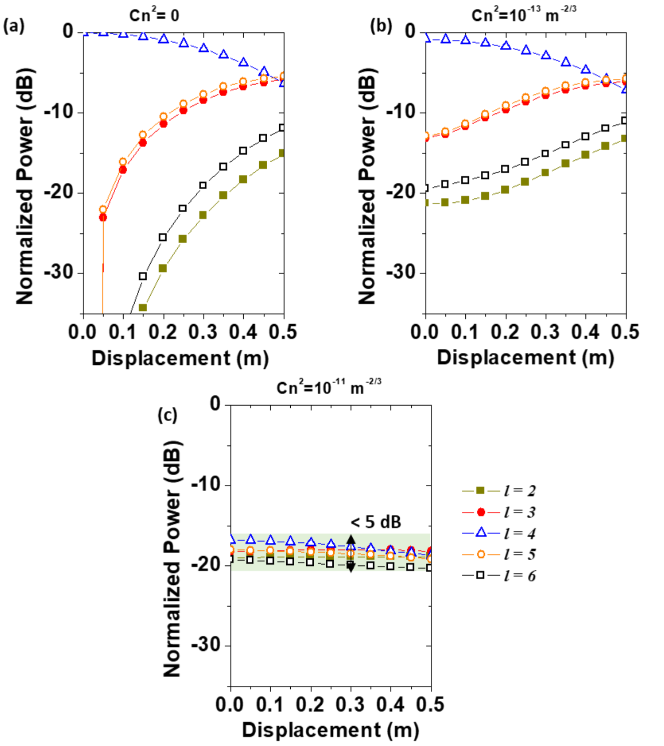

Figure 12 Effects of varying the displacement on the normalized power distribution on different OAM modes for 100-m links with (a) C_n^2 = 0 m$^{-2/3}$, (b) C_n^2 = 1 × 10^{-13} m$^{-2/3}$, and (c) C_n^2 = 1 × 10^{-11} m$^{-2/3}$. Other parameters are set as w_0 = 1 m, f = 5 THz, and OAM +4 is transmitted.

Acknowledgment

This work is supported by National Science Foundation (NSF) (ECCS-1509965); Vannevar Bush Faculty Fellowship sponsored by the Basic Research Office of the Assistant Secretary of Defense (ASD) for Research and Engineering (R&E) and funded by the Office of Naval Research (ONR) (N00014-16-1-2813); and Defense Security Cooperation Agency (DSCA 4440646262).

References

[1] J. Wang, J.-Y. Yang, I. M. Fazal, N. Ahmed, Y. Yan, H. Huang, et. al., “Terabit free-space data transmission employing orbital angular momentum multiplexing,” *Nature Photonics*, vol. 6, pp. 488-496, 2012.

[2] I. Djordjevic, “Deep-space and near-Earth optical communications by coded orbital angular momentum (OAM) modulation,” *Optical Express*, vol. 19, pp. 14277-14289, 2011.

[3] G. Gibson, J. Courtial, and M. Padgett, et. al, “Free-space information transfer using light beams carrying orbital angular momentum,” *Optics Express*, vol. 12, no. 22, pp. 5448-5456, 2004.

[4] Y. Yan, G. Xie, M. Lavery, et al., “High-capacity millimetre-wave communications with orbital angular momentum multiplexing,” *Nature Communications*, vol. 5, 2014

[5] F. Mahmouli, D. Walker, “4-Gbps uncompressed video transmission over a 60-GHz orbital angular momentum wireless channel.” *IEEE Wireless Communications Letters*, vol. 2, pp. 223–226, 2013.

[6] Z. Zhao, Y. Yan, L. Li, G. Xie, Y. Ren, et al., “A dual-channel 60 GHz communications link using patch antenna arrays to generate data-carrying orbital-angular-momentum beams,” *in Communications (ICC), 2016 IEEE International Conference* on, pp. 1-6, 2016.

[7] A.M. Yao and M.J. Padgett, “Orbital angular momentum: origins, behavior and applications,” *Advances in Optics and Photonics*, vol. 3, pp. 161-204, 2011.

[8] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, “Orbital angular momentum of light and the transformation of Laguerre- Gaussian laser modes,” *Physical Review A*, vol. 45, no. 11, pp. 8185- 8189, 1992.

[9] G. Xie, L. Li, Y. Ren, et. al., “Performance metrics and design considerations for a free-space optical orbital-angular-momentum–multiplexed communication link,” *Optica*, vol. 2, pp. 357-365, 2015.

[10] S. Fu and C. Gao, “Influences of atmospheric turbulence effects on the orbital angular momentum spectra of vortex beams,” *Photonic Research*, vol. 4, pp. B1-B4, 2016.

[11] Y. Ren, H. Huang, G. Xie, et. al., “Atmospheric turbulence effects on the performance of a free space optical link employing orbital angular momentum multiplexing,” *Optics Letters*, vol. 38, pp. 4062-4065, 2013.

[12] G. A. Tyler and R. W. Boyd, “Influence of atmospheric turbulence on the propagation of quantum states of light carrying orbital angular momentum,” *Optics Letters*, vol. 34, pp. 142-144, 2009.

[13] J. Karjalainen, M. Nekovee, H. Benn, W. Kim, J. Park, and H. Sungsoo, “Challenges and opportunities of mm-wave communication in 5G networks,” in *2014 9th International Conference on Cognitive Radio Oriented Wireless Networks and Communications* on, pp. 372-376, 2014.

[14] T. L. Frey, “The effects of the atmosphere and weather on the performance of a mm-wave communication link,” *Applied Microwave and Wireless*, vol. 11, pp. 76-81, 1999.

[15] N. Khalid and O. B. Akan, “Experimental throughput analysis of low-THz MIMO communication channel in 5G wireless networks,” *IEEE Wireless Communications Letters*, vol. 5, no. 6, pp. 616-619, 2016.

[16] D. M. Slocum, E. J. Slingerland, R. H. Giles, and T. M. Goyette, “Atmospheric absorption of terahertz radiation and water vapor continuum effects,” *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 127, pp. 49-63, 2013.

[17] L. Cang, H. K. Zhao, G. X. Zheng, “The Impact of Atmospheric Turbulence on Terahertz Communication,” *IEEE Access*, vol. 7, pp. 88685-88692, 2019.

[18] M. Cheng, L. Guo, J. Li, S. Liu and M. Cheng, “Effects of atmospheric turbulence on mode purity of orbital angular momentum millimeter waves,” in *2017 IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting*, pp. 1845-1846, 2017.

[19] J. Ma, R. Shrestha, L. Moeller and D. M. Mittleman, “Invited Article: Channel performance for indoor and outdoor terahertz wireless links,” *APL Photonics*, vol. 3, *(5)*, pp. 051601, 2018.

[20] G. Kim, T. Jeon and D. Grischkowsky, “910-m propagation of THz ps pulses through the Atmosphere,” *Optics Express*, vol. 25, pp. 25422-25434, 2017.